Research Journal of Applied Sciences, Engineering and Technology 4(7): 717-723, 2012
ISSN: 2040-7467

Submitted: September 25, 2011 Accepted: November 04, 2011 Published: April 01, 2012

Using Matlab with Quadrilateral Finite Elements in Analysis of Multilayered Nonhomogeneous Soils under Strip Footing

Ahmed Qasim Obaid Al-Ramthan
Department of Engineering Affairs, Baghdad University, Iraq

Abstract: Natural soils are often comprised of separate layers. Foundations of engineering structures are designed to transfer and distribute their loading to the underlying soil and/or rock. Therefore, the designer must ensure that the structure does not suffer from excessive displacements. In this research, it is intended to make a generalized numerical solution through a computer program has written in MATLAB to analyze a multilayered nonhomogeneous (Gibson-type) soil, where the stiffness contrast exists between layers, by determining the displacements and stresses under strip footing during applied incremental loading sequence. This research presents a plane strain two-dimensional finite element method. In this method, the soil is divided into several 4-node quadrilateral elements. The general behavior of a multilayered soil profile and the influences of strip foundations resting on soils having homogeneous (constant modulus with depth) to Gibson-type (linearly increasing modulus) profiles are studied. The influences of foundation size and embedment, incremental loading, and soil stratifications are considered in this study.

Key words: Finite element method, MATLAB, multilayered soils, nonhomogeneous soils, strip footing

INTRODUCTION

In nature, soil deposits are often formed in discrete layers. As a result, footings are usually supported by multilayered soil profiles, which influence the value of the settlements of the foundation. If a footing is placed on the surface of a layered soil and the thickness of the top layer is large compared with the width of the footing, then the displacement behavior of the footing can be estimated, to sufficient accuracy, using only the properties of the upper layer (Poulos *et al*., 2001).

It is commonly believed that the settlement criterion is more critical than the bearing capacity one in the designs of shallow foundations, especially for foundation width greater than 1.5 m, which is often the case. By limiting the total settlements, differential settlements and any subsequent distresses to the structure are limited. Generally, the settlements of shallow foundations such as pad or strip footings are limited to 25 mm (Terzaghi *et al*., 1996).

In general, the magnitude and distribution of the displacements and stresses in soil are predicted by using solutions that model soil as a linearly elastic, homogeneous and isotropic continuum. From the standpoint of practical considerations in engineering, anisotropic soils are often modeled as orthotropic or isotropic medium. Besides, the effects of deposition, overburden, desiccation, etc., can lead geotechnical media, which exhibit both nonhomogeneity and anisotropic deformability characteristics.

Desai (1968) considered the load deformation problem of a circular footing resting on homogeneous and two layered clay systems. Employing a plane strain approach, Radhakrishnan (1969) investigated the behavior of a continuous strip footing on homogeneous and layered clays.

Rowe and Booker (1982) presented a finite layer approach for the analysis of both layered and continuously varying elastic soil. The approach rested on the assumption that the soil deposit may be regarded as consisting of a number of distinct horizontal layers.

Douglas (1986) reported the existence of more than 40 different methods for estimating settlements in granular soils. All these methods recognize that the applied pressure, soil stiffness and the foundation width are the three most important variables affecting the settlements in granular soils.

Since many soils exhibit stiffness increasing with depth because of the increase in overburden stress, the displacements and stresses will be evaluated for a Gibson type soil. A footing resting on a nonhomogeneous elastic medium with modulus increasing with depth is a more generalized problem (Stark and Booker, 1997).

