

## A Unified 3D Mesh Segmentation Framework Based on Markov Random Field

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**Abstract:** 3D Mesh segmentation has become an important research field in computer graphics during the past decades. Many geometry based and semantic oriented approaches for 3D mesh segmentation has been presented. In this paper, we present a definition of mesh segmentation according to labeling problem. Inspired by the Markov Random Field (MRF) based image segmentation, we propose a new framework of 3D mesh segmentation based on MRF and use graph cuts to solve it. Any features of 3D mesh can be integrated into the segmentation framework. Experimental results show that the noise and over-segmentation are avoided. It also demonstrates that the proposed scheme has the capability of combining the geometric and topology information of the 3D mesh.

**Key words:** Gibbs random field, graph cuts, Markov random field, mesh segmentation

### INTRODUCTION

With the development of hardware and computer graphics, 3D object has become a kind of important multimedia content during the last decades. Triangular mesh is the most popular representation of 3D objects due to its adaptability in a wide range of computer graphics applications and processing operations, such as compression, simplification, approximation, indexing and watermarking (Dugelay *et al.*, 2008). Especially, 3D Mesh segmentation has attracted many research works and become an important issue in computer graphics. The focus of application mainly includes the following areas: texture mapping (Levy *et al.*, 2002), compression (Karni and Gotsman, 2000), metamorphosis (Gregory *et al.*, 1999; Zockler *et al.*, 2000), modeling (Funkhouser *et al.*, 2004), re-meshing and simplification (Garland *et al.*, 2001; Cohen-Steiner *et al.*, 2004), 3D shape retrieval (Zuckerberger *et al.*, 2002; Tierny *et al.*, 2009), collision detection (Li *et al.*, 2001), and skeleton extraction (Katz and Tal, 2003).

A lot of approaches in 3D mesh segmentation have been proposed in the last few years. However, to our knowledge, only a limited number of papers investigate random field model for mesh segmentation. Lavoué used MRF model to cluster the vertices with the roughness feature first, and then used the region growing method to segment the mesh (Lavoué and Wolf, 2008). Zouhar addressed the problem of 3D mesh segmentation for

categories of objects and modeled the label distribution using Conditional Random Field (CRF) to ensure semantic consistency in segmentation (Zouhar *et al.*, 2010). Inspired by their previous works, we directly segment 3D mesh through our MRF mesh segmentation model.

In this study we take mesh segmentation problem as a labeling problem whose solution is a set of labels assigned to faces, vertices or edges of the mesh according to the correlated features SDF (Shapira *et al.*, 2008), roughness (Lien *et al.*, 2006), AVG (Zhang *et al.*, 2005) and the connectivity of the mesh. A new unified 3D mesh segmentation framework integrating with MRF is proposed.

### LITERATURE REVIEW

**Markov random fields:** Markov Random Field (MRF) is a theory which can be used to state and analysis the spatial or temporal properties of physical phenomenon. It has been widely used in various areas of image processing, such as texture image synthesis, image segmentation (Levy *et al.*, 2002).

Many research topics in digital image processing, such as image segmentation, image restoration, image denoising, can be defined as the following labeling problem (Li, 2001): Given a set of initially labeled element  $S = \{1, 2, \dots, n\}$  and a continue or discrete set of labels  $L$ , we can assign a label  $f_i \in L$  for any element  $i \in S$ .

Then the set  $f = \{f_1, f_2, \dots, f_n\}$  is defined as a state and is a solution of the labeling problem. An element in  $S$  denotes a point or a region in Euclidean space, for example, a pixel in digital images, a vertex, an edge or a face in 3D mesh objects. We can also describe the labeling problem as a mapping:  $f: S \rightarrow L, f(i) = f_i$ . Then the set  $F = L \times L \times \dots \times L = L^n$  is the solution space of the labeling problem. Let  $X = \{X_1, X_2, \dots, X_n\}$  be a set of random variables defined over  $S$ , we call  $X$  a random field if the values of all  $X_i (1 \leq i \leq n)$  are belong to  $L$ . Obviously, any labeling problem can be regarded as a random field and each initial element is associated with a random variable. Therefore, solving a random field is equal to solving the labeling problem, there is a equivalence relation between labeling problem and random field.

