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Passivity of Switched Hopfield Neural Networks without Online Learning

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Abstract: As a continuation of our previous published results, in this study, we propose some results on passivity of switched Hopfield neural networks without online learning. First, a new matrix norm based condition for passivity of switched Hopfield neural networks is proposed. Second, a new passivity condition in the form of Linear Matrix Inequality (LMI) for switched Hopfield neural networks is proposed. In contrast to the existing result, the proposed conditions ensure asymptotic stability, but also passivity from the external input vector to the output vector without online learning.

Key words: Linear Matrix Inequality (LMI), passivity, switched hopfield neural networks

INTRODUCTION

Since Hopfield neural networks were introduced by Hopfield (Hopfield, 1984), they have been widely studied in theory and applications, including continuous-time and discrete-time settings. Meanwhile, they have been successfully applied to signal processing, pattern recognition, associative memories, and optimization problems, and so on (Gupta *et al*., 2003). Recently, by integrating the theory of switched systems (Lee *et al*., 2000; Daafouz *et al*., 2002) with Hopfield neural networks, switched Hopfield neural networks were introduced to represent several complex nonlinear systems efficiently (Huang *et al*., 2005; Yuan *et al*., 2006; Li and Cao, 2007; Lou and Cui, 2007). Some stability problems for these switched Hopfield neural networks were studied in (Huang *et al*., 2005; Yuan *et al*., 2006; Li and Cao, 2007; Lou and Cui, 2007).

The passivity theory (Willems, 1972; Byrnes *et al*., 1991) is a nice tool to deal with the stability of several nonlinear systems. The passivity framework is an appealing approach to the stability analysis of neural networks because we can obtain general conclusions on stability using only input-output characteristics. Recently, Ahn (Ahn, 2010) proposed a passivity condition for switched Hopfield neural networks. However, this work requires an online learning law. Unfortunately, it is impossible to guarantee the passivity of switched Hopfield neural networks without the online learning law. This paper provides an answer to the question of whether a passivity condition for switched Hopfield neural networks can be obtained without online learning. To the best of our knowledge, the passivity analysis of switched Hopfield neural networks without online learning has not been reported in the literature so far.

The objective of this study is to propose new passivity conditions for switched Hopfield neural networks without online learning. In contrast to the existing passivity condition (Ahn, 2010) for switched Hopfield neural networks, the conditions proposed in this paper do not require the online learning law.

NEW PASSIVITY CONDITIONS

Consider the following model of switched Hopfield neural networks (Huang *et al*., 2005):

$$\dot{x}(t) = A_\alpha x(t) + W_\alpha \phi\big(x(t)\big) + J(t), \qquad (1)$$

where, $x(t) = \left[x_1(t) \ldots x_n(t)\right]^T \in R^n$ is the state vector, $A = diag\{-a_1, \ldots, -a\} \in R^{n\times n}$ $(a_k > o, k = 1, \ldots, n)$ is the self-feedback matrix, $W \in R^{n\times n}$ is the connection weight matrix, $\phi\big(x(t)\big) = \left[\phi_1\big(x(t)\big) \ldots \phi_n\big(x(t)\big)\right]^T : R^n \to R^n$ is the nonlinear function vector satisfying the global Lipschitz condition with Lipschitz constant $L\phi > 0$, and $J(t) \in R$ is an external input vector, α is a switching signal which takes its values in the finite set I = {1, 2, ..., N}. The matrices ($A\alpha$, $W\alpha$) are allowed to take values in the finite set $\{(A_1, W_1), \ldots, (A_N, W_N)\}$ at an arbitrary time. In this study, we assume that the switching rule α is not known a priori and its instantaneous value is available in real time. Define the indicator function.

$\xi(t) = \left(\xi_1(t), \xi_2(t), \ldots, \xi_N(t)\right)^T$, where

$$\xi_i(t) = \begin{cases} 1, \text{when the switched system is described} \\ \quad \text{by the } i-th \text{ mode}(A_i, W_i) \\ o, \quad \text{otherwise} \end{cases}$$

with i = 1, ..., N. Therefore, the model of the switched Hopfield neural networks (1) can also be written as:

$$\dot{x}(t) = \sum_{i=1}^{N} \xi_i(t)\left[A_i x(t) + W_i\phi\left(x(t)\right) + J(t)\right] \qquad (2)$$

where, the relation $\sum_{i=1}^{N}\xi_i(t) = 1$ is satisfied under any switching rules.

In this study, we find conditions such that the switched Hopfield neural network (2) satisfies:

$$\int_0^t J^T(\tau)y(\tau)d\tau + \beta \geq \int_0^t \Phi\left(x(\tau)\right)d\tau, \quad \forall\, t \geq 0 \qquad (3)$$

where β is a nonnegative constant, $y(t) \in R^n$ is the output vector of the neural network (2), and $\Phi\left(x(t)\right)$ is a positive semi-definite storage function.

In the following theorem, a new passivity condition for the switched Hopfield neural network (2) is proposed without online learning.