The solutions of displacements and stresses for various types of applied loads to homogeneous and nonhomogeneous isotropic/anisotropic full/half-spaces have played an important role in the design of foundations. However, it is well known that a strip load solution is the basis of complex loading problems for all constituted materials. A large body of the literature was

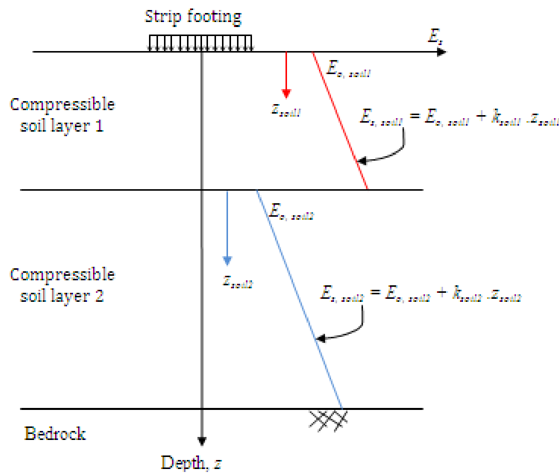


Fig. 1: Variation of soil modulus with depth

devoted to the calculation of displacements and stresses in isotropic media with the Young's or shear modulus varying with depth according to the linear law, the power law, and the exponential law, etc. (Wang *et al*., 2003).

For the purpose of analysis of natural soils, it may be convenient and reasonable to assume that the soil within each layer is homogeneous. If a footing is placed on the surface of a layered soil and the thickness of the top layer is large compared with the width of the footing, the displacement behavior of the footing can be estimated to sufficient accuracy using the properties of the upper layer only. However, if the thickness of the top layer is comparable to the footing width, this approach introduces significant inaccuracies and is no longer appropriate. This is because the zone of influence of the footing, including the potential failure zone, may extend to a significant depth, and thus two or more layers within that depth range will affect the behavior of the footing (Wang and Carter, 2002).

Estimation of foundation settlement (immediate, consolidation and creep) depends strongly on the calculated stresses in the underlying soil mass due to footing pressure. In theory, accurate stresses can be calculated for any geometry using numerical methods such as three-dimensional finite element or finite difference analysis (Hazzard *et al*., 2007).

The type of elastic nonhomogeneity is a useful approximation for modeling certain problems of geotechnical interest (Selvadurai, 1998).

Griffith and Fenton (2001) observed that the performance of foundations is considerably affected by the inherent spatial variability of the soil properties.

In practice, most foundations are flexible. Even very thick ones deflect when loaded by the superstructure loads (Bowles, 1996).

In this study, an elastic static loading problem for a continuously nonhomogeneous isotropic medium with Young's modulus varying linearly with depth is relevant.

METHODOLOGY

Generalized "Gibson-Type" profile: The elastic modulus for a generalized Gibson-type soil (Gibson, 1967) is expressed by:

$$E_s = E_o + kz \tag{1}$$

where E_s = the elastic soil modulus increasing linearly with depth; E_o = Young's modulus of elasticity of soil directly underneath the foundation base; k = linear rate of increase of elastic modulus with depth (units of E per unit depth); and z = depth.

Two layers of Gibson-type soils are considered in this study, as shown in Fig. 1.

Finite element analysis by MATLAB: The finite element method, which can handle very complex layer patterns, has been applied to this problem and reliable some research had been undertaken investigating the probabilistic analysis of the settlement of foundations supported on single-layered soil profiles for the calculated stresses (e.g., Boussinesq method).

The well-known Boussinesq equations apply strictly only to homogeneous soil deposits. The general theory of stresses and displacements in a two-layer system is developed in order to provide the engineer with a useful tool, which is more directly applicable to the analysis of actual conditions encountered in soil deposits (Burmister, 1945).

The case in the present paper is studied as a plane strain two-dimensional problem. The shape of elements used is the quadrilateral element because of its suitability to simulate the very important behavior of soils under strip footing.

The size of the finite element mesh in the horizontal direction is adjusted according to the width of the footing.

Table 1: Comparison with the theoretical results

Item considered	Smith and Griffiths (1988) Results	Author results
Ver. Disp. of Node 1	-0.8601E-05	- 8.6005657E-006
Ver. Disp. of Node 4	-0.3771E	- 05-3.7707115E-006
Hor. Disp. of Node 4	-0.5472E	- 07-5.4720226E-008
Ver. Stress at Elem. 1	0.8284	8.2847569E-001
Hor. Stress at Elem. 1	0.29075	2.9074293e-001

Table 2: The properties of soils

Parameters	Layer 1	Layer 2
E_o (kN/m^2)	12500	9000
k (kN/m^2/m)	2000	500
ν	0.4	0.3

It is generally recognized that the mesh boundaries should be set at a distance at least five times the loaded area to ensure that boundary effects do not influence the results (Desai and Abel, 1972).