Let  $X_i = f_i$  when the  $i$ th initial element has a state  $f_i$ ,  $P(f_i) = P(X_i = f_i)$  is the probability of the event  $X_i = f_i$ ,  $P(f) = P(X = f)$  is the probability of the joint event  $\{X_1 = f_1, X_2 = f_2, \dots, X_n = f_n\}$ . We define the neighborhood set  $N = \{N_i | i \in S\}$ , where  $N_i$  is the set of all initial elements adjacent to element  $i \in S$ . Then the following two properties follow immediately:

- $i \notin N_i$
- $i \in N_i \Leftrightarrow i' \in N_i$

Let  $X$  be a random field defined over the initial elements set  $S$  and  $N$  be the neighborhood of  $S$ , then  $X$  is a Markov Random Field (MRF) if the following two conditions are true:

- $P(f) > 0, \forall f \in F$
- $P(f_i | f_{s-(i)}) = P(f_i | f_{N_i})$

where  $f_{s-(i)} = \{f_i / i' \in S - \{i\}\}$ ,  $f_{N_i} = \{f_i / i' \in N_i\}$ .

We say that a random field  $X$  is Gibbs Random Field (GRF) if the status distribution function of  $X$  is Gibbs distribution:

$$P(f) = Z^{-1} \times e^{-\frac{1}{T}U(f)} \tag{1}$$

where

$$Z = \sum_{f \in F} e^{-\frac{1}{T}U(f)}, \quad U(f)$$

is an energy function and  $T$  is a constant.

Markov Random Field is a kind of Probability Graphical Model (PGM). We can use the undirected graphical model  $G = \{V, E\}$  to describe the statistical dependencies between those initial elements of MRF. Each vertex or node  $i \in V$  of graph  $G$  denotes one initial element  $i$  or random variable  $X_i$ . Each edge  $e = e_{ij} = \dots, (i, j) \in E$  exists if the elements  $i$  is adjacent to  $j$  namely the random variable  $X_i$  and  $X_j$  are statistical dependable.

Especially, each node  $i \in V$  of graph  $G$  is also corresponding to one observation random variable  $D_i$  whose value is some attribution information of element  $i$ . Let  $c$  be the completely connected graph of undirected graph  $G$ , then we say that  $c$  is a clique of graph  $G$  and  $C$  is the set of all cliques. Suppose that each node  $i \in V$  has an observation value  $d \in \{d_1, d_2, \dots, d_n\}$ , an optimize label set  $f^* = \{f^*_1, f^*_2, \dots, f^*_n\}$  according to the dependencies of nodes is necessary for solving MRF  $X$  to maximize the posterior probability  $P(f^* | d)$ . In order to define a unique random field with the conditional distribution of local attributions, The Hammersley-Clifford theorem proved the equivalence between MRF and GRF (Hammersley and Clifford, 1968). According to the theorem, the probability distribution of status of MRF  $X$  is Gibbs distribution. hence:

$$P(f) = Z^{-1} \times e^{-\frac{1}{T}U(f)}, \tag{2}$$

where,

$$Z = \sum_{f \in F} e^{-\frac{1}{T}U(f)}, \quad U(f) = \sum_{c \in C} V_c(f)$$

$Z$  is a normalized constant named as splitting function,  $U(f)$  is called energy function and is the sum of all potential energy functions  $V_c(x)$  defined over the sub-graph  $c$  of graph  $G$ . Obviously, the probability of realizing the status  $f$  is inversely proportional to the value of its energy function  $U(f)$ . We might also introduce some priori information into the MRF  $X$  by defining  $V_c(x)$ , such as the features of segmentation boundary in 3D mesh.

