

Study on Optimal Placement of Piezoelectric Actuators for a Whole Spacecraft Vibration Isolator

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Abstract: Position of actuators plays an important role in active vibration control, which affects not only the performance of vibration control but also the stability of the whole system, especially for flexible structures. In this paper, dynamic sensitivity analysis method was used to derive an optimization criterion for Piezoelectric Stack Actuator (PSA), this criterion was only related to the dynamic characteristics of the structure itself and the features of disturbances, but was not affected by initial conditions and control methods. Then by using this criterion, optimal placement of the piezoelectric actuator of a Whole-Spacecraft Vibration Isolator (WSVI) was studied, and vibration control effect of random position and optimal position was compared; simulation results verified the validity of the criteria, and showed that the optimized location of actuator could greatly enhance the actuation efficiency and vibration control effect.

Key words: Dynamic sensitivity, optimization criteria, piezoelectric stack actuator, whole-spacecraft vibration isolator

INTRODUCTION

Configuration and optimization of actuators and sensors have significant meanings in structural vibration control, which affect not only the vibration control effect, but also the control cost, performance of controller and even the stability of whole system (Crawley, 1994; Li *et al.*, 2001; Liu and Zhang, 2000; Frechker, 2003; Gupta *et al.*, 2010).

With the development of smart materials and structures, piezoelectric materials had been widely used for actuator in active vibration control in recent years (Crawley, 1994). On the study of piezoelectric actuators optimization, positions and numbers of piezoelectric actuators are the most important impact factors (Li *et al.*, 2001), and they correlate each other, which make the study complicated, so most studies only considered positions of actuators while numbers of actuators was considered less, and these studies all aimed only to specific problems rather than universal. For the optimization criteria, Liu and Zhang (2000), Frechker (2003) and Gupta *et al.* (2010) reviewed kinds of optimization criteria and their development, although these criteria had some generality, but were complex to implement, and the results obtained by using closed-loop design idea to study the optimal placement of actuators were usually affected by initial conditions, weight matrix and different control laws, which made the problem

complicated, and this couldn't improve the effects of vibration control. Most of the optimization criteria considered only the structure and actuators or sensors, there was little to consider the information of disturbance characteristics and output performances, and most people did research based on random distribution only, but in many cases the distribution of disturbance the structure suffered is known, and the output performances are determined by the designer in advance, so these information should be used in actuators optimization (Wang *et al.*, 2007).

This study begins with analyzing the inherent dynamic characteristics of structure, by using method of sensitivity analysis of dynamic characteristics, the affect of PSA's installed positions on inherent characteristics was studied, and then appropriate optimal criterion was recommended to determine the optimal positions of piezoelectric actuators in WSVI, finally, vibration control effects of optimal position and random position were compared in simulation.

METHODOLOGY

Dynamic sensitivity analysis and optimization criterion:

Dynamic sensitivity analysis: Sensitivity analysis of dynamic characteristics has a wide range of applications not only in structural vibration control, but also in

structural dynamic response analysis, model updating, structural damage detection, parameter optimization and other fields (Chen *et al.*, 2002; Xu, 2008). Fox and Kapour (1968) was the first to propose an exact expression for solving first derivative of eigenvalues and eigenvectors by using complete modal expansion, the basic idea was to differentiate system characteristic equation directly, and then compute by using modal orthogonality and symmetry of mass matrix and stiffness matrix, and solve the problem with a linear combination of complete modal sets. The defect of classical modal method is having to use all mode vectors when calculating eigenvector sensitivity, although modal truncation method can be used to take only a few modes in practice, but accuracy can not be guaranteed, so the modified modal method, iterative modal method and so on were developed later, but were all complicated for computing. Nelson (1976) proposed an easy way to solve the first derivative of eigenvalues and eigenvectors based on Fox method, which kept the ribbon nature of the coefficient matrix to simplify the computation, and only needed to use the self order mode, so was considered the most effective and most commonly used method in engineering applications.