In the present study, 4-noded quadrilateral elements with two degree of freedom at each node have been used to model soil. In each increment of the analyses, the stress-strain behavior of the soil is treated as being linear, and the relationship between stress and strain is assumed to be governed by the generalized Hooke's law of elastic deformations, which may be expressed as follows for conditions of plane strain case:

$$\begin{Bmatrix} \Delta\sigma_x \\ \Delta\sigma_y \\ \Delta\sigma_{xy} \end{Bmatrix} = \frac{E_s(1-v)}{(1+v)(1-2v)} \begin{bmatrix} 1 & \frac{v}{1-v} & 0 \\ \frac{v}{1-v} & 1 & 0 \\ 0 & 0 & \frac{1-2v}{2(1-v)} \end{bmatrix} \begin{Bmatrix} \Delta\varepsilon_x \\ \Delta\varepsilon_y \\ \Delta\varepsilon_{xy} \end{Bmatrix} \quad (2)$$

where $\Delta\sigma_x$, $\Delta\sigma_y$ and $\Delta\tau_{xy}$ = the increments of stress during a step of analysis; $D\varepsilon_x$, $D\varepsilon_y$ and $\Delta\gamma_{xy}$ = the corresponding increments of strain; E_s = the value of Young's modulus; and ν = the value of Poisson's ratio.

The author using MATLAB writes a computer program that used in the finite element analysis carried out during this research. The soil model that is considered in this work is nonhomogeneous, isotropic on primary loading with a different modulus. So, the behavior of the soil can be approximated by Gibson model (Gibson, 1967).

The sign convention for the stresses and the convention for numbering the nodes of elements are shown in Fig. 2. The program presents the results of analysis as the displacements of the nodal points, and the value of stresses developed at the centre of each element at the end of each solution increment.

Figure 3 is a flowchart that illustrates the main features of the solution procedure adopted in the finite element computer program used.

Verification of the computer program: The author has used this program in a different problem (Fig. 4) presented by another researcher (Smith and Griffiths, 1988).

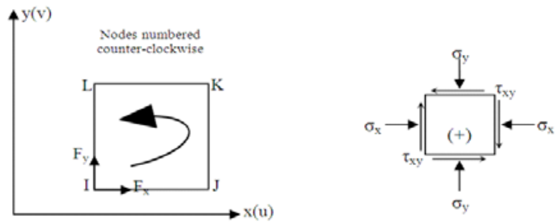


Fig. 2: Sign convention and element numbering

The results obtained by the program in this research were compared with results presented by Smith and Griffiths (1988). In all comparisons, excellent agreement was found between the present program results and those published, as shown in Table 1.

Problem geometry: The basic problem chosen for the parametric study shown in Fig. 5a, which involves soil strata of clayey soils, 30.0 m width, of a first layer (3.0 m thick) over a second layer (18.0 m thick) underlaid by bedrock and loads by strip sequence loadings (40, 45, 55, 70 kN/m^2) with base width equal to 3.0 m.

The finite element mesh (Fig. 5b) used consists of 2623 nodal points and 2520 quadrilateral two-dimensional elements. The nodal points along the bottom boundary of the mesh are assumed to be fixed both horizontally and vertically. The nodes on the right and left ends of the mesh are fixed in the horizontal direction while they are free to move in the vertical direction. All interior nodes are free to move horizontally and vertically.

The first layer properties (Küçükarslan, 2007) and the second layer properties (Das, 2006) are reported in Table 2. The behavior of soil materials is a nonhomogeneous elastic medium with modulus increasing linearly with depth.

