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Perspective of the Comparison between Fuzzy Dates Based on Geographical Information Systems

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Abstract: Many methods have been proposed for managing of fuzzy numbers. How ever, these methods just can apply to compare some types of fuzzy numbers and other comparing cases can just rank by their graph, intuitively. Therefore, using proper methods is important in proper conditions constructing ranking indexes since the centroid of fuzzy numbers is an important case. In this study, we present a modified method for ordering fuzzy numbers. Some applications, which constitute a step in the evaluation of the evolution of Reims during the domination of the Roman Empire, illustrate the use of the anteriority index.

Key words: Centroid point, distance method, fuzzy risk analysis, ranking methods, standard deviations

INTRODUCTION

In many applications, ranking of fuzzy numbers is an important component of the decision process. In addition to a fuzzy environment, ranking is a very important decision making procedure. More than 20 fuzzy ranking indices have been proposed since 1976. Various techniques are applied to compare the fuzzy numbers. Some of these ranking methods have been compared and reviewed by (Bortlan and Degani, 1985). (Chen and Hwang, 1972) thoroughly reviewed the existing approaches and pointed out some illogical conditions that arose among them. Among the existing ranking methods, centroid index methods are extensively studied and applied to many decision making problems. Recently, Saneifard (Saneifard *et al*., 2007) pointed out the drawback of the existing centroid index ranking method and proposed a new centroid index method for ranking fuzzy numbers based on the Center of Gravity (COG) point. They applied the COG based ranking method to a human selection problem based on Fuzzy Number Indicate Order Weighted Averaging operator (Saneifard and Asgari, 2011). However, the COG based ranking method presented by Chen and Hwang (1972). (Wang and Yang, 2006) stated that the results of Saneifard (Saneifard *et al*., 2007) and Chu (Chu and Tsao, 2002) were lack of accuracy. From (Murakami *et al.*, 1983; Chu and Tsao, 2002; Yager and Filev, 1993), one can see that some centroid index ranking methods have been proposed for ranking fuzzy numbers. There are some drawbacks in the existing centroid-index ranking methods, i.e., they can not rank correctly fuzzy numbers in some situations (Saneifard and Rasoul, 2011a, b).Thus, in this paper, we present a modified method for ranking generalized trapezoidal fuzzy numbers. The proposed method considers the centroid points and the standard deviations of generalized trapezoidal fuzzy numbers to deal with fuzzy-number ranking problems. We also use an example to compare the proposed method with the existing centroid-index ranking methods. The proposed method can overcome the drawbacks of the existing centroid-index ranking methods.

PRELIMINARIES

The basic definitions of a fuzzy number are given in (Heilpern, 1992; Kauffman and Gupta, 1991) as follow:

Definition 1: Let U be a set of objects (the universe) and define a mapping on U:

$$\tilde{A}: U \to [0,1], u \mapsto A(u)$$

Then $\tilde{A}$ is called a fuzzy set on U,A(u) is called the membership function of $\tilde{A}$ (or called the measure that u belongs to $\tilde{A}$). for $\forall \lambda \in [0,1]$, $\tilde{A}_\lambda = \{u|u \in U, A(u) \geq \lambda\}$ is called a λ cut of $\tilde{A}$; Supp $\tilde{A} = \{u|u \in U, A(u)>0\}$ and Ker $\tilde{A} = \{u|u \in U, A(u) = 1\}$ are called the support and the kernel of the fuzzy set $\tilde{A}$, respectively. If Ker $\tilde{A} \neq \emptyset$, then $\tilde{A}$ is called normal fuzzy set.

Definition 2: A real fuzzy number $\tilde{A}$ is fuzzy subset on the real axis R, the membership function $\mu_{\tilde{A}}(x)$ contains the following properties:

- $\mu_{\tilde{A}}(x)$ is a continuous mapping from R to the closed interval $[0,\omega], 0 \leq x\ \omega \leq 1$.
- $\mu_{\tilde{A}}(x)$ for $\forall x \in (-\infty, \alpha_1]$.
- $\mu_{\tilde{A}}(x)$ is strictly increasing on $(\alpha_1,\alpha_2]$.
- $\mu_{\tilde{A}}(x)$, for $\forall x \in (\alpha_2,\alpha_3]$ where ω is a constant and

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$0 \le \omega \le 1$.