Just as Li present an overview of the application of MRF (Li, 2001), the theory provides a convenient and consistent way of modeling context-dependent entities and correlated features (Li, 2001), and has been used in image processing extensively. However, MRF based 3D mesh segmentation is rarely presented.

**3D mesh segmentation:** Approaches for 3D mesh segmentation can be distinguished into geometry based and semantic oriented. The former is to identify parts of the object that are homogeneous with respect to a criterion based on geometric properties. The latter is to identify parts of the object that are meaningful under some semantic or perceptive criteria.

Geometry based approaches segment the 3D mesh into patches having homogeneous geometric properties and are typically used as pre-process for other geometry processing tasks. Curvature information is exploited by many works in this category. Developing on surface curvature, in Mangan and Whitaker (1999), Mangan and Whitaker (1999) present a watershed approach of 3D mesh segmentation which generalizes the morphological watersheds originally proposed for image segmentation.

While results in (Mangan and Whitaker, 1999) are negatively affected by the use of pseudo-curvatures, a fast marching watershed solution is reported. The actual principal curvatures and directions (Taubin, 1995) are used to improve the segmentation results. Principal curvature information for mesh segmentation is also exploited in (Ceccarelli *et al.*, 2005). In this case, segmentation of 3D mesh is achieved by way of a stochastic relaxation process determined by the mutual synchronization of a network of pulse-and-fire oscillators.

Several geometry based methods identify homogeneous patches by using clustering techniques, such as region growing (Lavoué *et al.*, 2005), iterative clustering (Cohen-Steiner *et al.*, 2004; Shatz *et al.*, 2006) or hierarchical clustering (Garland *et al.*, 2001). Inspired by the work about hierarchical face clustering on polygonal surfaces (Garland *et al.*, 2001), Attene *et al.* (2006) implemented a prototype with three fitting primitives including planes, spheres and cylinders and presented a hierarchical face clustering method for triangle meshes based on the prototype (Hammersley and Clifford, 1968). Many of the semantic oriented approaches introduce a bias which is assumed to help the segmentation algorithm to provide significant segmentations. Examples of bias are feature points (Liu and Zhang, 2007; Katz *et al.*, 2005), symmetry transformations (Podolak *et al.*, 2006), shape diameter (Shapira *et al.*, 2008), skeletons (Lien *et al.*, 2006) and Reeb graph defined with respect to the integral geodesic function (Zhang *et al.*, 2005).

MRF has been widely used in image processing, such as segmentation. Only a few algorithms has been presented for 3D object segmentation. However, according to MRF's theory, 3D mesh segmentation can be implemented with a labeling problem equivalent to MRF.

### DEFINITION OF MESH SEGMENTATION

Note that many image analysis and interpretation problems can be seen as labeling problems in which the solution to a problem is a set of labels assigned to image pixels or features (Li, 2001). A labeling problem is specified in term of a set of sites and a set of labels. In 3D labeling, a site often represents a point or a region in the Euclidean space such as a vertex, an edge, or a face of the mesh, a label set may be categorized as being continuous or discrete. We adopt the discrete case and regard mesh segmentation as a labeling problem in this paper.

Let  $M$  be a 3D mesh with a tuple  $M = \{V, E, F\}$  of vertices  $V = \{v_i \mid v_i \in \mathbb{R}^3, 1 \leq i \leq n\}$ , edges  $E = \{e_{ij} = (i, j) \mid v_i, v_j \in V, i \neq j\}$ , and faces which are usually triangles  $F = \{f_{ijk} = (i, j, k) \mid v_i, v_j, v_k \in V, i \neq j, j \neq k, k \neq i\}$  (Shamir, 2008). Suppose that  $S = \{1, 2, 3, \dots, m\}$  indexes a discrete set of sites, in which  $1, 2, 3, \dots, m$  are indices,  $L = \{1, 2, \dots, K\}$  be a set of  $K$  labels. Then mesh

segmentation can be regard as the process to assign a label  $f_i \in L$  to a site  $i \in S$ . The set  $f = \{f_1, f_2, \dots, f_m\}$  is called a labeling or a configuration (in random field theory) of the sites indexed  $S$  in terms of the labels in  $L$ , the set  $F = L \times L \times \dots \times L$  is called configuration space which is the set of all possible configurations. A configuration  $f$  in  $F$  segments the mesh model into different components, and two components of these components may be assigned the same label, but are disjoint in spatial.