RESULTS AND DISCUSSION

In this study, a model of two-layered system was analyzed under uniformly flexible strip loading with soil modulus increasing linearly with depth. In order to develop more knowledge about the behavior of multilayered soils under strip loading problems, a parametric study is performed by varying the basic problem parameters and comparing these results with the original basic problem results.

The results of increasing the incremental loading, soils having homogeneous (constant modulus with depth) to Gibson-type (linearly increasing modulus) profiles, foundation size (B) and embedment (D_f), and soil stratification are presented as follow:

For uniformly flexible strip loading area at surface, the vertical displacement along the surface of the model is shown in Fig. 6 and the contact settlement under the strip footing is shown in Fig. 7. The settlement at the center is much larger than the settlement at the edge of the

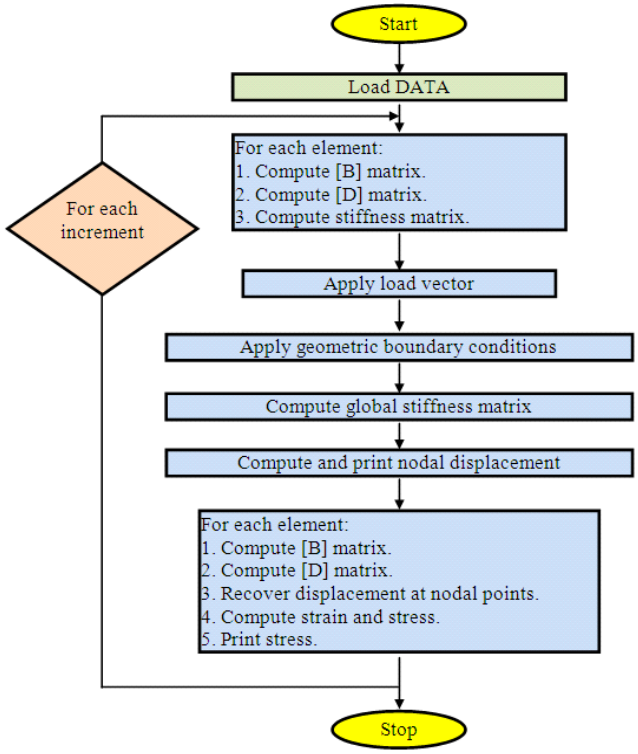


Fig. 3: Simplified flow chart of the finite element program

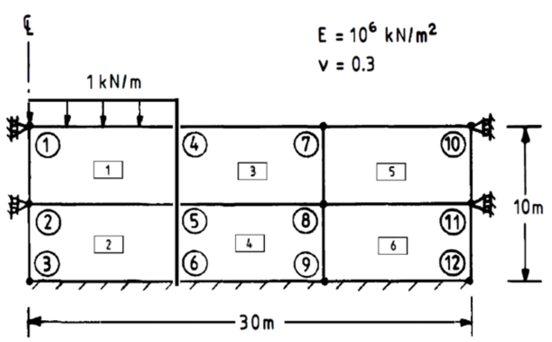
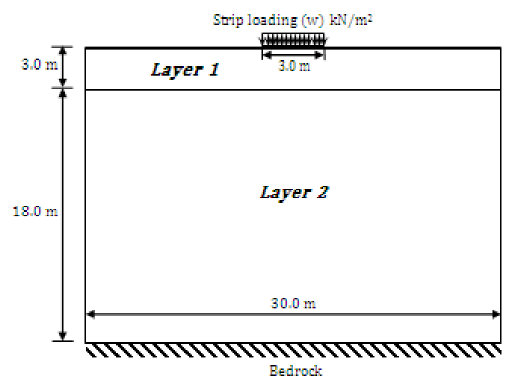


Fig. 4: Mesh and data for different problem (Smith and Griffiths, 1988)

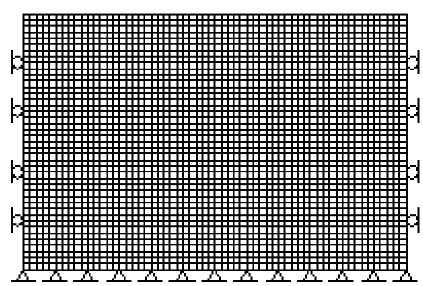
loaded area. These results agree with the results founded by Wu (1974) and Das (2006). Also, the vertical displacement increases in direct proportion to the pressure of the loaded area, as shown in Fig. 6 and 7, which agrees with that reported by Craig (1987).