Theorem 1: If the following condition is satisfied:

$$\|W_i\| < \sqrt{\frac{k_i - \eta_i - \|P\|}{L^2{}_\phi \|P\|}} \qquad (4)$$

$$\|P\| < k_i - \eta_i, \;\; k_i > \eta_i > 0, \;\; P = P^T > 0 \qquad (5)$$

where, P satisfies the Lyapunov inequality $A_i{}^T P + PA_i < -k_i I$ for i = 1, ..., N, then the switched Hopfield neural network (2) is passive from the external input vector $J(t)$ to the output vector $y(t)$ which is defined as $y(t) \overset{\Delta}{=} 2Px(t)$.

Proof: Consider the function V(t) = $x^T(t)$ Px(t). The time derivative of $V(t)$ satisfies:

$$\dot{V}(t) < \sum_{i=1}^{N}\xi_i(t)\left\{-k_i x^T(t)x(t) + 2x^T(t) \times PW_i\phi(x(t)) + 2x^T(t)PJ(t)\right\} \qquad (6)$$

By Young's inequality (Arnold, 1989), we have:

$$2x^T(t)PW_i\phi\left(x(t)\right) \leq x^T(t)Px(t) + \left(PW_i\phi\left(x(t)\right)\right)^T P^{-1}\left(PW_i\phi\left(x(t)\right)\right)$$

$$\leq \|P\|\,\|x(t)\|^2 + \|P\|\|W_i\|^2\|\phi(x(t))\|^2$$

$$\leq \|P\|\,\|x(t)\|^2 + L_\phi^2\|P\|\,\|W_i\|^2\|x(t)\|^2 \qquad (7)$$

By using (7), we obtain:

$$\dot{V}(t) < \sum_{i=1}^{N}\xi_i(t)\left\{-\left(k_i\|P\| - L_\phi^2\|P\|\|W_i\|^2\right) \times \|x(t)\|^2 + y^T(t)J(t)\right\}$$

$$= -\sum_{i=1}^{N}\xi_i(t)\left(k_i - \eta_i - \|P\| - L_\phi^2\|P\|\|W_i\|^2\right)\|x(t)\|^2$$

$$+ \sum_{i=1}^{N}\xi_i(t)\left\{-\eta_i\|x(t)\|^2 + y^T(t)J(t)\right\} \qquad (8)$$

If the following condition is satisfied:

$$k_i - \eta_i - \|P\| - L_\phi^2\|P\|\|W_i\|^2 > 0 \qquad (9)$$

for i = 1,..., N, we have:

$$\dot{V}(t) < -\sum_{i=1}^{N}\xi_i(t)\eta_i\|x(t)\|^2 + y^T(t)J(t) \qquad (10)$$

Integrating both sides of (10) from 0 to t gives:

$$V(t) - V(0) < -\int_0^t\sum_{i=1}^{N}\xi_i(\tau)\eta_i\|x(\tau)\|^2 d\tau + \int_o^t y^T(\tau)J(\tau)d\tau. \qquad (11)$$

Let β = V(0). Since $V(t) \geq 0$,

$$\int_0^t y^T(\tau)J(\tau)d\tau+\beta$$

$$>\int_0^t \sum_{i=1}^N \xi_i(\tau)\eta_i\|x(\tau)\|^2 d\tau+V(t)$$

$$\geq \int_0^t \sum_{i=1}^N \xi_i(\tau)\,\eta_i\,\|x(\tau)\|^2\,d\,\tau \tag{12}$$

Let $\Phi\left(x(\tau)\right)=\sum_{i=1}^N \xi_i(\tau)\eta_i\|x(\tau)\|^2\geq 0$. The relation (12) satisfies the passivity definition (3).

Therefore, the switched Hopfield neural network (2) is passive from the external input vector $J(t)$ to the output vector $y(t)$ under the condition (9), which is rewritten as

$$\|W_i\|^2<\frac{k_i-\eta_i-\|P\|}{L_\phi^2\|P\|},$$

$$\|P\|< k_i-\eta_i,\quad k_i>\eta_i$$

This completes the proof.

Corollary 1: (Zero-input State Response) When $J(t)=0$, the condition (4)-(5) ensures that the switched Hopfield neural network (2) is asymptotically stable.

Proof: When $J(t)=0$, from (10), we have:

$$\dot V(t)<-\sum_{i=1}^N \xi_i(t)\,\eta_i\,\|x(t)\|^2$$

$$<0,\qquad \forall\, x(t)\neq 0 \tag{13}$$

This relation ensures that the switched Hopfield neural network (2) is asymptotically stable from Lyapunov stability theory. This completes the proof.

If the switched Hopfield neural network (2) is passive, the external input vector $J(t)=-\gamma(y(t))$ satisfying $\gamma(0)=0$ and $y^T(t)\mu(y(t))>0$ for each nonzero $y(t)$ asymptotically stabilizes the switched Hopfield neural network (2). For example, a pure gain output feedback $J(t)=-\mu y(t)$ $(\mu>0)$ can stabilize the switched Hopfield neural network (2).