The basic idea of Nelson method is to make the eigenvector derivative be expressed as sum of general solution of homogeneous equation and particular solution of non-homogeneous equation, and then solve the two parts individually. That is:

$$\frac{\partial \varphi_r}{\partial P_m} = \varphi_r' = v_r + c_r \varphi_r \tag{1}$$

where φ_r denotes r -th eigenvector, P_m denotes m -th parameter, v_r is a particular solution of the eigenvector derivative, c_r is the undetermined coefficient for general solution, and we can get the particular solution by solve the following equation

$$F_r v_r = G_r \tag{2}$$

where,

$$F_r = K - \lambda_r M$$

$$G_r = \left[\frac{\partial K}{\partial P_m} - \frac{\partial \lambda_r}{\partial P_m} M - \lambda_r \frac{\partial M}{\partial P_m} \right] \varphi_r$$

K and M are stiffness matrix and mass matrix, λ_r denotes r -th eigenvalue, the coefficient matrix F_r is a rank of $n-1$ of singular matrix, and can not be inversed directly, Nelson gave a way to eliminate the

singularity through rebuilding matrix F_r and G_r , get a particular solution V_r , and then obtain the undetermined coefficient c_r by:

$$c_r = -\varphi_r^T M v - \frac{1}{2} \varphi_r^T \frac{\partial M}{\partial P_m} \varphi_r \tag{3}$$

For small damping system, the difference of natural frequency and modal frequency is slight, so the small damping system is often simplified to undamped system for characteristic sensitivity analysis in engineering, only consider the impact of the mass and stiffness changes on natural frequencies and mode shapes. The impact of piezoelectric stack installation on mass matrix can be neglected. Thus, the first order sensitivity of the natural frequency to stiffness is:

$$\frac{\partial \omega_r}{\partial k_{ij}} = \begin{cases} \varphi_{ir} \varphi_{jr} / \omega_r & i \neq j \\ \varphi_{ir}^2 / 2 \omega_r & i = j \end{cases} \tag{4}$$

where, ω_r is the r -th order natural frequency of the structure, k_{ij} is the element of structural stiffness matrix, φ_{ir} is the i -th element of mode shape normalized to mass corresponding to inherent frequency ω_r , and the first order sensitivity of inherent mode shape to stiffness could be obtained directly by Eq. (3).

Optimization criterion: Because the displacement output of piezoelectric stack is very small, its stiffness is very large relative to the general isolation devices, so it has a great impact on the stiffness near the installation location, especially the stiffness in the direction the piezoelectric stacks actuating, thus affects the overall stiffness matrix, and the actuators just act by changing the stiffness of the structure for vibration reduction after its installation.

For the request of vibration suppression, actuators should be arranged in the locations that affect the natural frequencies and mode shapes of the structure greatly, so to improve the efficiency of actuators; but to meet the requirements of structural design, the changes of some natural frequencies can not be too large, in addition, for the objectives of vibration reduction, not all modes need to be controlled, because not all modes have contribution to the vibration reduction objectives, but controlling all modes will increase control costs. Therefore, the optimization target can be defined as follows

$$J = \left(\sum_{r=1}^m W_{or} |P_{or}| \right) \cdot \left(\sum_{r=1}^m W_{or} |P_{or}| \right) \tag{5}$$

and the corresponding optimization criterion is

$$\max J \rightarrow \text{node number} \tag{6}$$

RESULTS AND DISCUSSION

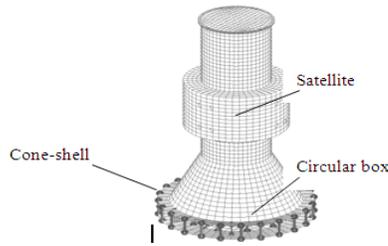


Fig. 1: Structure of whole-spacecraft and vibration isolator model

where $P_{\omega_r} = \frac{\partial \omega_r}{\partial k_{ij}}$ and $P_{\phi_r} = \frac{\partial \phi_r}{\partial k_{ij}}$ respectively denote the sensitivity of natural frequencies and sensitivity of mode shapes of the original structure, W_{ω_r} and W_{ϕ_r} respectively denote corresponding weighting coefficients, W_{ω_r} and W_{ϕ_r} are all vectors, and K_{ij} contains the related information of node number.