The vertical stress contours throughout the soil under the strip loadings (40, 45, 55, 70 kN/m^2) with base width (B) equal to (3.0 m) are shown in Fig. 8. It can be seen that the vertical stress values along the depth of the layer decrease throughout the layer for each increment and increase throughout the loading sequence stages.

From the settlements at surface (Fig. 9), it can be seen that the settlements for soils with modulus increasing



(a)



(b)

Fig. 5: The basic problem for the parametric study

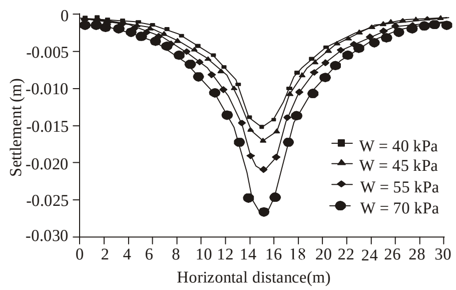


Fig. 6: Settlements along the top of model under the strip loadings (40, 45, 55, 70 kN/m^2) with base width equal to (3.0 m)

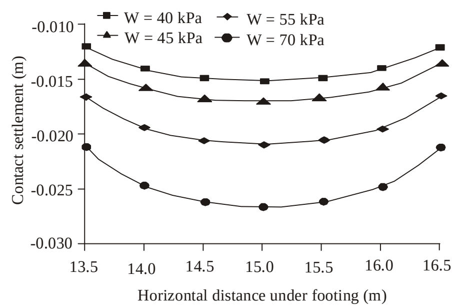


Fig. 7: Contact settlements under the strip loadings (40, 45, 55, 70 kN/m^2) with base width equal to (3.0 m)

linearly with depth are less than the settlements for soils with constant modulus ($E_{s,\ soil1} = E_{o,\ soil1}$ and $E_{s,\ soil2} = E_{o,\ soil2}$). The results agree with that mentioned by Terzaghi in Wu's book (Wu, 1974).

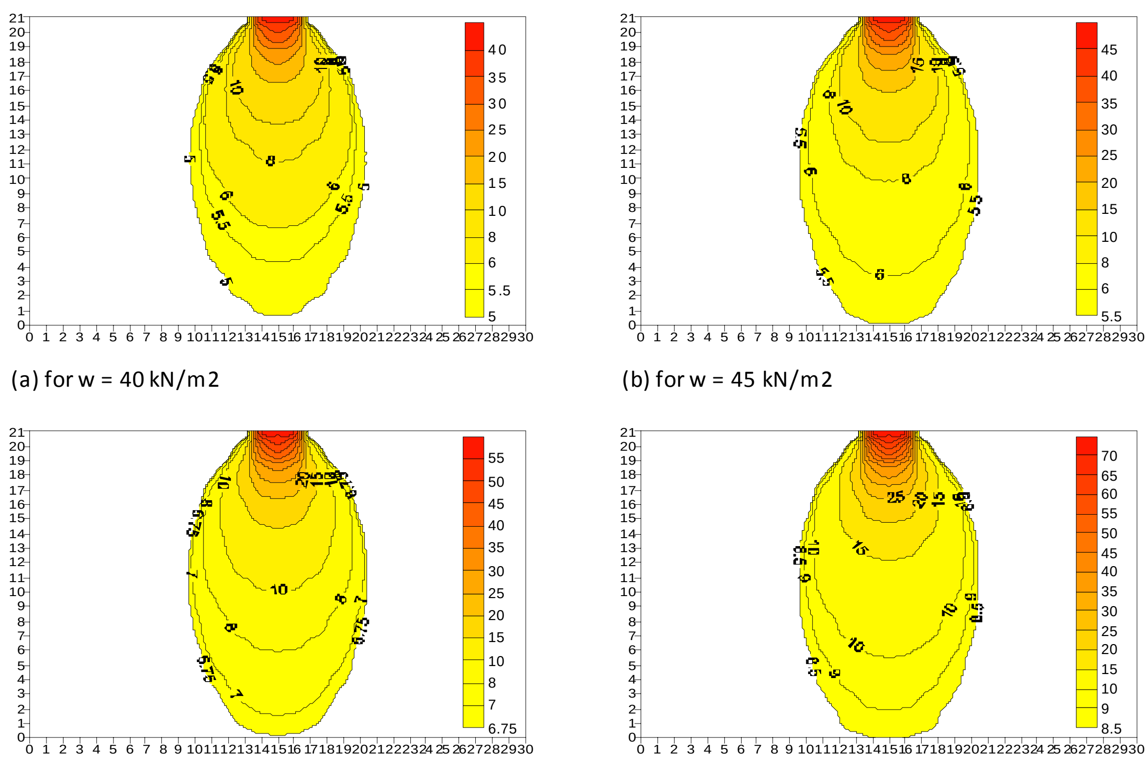
The immediate settlement at the center of the loaded area decreases when the thickness of the first (upper) layer increases, as shown in Fig. 10, where the top layer is stronger than the bottom layer. In addition, the effect of the stiff (top) layer will be to reduce the stress concentration in the lower layer, as shown in Fig. 11. These results agree with that discussed by Das (2008 and 2009).