Because each site (either a vertex, an edge or a face) is assigned a unique label in mesh segmentation, the assign process  $f_i = f(i)$  can be regarded as a mapping from  $S$  to  $L$ , i.e.,  $f: S \rightarrow L$ , and  $f(S)$  is a configuration in  $F$ . Now, we can define mesh segmentation as follow:

**Definition I:** Let  $M = \{V, E, F\}$  be a 3D mesh model, and  $S = \{1, 2, 3, \dots, m\}$  is the set of the indices of vertices, or edges, or faces,  $L = \{1, 2, \dots, K\}$  is the set of labels. A segmentation of  $M$  is a configuration in  $F$  obtained by  $f: S \rightarrow L$ .

**Remark I:** The kernel question in our definition of mesh segmentation model is which label will be assigned to each site such that the configuration can induce the best segmentation. Obviously, this heavily relies on the correlated features of the site (e.g., curvature, roughness or SDF), the spatial dependencies between sites in  $S$  and some other geometric constraints. To evaluate the configurations in configuration space  $F$ , we define an energy function of  $f: E(f): F \rightarrow \mathbb{R}$ . Then we can pose a mesh segmentation problem as an optimal problem in the following manner:

**Definition II:** Given a 3D mesh model  $M = \{V, E, F\}$ , the set of sites  $S = \{1, 2, 3, \dots, m\}$  and the set of labels  $L = \{1, 2, \dots, K\}$ , then mesh segmentation can be viewed as the optimal problem to find a configuration  $f$  in configuration space  $F$  such that the energy function  $E(f)$  will be minimized (or maximized).

**Remark II:** As mentioned above,  $S$  can be a set of indices of each one of faces, vertices or edges and the configuration of the sites in terms of the labels in  $L$  induced a segmentation of  $M$ . Mesh segmentation processes, usually assign labels to the faces of the mesh, i.e.,  $S$  indexes the set of faces (we denote this  $S = F$ ). If  $S = V$  or  $S = E$ , then some faces will have vertices or edges with different labels, and must join into one of the adjacent components.

**MRF clustering based segmentation framework:** As the sites of a 3D mesh model are spatial dependent, if consider the features information only in the segmentation

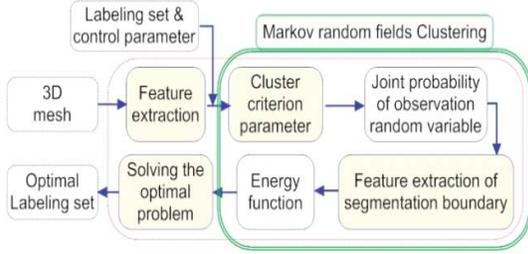


Fig. 1: Workflow of our framework based on MRF clustering

model, the segmentation induced by the configuration may be noisy and may be an over-segmentation. Research shows that the edge with a concave dihedral angle across it is a better candidate for the boundary between two components. As above, we must take some geometric constraints into account in our mesh segmentation model, to consider both feature and spatial dependency and smooth the boundaries between components.

Note that in the random field theory, a random variable whose realization is a configuration  $f \in F$  can be posed as a random field. We choose the hierarchical GRF model (i.e., hierarchical two-level Gibbs model) to model the mesh segmentation problem: the lower-level Gibbs distribution and the higher-level Gibbs distribution.