In Eq (5) of optimization target, the weight coefficient can be selected according to different characteristics of disturbances and different vibration reduction requirements, for the modes natural frequency can not be changed too much and modes do not need to be controlled, take W_{ω_r} as a small number or even zero; for the modes in the band of disturbance frequencies and need to be controlled, we can take w_{ω_r} as a large number. The selection of w_{ϕ_r} is related to the modes that need to be controlled and the response of control points, also we can simply set the components of mode shape which mode need to be controlled corresponding to the response concerned of the control points to 1 and others to zero, the all elements of W_{ϕ_r} corresponding to the modes need not to be controlled are set to zero. The subscript i of $-$ is taken as the location in the stiffness matrix corresponding to the component of DOF (degree of freedom) the PSA actuating, then subscript i contains the node number, that is the location information of the node; the subscript j is taken to be the location of certain DOF of any nodes in the stiffness matrix, then j relates to the node number too, if we select certain k_{ij} , we can determine the position where piezoelectric stack actuator should be installed.

In the optimization criterion above, we only used the inherent dynamic characteristics such as mass matrix, stiffness matrix, modal frequencies and mode shapes before installation of piezoelectric actuators, and the frequency distribution of disturbance was also taken into account in weight coefficients, which was easy to be selected according to different excitations and different reduction requirements. This criterion isn't affected by initial conditions and control methods and is suitable for any structure.

Placement optimization for WSVI: During the Eleventh Five-Year Plan period of China, we proposed a new circular Whole-Spacecraft Vibration Isolator (WSVI) (Liu *et al.*, 2010), the structure of whole-spacecraft and vibration isolator model is shown in Fig. 1.

The WSVI was in the form of series connected with a circular box and cone-shell adapter, spacecraft and other payloads connect with WSVI still through the top box of the cone-shell; the bottom of WSVI is connected with the rocket, the bottom box of cone-shell is connect with top surface of circular box, and beneath it internal sets of ribs are connected with the top and bottom surface of circular box, and between every set of ribs can paste viscoelastic damping material to increase structural damping; in order to facilitate rib's installation and removal, inner ring of circular box uses flange to connect, and the out ring connect with bolts. The improved WSVI mainly depend on the deformation of the upper stage of the isolator and the shear deformation of the viscoelastic material pasted between sets of ribs to consume the vibration energy, in addition, improved rib structure is more conducive to the damping effect of the viscoelastic material.

To compensate the lack of passive vibration isolation in low frequency, improve the vibration isolation frequency band and vibration isolation effect, the new WSVI uses integrated passive-active vibration isolation, and according to the structure of WSVI and the stringent requirement for mass of space launch, we use piezoelectric stack actuator for active control.

As the structure of whole-spacecraft and isolator shown in Fig. 1, the feasible region for piezoelectric stacks arranged is between upper and lower panels of the circular box isolator, and the actual region for analysis is the up board of circular box isolator and the actuating direction of PSA is perpendicular to the up board, and the size for installation of PSA is constrained by structure, so it is not possible to increase the PSA's displacement output by increasing the height of PSA, we should find out suitable places with the limited conditions to ensure the efficiency of PSA and get better vibration control effect.

Firstly, we use finite element analysis software NASTRAN to make structure modal analysis of the whole-spacecraft to get modal frequencies and mode shapes normalized to mass, there are six global modes in less than 100 Hz, they are three lateral modes, two longitudinal modes and one torsion mode, the mode shape and modal frequency of each mode are shown in Fig. 2. The control objective is to reduce the vibration amplitude of the first two lateral modes at resonant peaks of the antenna and barycentre of the satellite, the disturbance frequency distribution band considered is 0-200Hz, and the excited location is base connected with the bottom of the isolator.