The immediate settlement at the center of the loaded area is reduced when the strip footing is placed at some depth ($D_f \leq B$) in the ground, as shown in Fig. 12. These results agree with that mentioned by Fox in Bowels' book (Bowles, 1996).

The vertical displacement (immediate settlement) increases in direct proportion to the width of the loaded area (size of the footing) at surface, as shown in Fig. 13, which agrees with that reported by Wu (1974) and Craig (1987).

CONCLUSION

The results obtained from this study can lead that a generalized numerical solution through a computer



(a) for w = 40 kN/m2

(b) for w = 45 kN/m2

(c) for w = 55 kN/m2

(d) for w = 70 kN/m2

Fig. 8: Vertical stress contours for the soil model throughout the loading sequences

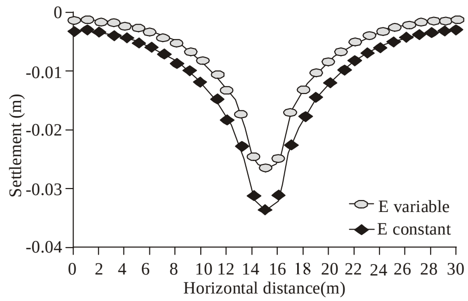


Fig. 9: Settlements along the horizontal distance at surface under the strip loading (70 kN/m^2)

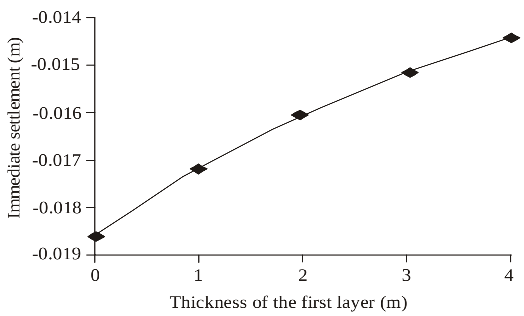


Fig. 10: Immediate settlements at the center of the strip loading (40 kN/m^2) with different thickness of the first (top) layer

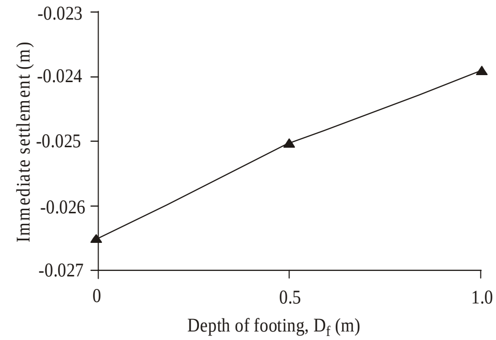


Fig. 12: Immediate settlements at the center of the strip loading (70 kN/m^2) with different depth of footing