**Overview of the proposed framework:** In this study, we present a MRF based framework of arbitrary polygonal meshes segmentation. In order to understand the framework more clearly, here, we overview it systematically. Figure 1 illustrates the workflow of our mesh segmentation framework based on MRF clustering.

Firstly, the input parameters of our proposed framework are a label set  $L = \{1, 2, \dots, K\}$ , an interactive control parameter  $\beta$ , a 3D mesh model  $M = \{V, E, F\}$  and its property set  $d = \{d_1, d_2, \dots, d_N\}$ . The output parameters is the optimal status set  $f^* = \{f^*_1, f^*_2, \dots, f^*_N\}$ .

Secondly, the cluster criterion parameter including the mean  $\mu = \{\mu_1, \mu_2, \dots, \mu_K\}$  and standard variance  $\sigma = \{\sigma_1, \sigma_2, \dots, \sigma_K\}$  can be designed for each cluster. This is the lower-level distribution in our proposed framework. Thirdly, the joint probability of observation random variable can be deduced by:

$$P(d|f) = \prod_{i=1}^N P(d_i / f_i) = \prod_{i=1}^N N(d_i / \mu_{f_i}, \sigma_{f_i}^2) \quad (3)$$

$$= \prod_{i=1}^N \frac{1}{\sqrt{2\pi\sigma_{f_i}^2}} \exp\left(-\frac{(d_i - \mu_{f_i})^2}{2\sigma_{f_i}^2}\right)$$

Fourthly, the feature of segmentation boundaries can be defined with energy potential functions  $V_{(i,j)}(f)$ . It states a higher-level distribution in the proposed framework. Then, the distribution function of the status of Markov

Random Field can be expressed by Eq. (4) according to Hammersley-Clifford theorem:

$$P(f) = \frac{1}{Z} \exp\left(-\beta \sum_{(i,j) \in C} V_{(i,j)}(f)\right), Z = \sum_f e^{-U(f)} \quad (4)$$

Finally, we define the objective function  $E(f): F \rightarrow R$  according to the second and the third steps, where:

$$E(f) = \sum_{i=1}^N -\ln P(d_i | f_i) + \beta \sum_{(i,j) \in E} V_{(i,j)}(f)$$

The optimal status  $f^*$  can be obtained by solving the optimization problem  $f^* = \min_{f \in F} (E(f))$ .

**Definition of higher-level distribution:** To express the spatial dependencies between sites in  $S$  of the mesh  $M$ , we build the dual graph  $G$  of  $M$  by representing each site in  $S$  by a node in  $G$  and defining the edges in  $G$  by adjacency relation in  $M$  of the sites of  $S$  like in (Shamir, 2008). For instance, if  $S = F$  then each node in  $G$  represents a face in  $M$  and each edge connects two adjacency faces.

Then we define the neighborhood system on the graph  $G$ : the neighborhood of site (i.e., node)  $i$  is  $N_i = \{j | i, j \text{ is connected by an edge in } G\}$ , and the spatial dependencies between sites can be express as follow: site  $i$  depends on site  $j$  in spatial if site  $j$  is a neighbor of site  $i$ . This neighborhood system defines a set of cliques, in which a clique is a fully connected sub graph.

The higher-level distribution determines the prior information of a configuration of the random field on the spatial dependencies between sites through the distribution of the configuration  $P(f)$ . According to the Hammersley-Clifford theorem (Hammersley and Clifford, 1968),  $P(f)$  is a Gibbs distribution:

$$P(f) = \frac{1}{Z} \exp(-U(f)) \quad (5)$$

where  $U(f) = \sum_{c \in C} V_c(f)$  is a energy function defined as the sum of energy potentials function over all possible cliques  $C$ ,  $V_c(f)$  is the energy potential function for the configuration  $f$  defined by spatial dependencies in the clique  $c$ , and  $Z = \sum_f e^{-U(f)}$  is a normalization constant.