- $\mu_{\tilde{A}}$ (x), \is strictly decreasing on (α_3, α_4].
- $\mu_{\tilde{A}}$ (x) = 0, for $\forall$ x $\in$(α_1,+∞).

where α_1, α_2, α_3 ,α_4 are real numbers. We may let α_1 = +∞, , or $\alpha_1 = \alpha_2$, or $\alpha_2 = \alpha_3$, or $\alpha_3 = \alpha_4$, or α_4 = +∞. A is a fuzzy number, if $\forall \lambda \in [0,1]$ $\tilde{A}_\lambda$ is bounded, then Ã is called a bounded fuzzy number; If Supp Ã is a positive real number set, then Ã is called a positive fuzzy number; if Supp Ã is a negative real number set, then Ã is called a negative fuzzy number. All the fuzzy numbers can be denoted simply as F(R).

Traditional centroid methods: The traditional centroid method is very useful to deal with defuzzy function problems (Wierman, 1997; Yager and Filev, 1993) and fuzzy ranking problems (Yager, 1982). The formulas for calculating the centroid ($x^*_{\tilde{A}}$, $y^*_{\tilde{A}}$) of a fuzzy number Ã is shown as follow (Murakami *et al*., 1983):

$$x^*_{\tilde{A}} = \frac{\int_0^1 x\mu_{\tilde{A}}(x)dx}{\int_0^1 \mu_{\tilde{A}}(x)dx} \tag{1}$$

$$y^*_{\tilde{A}} = \frac{\int_0^1 x\mu_{\tilde{A}}(x)d\mu_{\tilde{A}}(x)}{\int_0^1 x\mu_{\tilde{A}}(x)dx} \tag{2}$$

where $\mu_{\tilde{A}}$ is the membership function of the fuzzy number Ã, $\mu_{\tilde{A}}$ (x) indicates the membership value of the element x in Ã and $\mu_{\tilde{A}}$ (x) $\in$[0, 1].

A simple center of gravity method (SCGM): In Saneifard *et al*. (2007) and Chen and Hwang (1972). presented a method, called the Simple Center-of-Gravity Method (SCGM), to calculate the centroid point of a generalized trapezoidal fuzzy number based on the concept of the medium curve (Subasic and Hirota, 1998). Let Ã be a generalized trapezoidal fuzzy number, where Ã = (a_1, a_2, a_3, a_4; $w_{\tilde{A}}$) The SCGM method for calculating the centroid point ($\hat{x}_{\tilde{A}}$, $\hat{y}_{\tilde{A}}$) of the generalized trapezoidal fuzzy number Ã, is as follows:

$$\hat{y}_{\tilde{A}} = \begin{cases} \dfrac{w_{\tilde{A}} \times (\dfrac{\alpha_3 - \alpha_2}{\alpha_4 - \alpha_1})}{6}, & if \alpha_1 \ne \alpha_4 \; and \; 0 < w_A \le 1 \\ \dfrac{w_{\tilde{A}}}{2}, & if \alpha_1 \ne \alpha_4 \; and \; 0 < w_A \le 1 \end{cases} \tag{3}$$

$$\hat{x}_{\tilde{A}} = \frac{\hat{y}_{\tilde{A}}(a_3 + a_2) + (a_4 + a_1)(w_{\tilde{A}} - \hat{y}_{\tilde{A}})}{2w_{\tilde{A}}} \tag{4}$$

Some methods of ranking fuzzy numbers based centroid: In this section, we briefly review four existing centroid-index ranking methods from others method. In (Yager, 1982), Yager presented a centroid-index ranking method which calculates the value $x^*_{\tilde{A}}$ of a fuzzy number Ã as follows:

$$x^*_{\tilde{A}} = \frac{\int_0^1 w(x)\mu_{\tilde{A}}(x)dx}{\int_0^1 \mu_{\tilde{A}}(x)dx(x)} \tag{5}$$