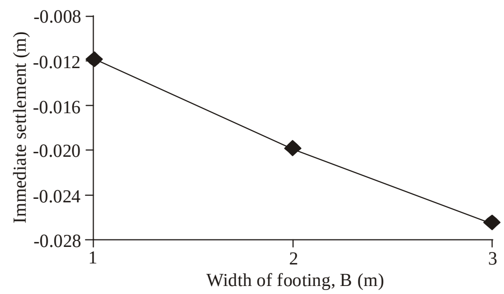
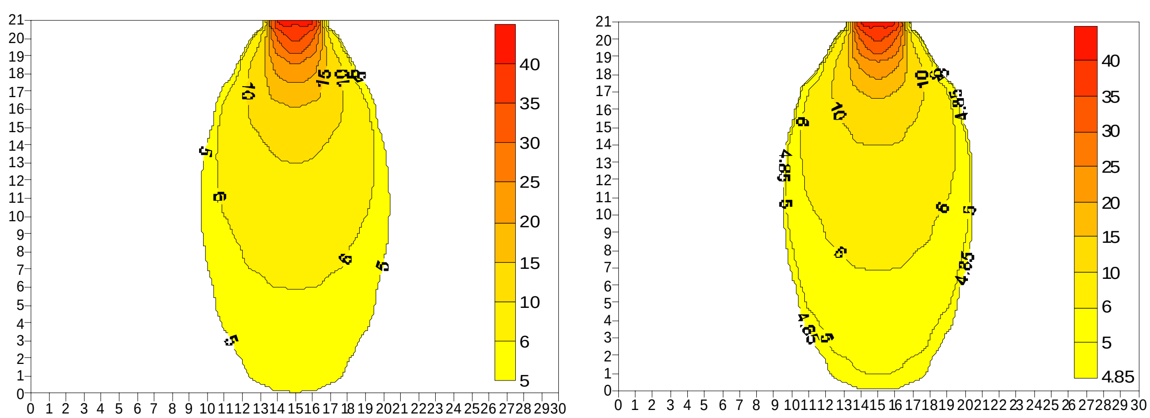


Fig. 13: Immediate settlements at the center of the strip loading (70 kN/m^2) with different width of footing



(a) thickness of first (top) layer = 0 m

(b) thickness of first (top) layer = 4 m

Fig. 11: Vertical stress contours for the soil model for the strip loading (40 kN/m^2)

program, written in MATLAB, can simulate the analysis of multilayered nonhomogeneous (Gibson-type) soils that had a soil modulus increasing linearly with depth and loaded with incremental strip loading.

Displacements and stresses can be calculated with knowledge of soil stiffness beneath the footing, rate of increase of soil stiffness with depth, soil Poisson's ratio, depth to an incompressible layer, and footing width. This study shows how the computer solutions may be used to improve the prediction of settlements and stresses beneath a strip footing resting on multilayered Gibson-type soils.

The immediate settlement at the center is much larger than the settlement at the edge of the strip loaded area. The immediate settlement increases in direct proportion to the pressure of the strip loaded area. The vertical stress values (stress bulb) under the strip loading area decrease throughout the layers for each increment and increase throughout the loading sequence stages. The vertical displacements for soils with moduli increasing linearly with depth (Gibson-type) are less than the vertical displacements for the same soils with constant moduli,

which leads to that the soils with (Gibson-type) modulus, are more approximate simulation for soil modulus. The immediate settlement decreases when the thickness of the upper (stronger) soil layer overlaying a weaker soil layer increases and the stress bulb of strip foundation reduces when the thickness of top (stronger) soil layer increases. The immediate settlement of the strip loaded area decreases when the embedment of strip footing increases and the immediate settlement increases with the increasing of foundation size. The results compare approvingly with available published analytical and numerical solutions.