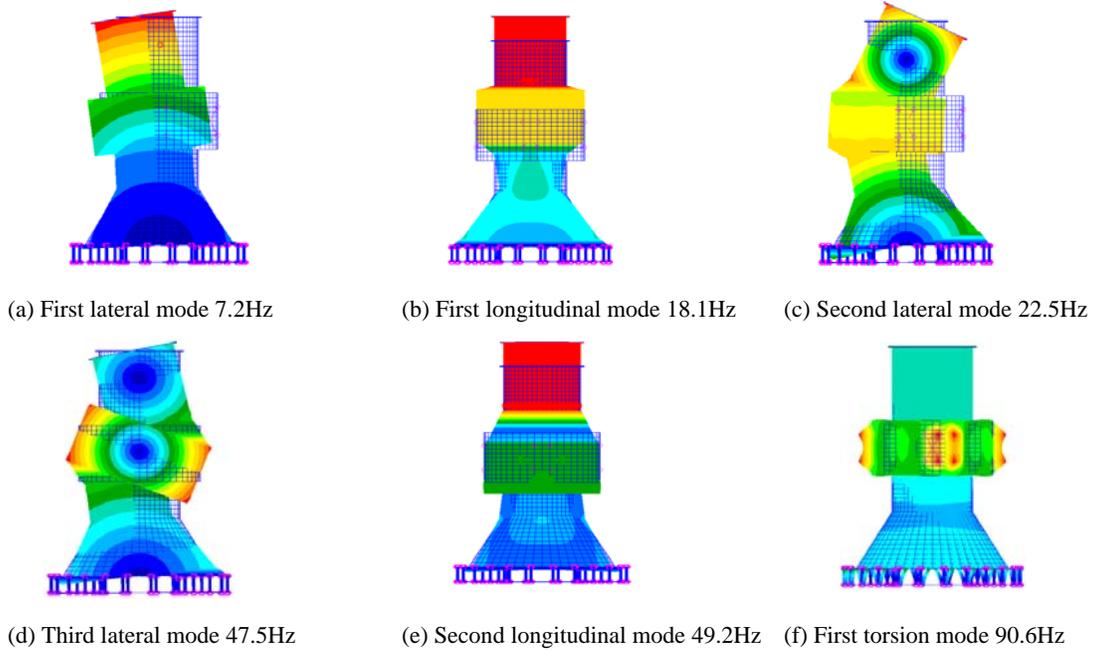


Fig. 2: The first six modes and frequencies of whole-spacecraft and isolator

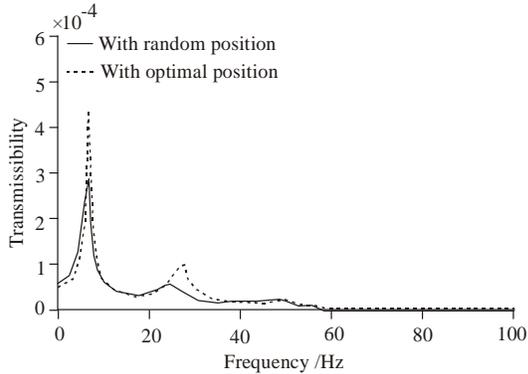


Fig. 3: Transmissibility curve comparison for random position and optimal position from PSA to antenna

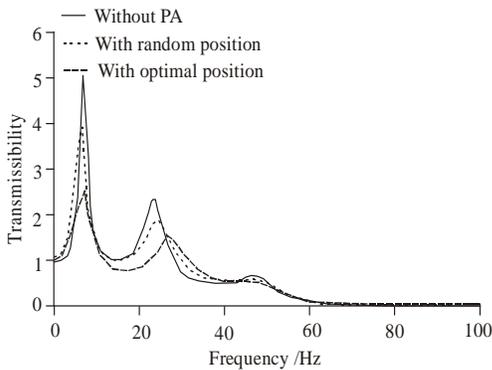


Fig. 4: Control effect comparison of lateral vibration of antenna for random position and optimal position

Table 1: Corresponding node number of optimal positions for different cases

Cases	Node number
Just lateral modes	663
Without torsion mode	663
With all modes	663
Just first lateral mode	634
With first two lateral modes	663
With all lateral modes	663
Controlling first DOF of antenna	663
Controlling second DOF of antenna	658
With proportion 1:1	663
With proportion 1:10	663
With proportion 10:1	663
Just control antenna	663
Just control barycentre	634
Control antenna and barycentre	634