In our case, we using potts model to define the energy potential functions  $V_{(i,j)}(f)$  on 2-site cliques (i.e., edges in graph  $G$ ):

$$V_{(i,j)}(f) = \begin{cases} 1, & \text{if } f_i \neq f_j \\ 0, & \text{else} \end{cases} \quad (6)$$

where  $\{i, j\}$  is an edge in graph  $G$ . And define the energy potential functions on other cliques equal to constant zero. Now, we can get the prior information of configurations:

$$P(f) = \frac{1}{Z} \exp \left( -\beta^* \sum_{(i,j) \in C} V_{(i,j)}(f) \right) \quad (7)$$

where  $\beta$  is an interaction coefficient controlling the weight of the prior information.

**Definition of lower-level distribution:** To assign  $K$  labels to the set of sites  $S$  based on the correlated feature values, we can select one piecewise probability density function to approximate the histogram of feature values. We denote the random variable whose realization is the feature value of site  $i$  of the mesh as  $x_i$ , denote the family of these random variables on  $S$  as  $X = \{x_1, x_2, \dots, x_N\}$ , and denote the joint event  $\{x_1 = x_1, x_2, \dots, x_N = x_N\}$  as  $x = \{x_1, x_2, \dots, x_N\}$ . Then we can express the density  $P(x_i)$  as Eq. (8):

$$P(x_i) = \sum_{k=1}^k \pi_k N(x_i / \mu_k, \sigma_k) \quad (8)$$

where,  $\mu_k$  is the mean value,  $\sigma_k$  is the standard deviation,  $\pi_k$  is the weight of the  $k$ -th piecewise probability density function,  $\sum_{k=1}^k \pi_k = 1$ ,  $K$  is the number of labels, and  $N(x_i | \mu_k, \sigma_k)$  is the probability of  $x_i$  conditioned on label  $k$ :

$$N(x_i | \mu_k, \sigma_k) = \frac{1}{\sqrt{2\pi}\sigma_k} \exp \left( -\frac{(x_i - \mu_k)^2}{2\sigma_k^2} \right) \quad (9)$$

Therefore, the likelihood function for the sites set  $S$  is:

$$\prod_{i=1}^N \left( \sum_{k=1}^K \pi_k N(x_i | \mu_k, \sigma_k) \right) \quad (10)$$

the probability of assigning a label to a site can be described as follow:  $P(x_i | f_i)$  obey a normal distribution  $N(\mu, \sigma)$ , and each label  $k \in L = \{1, 2, \dots, k\}$  is represented by its mean value  $\mu_k$  and standard deviation  $\sigma_k$ :

$$N(\mu_k, \sigma_k) = \frac{1}{\sqrt{2\pi}\sigma_k} \exp \left( -\frac{(x - \mu_k)^2}{2\sigma_k^2} \right) \quad (11)$$

We can estimate the weights  $\pi = \{\pi_1, \dots, \pi_k\}$ , the mean values  $\mu = \{\mu_1, \dots, \mu_k\}$ , and the standard deviations  $\sigma = \{\sigma_1, \dots, \sigma_k\}$  by maximizing the log of likelihood function (10):

$$\ln p(X | \mu, \sigma) = \sum_{i=1}^N \ln \left( \sum_{k=1}^K \pi_k N(x_i | \mu_k, \sigma_k) \right) \quad (12)$$

Then we can get the density of  $X$  conditioned on the configuration  $f$ :

$$P(X | f) = \prod_{i=1}^N P(x_i | f_i) = \prod_{i=1}^N N(x_i | \mu_k, \sigma_k) \quad (13)$$