REFERENCES

Bowles, J.E., 1996. Foundation Analysis and Design. 5th Edn., McGraw-Hill Companies.

Burmister, D.M., 1945. The general theory of stresses and displacements in layered systems. J. Appl. Phys., 16(2): 89-94.

Craig, R.F., 1987. Soil Mechanics. 4th Edn., Van Nostrand Reinhold Co. Ltd, UK.

Das, B.M., 2006. Principles of Geotechnical Engineering. 5th Edn., Thomson Ltd, Canada.

Das, B.M., 2008, Advanced Soil Mechanics. 3rd Edn., Taylor and Francis Group.

Das, B.M., 2009. Shallow Foundations: Bearing Capacity and Settlement. 2nd Edn., Taylor and Francis Group.

Desai, C.S., 1968. Solution of stress deformation problems in soil and rock mechanics using finite element methods. Ph.D. Thesis, The University of Texas at Austin, Texas, USA.

Desai, C.S. and J.F. Abel, 1972. Introduction to the Finite Element Method. Van Nostrand Reinhold, London.

Douglas, D.J., 1986. State-of-the-art. Ground Eng., 19(2): 2-6.

Gibson, R.E., 1967. Some results concerning displacements and stresses in a non-homogeneous elastic half-space. Geotechnique, 17: 58-67.

Griffith, D.V. and G.A. Fenton, 2001. Bearing capacity of spatially random soil: The undrained clay prandtl problem revisited. Geotechniqe, 51(4): 351-359.

Hazzard, J.F., T.E. Yacoub, S. Vijayakumar and J.H. Curran, 2007. Stresses Under Footings in Multilayered Soils: A Comparative Study. Proceedings the 60th Canadian Geotechnical Conference and 8th Joint CGS/IAH-CNC Groundwater Specialty Conference; Ottawa, Canada.

Küçükarslan, S., 2007. The Effect of a Transient Dynamic Loaded Pile on a Neihbouring Pile for Nonhomogeneous Soils. International Earthquake Symposium, Kocaeli, Turkey.

Poulos, H.G., J.P. Carter and J.C. Small, 2001. Foundations and Retaining Structures -Research and Practice. Proceeding the 15th International Conference on Soil Mechanics and Geotechnical Engineering, Istanbul, Turkey, 4: 2527-2606.

Radhakrishnan, N., 1969. Solution of plane strain problems in soil mechanics using the method of finite elements. Ph.D. Thesis, The University of Texas at Austin, Texas, USA.

Rowe, R.K. and J.R. Booker, 1982. Finite layer analysis of nonhomogeneous soil. Journal of Engineering Mechanics Division, Proceedings of the American Society of Civil Engineers, ASCE, Vol. 108, No. EM1.

Selvadurai, A.P.S., 1998. Lan Q. Axisymmetric mixed boundary value problems for an elastic halfspace with a periodic nonhomogeneity. Int. J. Solids Structure, 35: 1813-1826.

Smith, I.M. and D.V. Griffiths, 1988. Programming the Finite Element Method. 2nd Edn., John Wiley & Sons Ltd.

Stark, R.F. and J.R. Booker, 1997. Surface displacements of a non-homogenous elastic half-space subjected to uniform surface tractions. Inter. J. Numer. Anal. Methods Geomech., 21(6): 361-378.

Terzaghi, K., R.B. Peck and G. Mesri, 1996. Soil Mechanics in Engineering Practice. 3rd Edn., John Wiley & Sons Ltd.

Wang, C.D., C.S. Tzeng, E. Pan and J.J. Liao, 2003. Displacements and stresses due to a vertical point load in an inhomogeneous transversely isotropic half-space. Inter. J. Rock Mech. Min. Sci., 40: 667-685.

Wang, C.X. and J.P. Carter, 2002. Deep penetration of strip and circular footings into layered clays. Inter. J. Geomechanics, 2(2): 205-232.

Wu, T.H., 1974. Soil Mechanics. 7th Printing, Allyn and Bacon Inc.