Table 2: Parameters of piezoelectric actuator

Parameter	Value
Piezoelectric constant d_{33}	4×10^{10} m/v
Equivalence elastic modulus E	7.64×10^9 Pa
Sectional area A	3.14×10^4 m ²
Poisson's ratio	0.3
Length of PSA	0.08 m

There are total of 9557 analysis nodes, and 672 nodes the piezoelectric stack can be arranged, each node has six analysis DOF, but the only direction for piezoelectric stack actuators actuating is the vertical of whole-spacecraft. Compute dynamic sensitivity of whole-spacecraft to the nodes on which piezoelectric stack can be arranged by the method described above, get the sensitivity of modal frequencies and sensitivity of mode

shapes of each node, and compute optimization indexes according to Eq. , determine the installation region of PSA according to Eq. (6). Table 1 lists corresponding node numbers the maximum optimization indexes according to different mode types, different orders of modes, different DOF of control nodes, different weighting coefficients ratio of frequency and mode shape and different control node.

By comparing and analyzing, we can see that for certain DOF of control node, though mode types and the ratio of weighting coefficients have affection on optimization indexes, but they do not affect the optimal location selection of actuators; modal orders and DOF of control nodes have larger affection on the optimal location selection, the optimal location is node 634 when just take the first order lateral mode, the best location becomes node 663 when taking the first two lateral modes or taking all lateral modes; the optimal location is node 663 when choosing the first DOF of control node, the optimal location changes to node 658 when choosing the second DOF of control node; and when considering different control node, the optimal location is also different , when choosing the antenna as control node, the optimal location is node 663, and when choosing the barycentre as control node, the optimal location is node 634. These above all just confirms our previous idea that the characteristics distribution of disturbance and concerned output performances affect the optimal location of actuators, therefore, characteristics distribution of disturbance and different output performance requirements must be considered during the placements optimization of actuators.

Then take lateral vibration control of antenna of the satellite as an example, selecting the optimal position determined according to the above optimization criteria and random position for simulation analysis. We use single PSA, because PSA can only carry axial pressure, we use ROD element for simulation, and simulate the inverse piezoelectric effect in accordance with the principle of thermo-elastic equivalence, selecting coefficient of thermal expansion α and d_{33} is piezoelectric constant of piezoelectric materials, t is thickness of single piezoelectric patch, the size and material properties of piezoelectric element are lists in Table 2. Figure 3 shows the transmissibility comparison from PSA's excitation to antenna when PSA being installed at random position and optimal position, transmissibility of PSA installed at optimal position obviously higher than randomly selected position, which means that with same excitation voltage, PSA installed at optimal position is more efficient for vibration control of antenna. Figure 4 shows the vibration control effect comparison for PSA installed at random position and optimal position of antenna when using optimal control and selecting same weighting function, we can see that

the vibration control effect of PSA installed at optimal position is much better than PSA installed at random position when controlling lateral vibration of antenna.

CONCLUSION

Whole-spacecraft vibration isolator integrated passive and active vibration isolation can isolate vibration transmitted from rocket to satellite in a wide frequency band, and can compensate the lack of passive vibration isolation in some cases, but the optimal placement of actuators is also a important issue. This paper established relevant optimization criteria through the use of the sensitivity of modal frequencies and the sensitivity of mode shapes, which is only affected by the inherent characteristics of the system and the external disturbances, but not affected by initial conditions and control laws, and for different distribution of disturbance and different output performance control, the optimal placements of actuators can be selected by adjusting different weighting function.

By computing the best locations of PSA for a whole-spacecraft, comparing and analyzing the affections of mode types, mode orders, DOF of control nodes, weighting coefficient ratio of frequency and mode shape on optimization indexes of each node when controlling the lateral vibration of antenna, results verified the validity of the optimization criterion, and indicated that different characteristics distribution of disturbance and different output performance requirements must be considered when configuring optimal displacement of actuators.

We compared the control effect of antenna's lateral vibration for PSA installed at random position and optimal position, PSA installed at optimal position could greatly enhance the actuator's actuation efficiency.

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