**Definition of energy function:** In conclusion, given the feature values of all sites, our goal is to find the most probable configuration  $f^*$  to maximize the posterior probability  $p(f^* | x)$ . According to the Bayesian rule:

$$p(f | x) = \frac{p(f) p(x | f)}{p(x)} \quad (14)$$

As the feature values of all sites is given,  $p(x)$  is constant, we can express  $f^*$  as follow:

$$p(f | x) \propto p(f) p(x / f) \quad (15)$$

$$f^* = \arg \max_f (P(f) P(x / f)) \quad (16)$$

Then, we denote  $U(f | x)$  as Eq. (17):

$$U(f | x) = \beta U(f) + U(x | f) \quad (17)$$

where,

$$U(x / f) = -\sum_{i=1}^N \left( \ln \left( \sqrt{2\pi} \sigma_{f_i} \right) + \frac{(x_i - \mu_{f_i})^2}{2\sigma_{f_i}^2} \right) \quad (18)$$

Now the posterior probability is:

$$P(f | x) \propto \exp(-U(f | x)) \quad (19)$$

Then we define the energy function  $E(f)$  as:

$$E(f) = U(f | x) = \beta U(f) + U(x / f) \quad (20)$$

and  $f^*$  is:

$$f^* = \arg \min_f (E(f)) \quad (21)$$

## EXPERIMENTAL RESULTS AND ANALYSIS

In this study, we will instance the framework of mesh segmentation for 3D triangular mesh to validate the effectiveness of the proposed framework. Firstly, we select  $S = F$  and use the Shape Diameter Function (SDF)

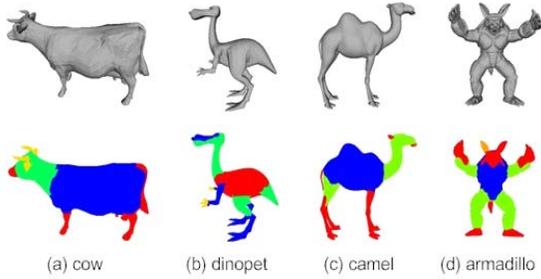


Fig. 2: Original 3D mesh model and the result of our algorithm with parameters: (a)  $\beta = 7.0$ ; (b)  $\beta = 8.0$ ; (c)  $\beta = 15.0$ ; (d)  $\beta = 10.0$

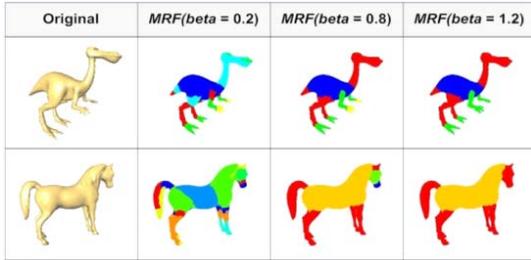


Fig-3: Segmentation results with different control parameters

Table 1: Processing time of our 3D mesh Segmentation algorithm based on MRF

Name	Faces Num	Cluster Num	Time EM (s)	Time GC (s)
Cow	5804	3	0.1	0.05
Dinopet	8996	5	0.21	0.20
Camel	19536	4	0.45	0.34
Armadillo	52000	5	6.03	1.13

(Shapira *et al.*, 2008) as the feature of all triangles. To assign K labels to the set of sites S based on the correlated feature values, K-Gaussian mixture model is used to fit the histogram of feature values. Secondly, EM (Expectation-Maximization) algorithm is used to estimate the weights  $\pi = \{\pi_1, \dots, \pi_k\}$ , the mean values  $\mu = \{\mu_1, \dots, \mu_k\}$ , and the standard deviations  $\sigma = \{\sigma_1, \dots, \sigma_k\}$  by maximizing Eq. (10). Thirdly, we can inject the prior information smoothness of boundaries by defining a cost function for all edges in the mesh M (or a weight function for all edges in the graph G) when the set of sites  $S = F$ . According to the minimum rule of human visual perception theory (Biederman, 1987), the edge with a concave dihedral angle is better for a boundary. Given an edge  $\{i, j\}$ , and  $\theta_{\{i, j\}}$ , the angle between the normal vectors of the two faces which are adjacent to the edge  $\{i, j\}$ , we can define the cost function to:

$$Cost(i, j) = \ln \left( \alpha \frac{\theta_{\{i, j\}}}{\pi} \right) \quad (22)$$