Corollary 2: (Nonzero-input State Response) If the external input vector J(t) is selected as:

$$J(t)=-\mu y(t)=-2\mu Px(t),\qquad \mu>0 \tag{14}$$

the switched Hopfield neural network (2) is asymptotically stable.

Proof: For $J(t)=-\mu y(t)$, the time derivative of $V(t)$ satisfies:

$$\dot V(t)<-\sum_{i=1}^N \xi_i(t)\eta_i\|x(t)\|^2-\mu y^T(t)y(t)<0,\quad \forall\, x(t)\neq 0 \tag{15}$$

from (10). This guarantees asymptotic stability from Lyapunov stability theory. This completes the proof.

In the next theorem, a new LMI based passivity condition for the switched Hopfield neural network (2) without online learning is proposed. The condition in the form of LMI can be facilitated readily via standard numerical algorithms (Boyd *et al*., 1994; Gahinet *et al*., 1995). Hence, this condition is computationally attractive.

Theorem 2: If there exist positive symmetric matrices P, S, and a positive scalar ε such that:

$$\begin{bmatrix} A_i^T P+PA_i+\varepsilon L_\varphi^2 I+S & PW_i \\ W_i^T P & -\varepsilon I \end{bmatrix}<0 \tag{16}$$

for I = 1, ..., N, then the switched Hopfield neural network (2) is passive from the external input vector J(t) to the output vector y(t) which is defined as y(t) = 2 Px(t).

Proof: Consider the function $V(t)=x^T(t)Px(t)$. By Young's inequality (Arnold, 1989), for any positive scalar ε, the following relation is satisfied:

$$\varepsilon\left[L_\phi^2 x^T(t)x(t)-\phi^T(x(t))\phi(x(t))\right]\geq 0 \tag{17}$$

By using (17), the time derivative of $V(t)$ is :

$$\dot V(t)\leq\sum_{i=1}^N \xi_i(t)\Big\{x^T(t)\left[A_i^T P+PA_i\right]x(t)$$

$$+2x^T(t)PW_i\phi(x(t))+2x^T(t)PJ(t)$$

$$+\varepsilon\left[L_\phi^2 x^T(t)x(t)-\phi^T(x(t))\phi(x(t))\right]\Big\}$$

$$=\sum_{i=1}^N \xi_i(t)\begin{bmatrix} x(t) \\ \phi(x(t))\end{bmatrix}^T\begin{bmatrix} A_i^T P+PA_i+\varepsilon L_\phi^2 I+S & PW_i \\ W_i^T P & -\varepsilon I\end{bmatrix}\begin{bmatrix} x(t) \\ \phi(x(t))\end{bmatrix}$$

$$+\sum_{i=1}^N \xi_i(t)\left\{-x^T(t)Sx(t)+J^T(t)y(t)\right\} \tag{18}$$

If the LMI (16) is satisfied, we have:

$$\dot{V}(t) < - x^T(t) Sx(t) + J^T(t) y(t) \tag{19}$$

Integrating both sides of (19) from 0 to t gives:

$$V(t) - V(0) < -\int_0^t x^T(\tau) Sx(\tau) d\tau + \int_0^t y^T(\tau) J(\tau) d\tau \tag{20}$$

Let $\beta = V(0)$. Since $V(t) \geq 0$

$$\int_0^t y^T(\tau) J(\tau) d\tau + \beta > \int_0^t x^T(\tau) Sx(\tau) d\tau + V(t) \geq \int_0^t x^T(\tau) Sx(\tau) d\tau \tag{21}$$

The relation (21) satisfies the passivity definition (3). This completes the proof.

Corollary 3: (Zero-input State Response) When, J(t) = 0 the LMI condition (16) ensures that the switched Hopfield neural network (2) is asymptotically stable.

Proof: When $J(t) = 0$ from (19), we have:

$$\dot{V}(t) < - x^T(t) Sx(t) < 0, \quad \forall x(t) \neq 0 \tag{22}$$

This inequality ensures that the switched Hopfield neural network (2) is asymptotically stable from Lyapunov stability theory. This completes the proof.

Corollary 4: *(*Nonzero-input State Response) If the external input vector $J(t)$ is selected as:

$$J(t) = -\mu y(t) = -2\mu Px(t), \quad \mu > 0 \tag{23}$$

then the LMI condition (16) ensures that the switched Hopfield neural network (2) is asymptotically stable

Proof: For $J(t) = -\mu y(t)$, the time derivative of $V(t)$ satisfies

$$\dot{V}(t) < -x^T(t) Sx(t) - \mu y^T(t) y(t) < 0, \forall x(t) \neq 0 \tag{24}$$

from (19). This guarantees asymptotic stability from Lyapunov stability theory. This completes the proof.

CONCLUSION

In this study, we have proposed new passivity conditions for switched Hopfield neural networks based on matrix norm and LMI. These conditions ensured asymptotic stability, but also passivity from the external input vector to the output vector. The conditions proposed in this paper did not require any online learning law.

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