where w is a weight-function, w:X→[0,1] and w(x) denotes the grade of importance of the value x, where x--X. The larger value of $x^*_{\tilde{A}}$, the better ranking of Ã .When w(x) = x, the value $x^*_{\tilde{A}}$ shown in (5) becomes the traditional centroid of (1). In [10], Murakami *et al*. (1983) presented a centroid-index ranking method for ranking fuzzy numbers. The centroid point of a fuzzy number Ã is ($x^*_{\tilde{A}}$, $y^*_{\tilde{A}}$) where $x^*_{\tilde{A}}$ the same as is (1) and $y^*_{\tilde{A}}$ is the same as (2). The larger value of $x^*_{\tilde{A}}$ and (or) $y^*_{\tilde{A}}$ of a fuzzy number, the better ranking of the fuzzy number Ã. Note that there are n fuzzy numbers $\tilde{A}_1$, $\tilde{A}_2$, … , and $\tilde{A}_n$ to be ranked based on ($x^*_{\tilde{A}i}$, $y^*_{\tilde{A}i}$), where 1 ≤ I ≤ n The fuzzy number Ã Ã Ãk is said to be optimal if $x^*_{\tilde{A}k}$ max [$x^*_{\tilde{A}k}$] or $y^*_{\tilde{A}k}$= max [$y^*_{\tilde{A}k}$]. In (Saneifard *et al*., 2007), Saneifard presented a centroid-index ranking method for ranking fuzzy numbers. Assume that the trapezoidal fuzzy number Ã = (α_1, α_2, α_3, α_4;1) with the membership function $f_{\tilde{A}}$ is as follows:

$$f_{\tilde{A}}(x) = \begin{cases} \dfrac{x - \alpha_1}{\alpha_2 - \alpha_1}, & \alpha_1 \le x \le \alpha_2 \\ 1, & \alpha_2 \le x \le \alpha_3 \\ \dfrac{x - \alpha_4}{\alpha_3 - \alpha_4} & \alpha \le x \le \alpha_4 \\ 0 & otherwise \end{cases} \tag{6}$$

where $f_{\tilde{A}}$; [α_1, α_2]→[0,1] is continuous and strictly increasing; $f^R_{\tilde{A}}$; [α_3, α_4]→[0,1] is continuous and strictly decreasing:

$$f^L_{\tilde{A}}(x) = \frac{x - \alpha_1}{\alpha_2 - \alpha_1} \tag{7}$$

and

$$f^R_{\tilde{A}}(x) = \frac{x - \alpha_4}{\alpha_3 - \alpha_4} \quad . \tag{8}$$

The inverse functions $g^L_{\tilde{A}}$ and $g^R_{\tilde{A}}$ of $g^l_{\tilde{A}}$ and $F^R_{\tilde{A}}$, respectively are shown as follows:

$$g^L_{\tilde{A}}(y) = a_1 + (a_2 - a_1)y \tag{9}$$

$$g_{\tilde{A}}^{R}(y) = \alpha_4 + (\alpha_3 - \alpha_4)y \tag{10}$$

In (Saneifard *et al*., 2007), Saneifard transforms formulas (1) and (2) into the follows formulas:

$$\bar{x}_{\tilde{A}} = \frac{\int_{\alpha_1}^{\alpha_2}(xf_{\tilde{A}}^{L})dx + \int_{\alpha_2}^{\alpha_3}xda + \int_{\alpha_3}^{\alpha_4}(xf_{\tilde{A}}^{R})dx}{\int_{\alpha_1}^{\alpha_2}(xf_{\tilde{A}}^{L})dx + \int_{\alpha_3}^{\alpha_2}dx + \int_{\alpha_3}^{\alpha_4}(xf_{\tilde{A}}^{R})dx} \tag{11}$$

$$\bar{y}_{\tilde{A}} = \frac{\int_0^1(yg_{\tilde{A}}^{L})dy + \int_0^1(yg_{\tilde{A}}^{R})dy}{\int_0^1(g_{\tilde{A}}^{L})dy + \int_0^1(g_{\tilde{A}}^{R})dy} \tag{12}$$

The ranking value R(Ã) of the fuzzy number Ã is defined as:

$$R(\tilde{A}) = \sqrt{\bar{x}_{\tilde{A}}^2 + \bar{y}_{\tilde{A}}^2} \quad . \tag{13}$$

The larger the value of R(Ã) the better the ranking of Ã.

A new ranking method for generalized trapezoidal fuzzy numbers: In this section, we present a new approach for ranking generalized trapezoidal fuzzy numbers based on the distance method. The new ranking method not only considers the centroid point of a generalized trapezoidal fuzzy number, but also the standard deviation of a generalized trapezoidal fuzzy number. Assume that there are n generalized trapezoidal fuzzy numbers, $\tilde{A}_1$, $\tilde{A}_2$, … , and $\tilde{A}_n$ where $\tilde{A}_j = (\alpha_{1j}, \alpha_{2j}, \alpha_{3j}, \alpha_{4j}; w_{\tilde{A}i})$.$1 \le j \le n, 0 < w_{\tilde{A}i} \le 1, 0 \le \alpha_{1j}, \le \alpha_{2j}, \le \alpha_{3j}, \le \alpha_{4j} \le k$) and k is any real value, the value $w_{\tilde{A}i}$ denotes the maximum membership value of the generalized trapezoidal fuzzy number $\tilde{A}_j$, where $1 \le j \le n$. The proposed method for ranking generalized trapezoidal fuzzy numbers $\tilde{A}_1$, $\tilde{A}_2$, … , and $\tilde{A}_n$ is now presented as follows:

Step 1: For each generalized trapezoidal fuzzy number $\tilde{A}_j$ is not a standardized generalized trapezoidal fuzzy number, where the span of discourse of the generalized trapezoidal fuzzy number $\tilde{A}_j$ is [0,k], then form of the generalized trapezoidal fuzzy number $\tilde{A}^*_j$ is shown as follows:

$$\begin{aligned}\tilde{A}^*_j &= \left(\frac{\alpha_{1j}}{k}, \frac{\alpha_{2j}}{k}, \frac{\alpha_{3j}}{k}, \frac{\alpha_{4j}}{k}; w_{\tilde{A}^*_j}\right) \\ &= (\alpha^*_{1j}, \alpha^*_{2j}, \alpha_{*3}j, \alpha^*_{4j}, w_{\tilde{A}^*_j})\end{aligned} \tag{14}$$

where $w_{\tilde{A}^*j} = w_{\tilde{A}j}$, $0 \le \alpha^*_{1j} \le \alpha^*_{2j} \le \alpha^*_{3j} \le \alpha^*_{4j} \le 1$ or $-1 \le \alpha^*_{1j} \le \alpha^*_{2j} \le \alpha^*_{3j} \le \alpha^*_{4j} \le 1$, and $1 \le j \le n$

Step 2: Use (3) and (4) to calculate the centroid point ($\hat{x}_{\tilde{A}^*j}$, $\hat{y}_{\tilde{A}^*j}$) of each standardized generalized trapezoidal fuzzy number $\tilde{A}^*_j$ so that $1 \le j \le n$

Step 3: Calculate the standard deviation $S^*_{\tilde{A}j}$ of each standardized generalized trapezoidal fuzzy number $\tilde{A}^*_j$ as follow:

$$sA^*_j = \sqrt{\frac{\sum_{i=1}^{4}(a^*_{ij} - \bar{a}_j)^2}{4-1}} = \sqrt{\frac{\sum_{i=1}^{4}(a^*_{ij} - \bar{a}_j)^2}{3}} \tag{15}$$

where $\bar{\alpha}_j$ denotes the mean value of $a^*_{1j} \le a^*_{2j} \le a^*_{3j}$ and $a^*_{4,j}$, $\bar{\alpha}_j = (\alpha^*_{1j} \le \alpha^*_{2j} \le \alpha^*_{3j} \le \alpha^*_{4j})/4$, as $1 \le j \le n$. The standard deviation $S^*_{\tilde{A}j}$ indicates the degree of dispersion of the standardized generalized trapezoidal fuzzy number $\tilde{A}^*_j$as $1 \le j \le n$.

Step 4: Use the standard deviation $S^*_{\tilde{A}j}$ and the value $\hat{y}^*_{\tilde{A}j}$ of the centroid point ($\hat{x}_{\tilde{A}j}$, $\hat{y}_{\tilde{A}j}$) to derive a new value $\hat{y}\, s^*_{\tilde{A}j}$ shown as follow:

$$\hat{y}s^*_{\tilde{A}_j} = \frac{w_{\tilde{A}^*_j}}{2} - \hat{y}_{\tilde{A}^*_j} \times s_{\tilde{A}^*_j} \tag{16}$$

where $0 < \hat{y}s_{\tilde{A}^*j} \le 0.5$ and $1 \le j \le n$ prepared to obtain a new point ($\hat{x}_{\tilde{A}^*j}$, $\hat{y}_{\tilde{A}^*j}$).