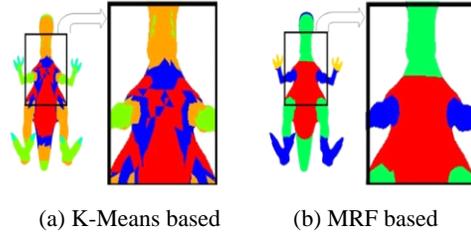


Fig. 4: Comparison with K-Means algorithm

where  $\alpha = 1$  for the concave dihedral angle, and  $\alpha$  equal to a small positive constant for the convex dihedral angle. Then, we can modify the energy potential functions  $V_{(i, j)}$  (f) as the following and substitute it with Eq. (7):

$$V_{(i, j)}(f) = \begin{cases} Cost(i, j), & \text{if } f_i \neq f_j \\ 0, & \text{else} \end{cases} \quad (23)$$

Finally, we use graph cuts algorithm (Kolmogorov and Zabini, 2004; Boykov *et al.*, 2001; Boykov and Kolmogorov, 2004) to minimize the energy function E(f).

**Run time analysis of the proposed framework:** In order to demonstrate the efficiency of our algorithm instanced from our framework for 3D mesh segmentation based on MRF clustering, we conducted experiments to segment different 3D meshes with different numbers of faces and for different numbers of labels, according to their SDF. Table 1 details the processing times for the different 3D meshes which are presented in the Fig. 2, the first row are original 3D mesh models, and the second row are the result of our 3D mesh segmentation algorithm.

The segmentation results show that our algorithm instanced from the proposed framework can achieve visual meaningful segmentation. The main reason is that the proposed framework provide a unified channel for the features of triangles to integrate the perceptual principles into boundary of each component.

**Different control parameters for mesh segmentation:**

The segmentation results depend on the control parameter  $\beta$ . Here, we select different parameters to validate the effectiveness of the mesh segmentation. Figure 4 shows the experimental results with different control parameters.

Obviously, different parameters  $\beta$  can achieve different visual meaningful segmentation results. More details with the experimental results with different control parameters. More components depend on minor  $\beta$  and some components will be merged with other neighbor sub-parts when the initial segmentation number is given. The final segmentation number might be less than the initially given number if a larger control parameter  $\beta$  is configured. The second column, the third column and the last column illustrate the segmentation phenomena.

**Comparison with k-means clustering based segmentation:** Based on MRF and the two level distributions, the proposed framework provides a bridge for the features of mesh and the boundary of segmentation components. It is a clustering based framework. Therefore, We also compare our algorithm instanced from MRF clustering based framework with the method based on K-Means clustering. Figure 3 illustrates the clustering of the Blade model (40K vertices) according to its SDF, where we set  $K = 3$ . Obviously, the noise introduced by the K-Means classification has been almost entirely removed by using our 3D mesh segmentation algorithm based on MRF clustering. Figure 3 also shows that our algorithm achieves visual meaningful segmentation.

### CONCLUSION AND RECOMMENDATION

This study presents a definition of 3D mesh segmentation and a new 3D mesh segmentation framework based on MRF clustering. The framework provides a unified segmentation for 3D polygonal mesh in a global optimization process. Both feature values of items of 3D object and spatial constraints in the labeling process are integrated by using a two-level Gibbs Random Field to model the geometric and topology information of 3D mesh. Experimental results demonstrate the efficiency and validity of our scheme. Experiment results show that our proposed framework can achieve visual meaningful segmentation if some perceptual features and perceptual rules are used to instance the framework. We will use different features of 3D mesh for  $S = F$ ,  $S = V$  and  $S = E$  in the future to validate and demonstrate the universality of the framework. Different methods for selecting cluster criterion parameters, defining the feature of segmentation boundary and solving the optimal problem will also be developed to further demonstrate the validity of our MRF clustering based framework.

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