Step 5: Use the new point ($\hat{x}_{\tilde{A}^*j}$, $\hat{y}s_{\tilde{A}^*j}$) to calculate the ranking value Score ($\tilde{A}^*_j$) of the standardized generalized trapezoidal fuzzy numbers $\tilde{A}^j_j$ where $1 \le j \le n$, as follows:

$$\begin{aligned}Rank(\tilde{A}_j) &= \sqrt{(\hat{x}_{\tilde{A}^*_j} - L)^2 + (\hat{y}^s_{\tilde{A}^*_j} - 0)^2} \\ &= \sqrt{(\hat{x}_{\tilde{A}^*_j} - L)^2 + (\hat{y}^s_{\tilde{A}^*_j})^2}\end{aligned} \tag{17}$$

where $L = \min [a^*_{ij}]_{i=1,\dots,4\ j=1,2,\dots,n}$ denotes the minimum value of the values $a^*_{ij\ i=1,\dots,4\ j=1,2,\dots,n}$. From (18), we can gain that $Rank(\tilde{A}^*_j)$ can be considered as the Euclidean distance between the point ($\hat{x}_{\tilde{A}^*j}$, $\hat{y}_{\tilde{A}^*j}$) and the point $(\min[\alpha^*_{ij}]_{i=1,\dots,4\ j=1,2,\dots,n}, 0)$. In (18), we assume that the value $\alpha^*_{ij\ i=1,\dots,4\ j=1,2,\dots,n}$ is the smallest value among the values $\alpha^*_{ij\ i=1,\dots,4\ j=1,2,\dots,n}$. From (18), we can see that the larger value of $Rank(\tilde{A}^*_j)$, the better ranking of $\tilde{A}^*_j$, as $1 \le j \le n$ In the following, we use an example to illustrate the ranking process of generalized trapezoidal fuzzy numbers.

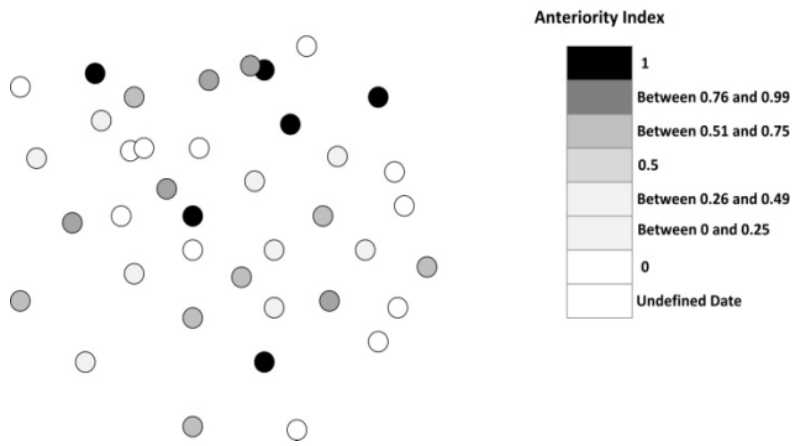

Fig. 1: Map of "BDRues" objects according to the anteriority of a reference date to "BDRues" object activity periods-the referenc date D_{ref} is a triangular fuzzy number

An Application: In the following application, we use a geodatabase of street excavations to estimate the possibility of a street to have been in activity after a given date. In the SIGRem project, information about Roman street excavations is stored in the ''BDRues'' geodatabase with a fuzzy representation of their dates. Our goal is to obtain a visualization of the comparison between these fuzzy dates and a given fuzzy date. For example, archaeologists and historians want to know which streets were built after the first century. This application uses a GIS software to localize excavations, and assigns a color to each excavation (Fig. 1). According to a reference date D_{ref}, the query is ''How anterior is D_{ref}, to the objects from the database?''. Thus, for each excavation e_i in the database, the anteriority between D_{ref}, and the excavation activity period e_id: Rank(D_{ref}) , Rank(e_id) are computed and we assign it a color in accordance with this value. The example of the map shown on Fig. 1 allows us to visualize the query results obtained by using the triangular fuzzy number (0, 200, 400) for D_{ref}. Those values could be interpreted as qualities of anteriority. In this map, an object is white if its date is not defined. Experts use this kind of visualization mode to select data in their diagnosis processes. During their prospection, experts evaluate which kind of objects they expect to find in a specific site. This application helps archaeologists to determine the evolution of the city during the Roman period. The anteriority index will guide archaeologists in their expertise processes.

CONCLUSION

In this study, we proposed a new centroid index method for evaluating fuzzy datum. First we briefly introduce some existing centroid index ranking methods of fuzzy numbers. The proposed method considers the centroid points and minimum crisp value of fuzzy numbers. It can overcome the drawback of the previous centroid methods. In this study, we apply our approach to one case: we use our approach to know which streets were present after a given date. This application allows us to evaluate the evolution of the city during the domination of the Roman Empire.

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