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Bayesian Double Sampling Plan under Gamma-Poisson Distribution

[1]Saminathan Balamurali, [2]Muhammad Aslam and [3]Chi-Hyuck Jun
[1]Departments of Mathematics, Kalasalingam University, Krishnankoil 626190, TN, India
[2]Department of Statistics, Forman Christian College University, Lahore, Pakistan
[3]Department of Industrial and Management Engineering, POSTECH,
Pohang 790-784, Republic of Korea

Abstract: The present study proposes a Bayesian double sampling plan for the inspection of attribute quality characteristics under the gamma-Poisson distribution. The design parameters of the proposed plan such as the sample sizes and the acceptance numbers are determined for specified two points on operating characteristics curve such as acceptable quality level and limiting quality level along with the corresponding producer's and the consumer's risks. The optimal parameters are determined using the minimum average sample number criteria. Extensive tables are provided for selection of parameters of the proposed plan for selected combinations of two quality levels and the results are explained with examples.

Key words: Acceptable quality level, double sampling, gamma-Poisson distribution, limiting quality level, producer's risk and consumer's risk

INTRODUCTION

Acceptance sampling is one of the major areas of statistical quality control. Inspection of the raw material and final product is very necessary to ensure the good quality. It is well known that the acceptance sampling plans are used to reduce the cost of inspection. One of the applications of sampling plans in that situation where the product under inspection is destructive or the cost of the life test or inspection of complete product is very high or the inspection error is too high and the product liability risks are serious.

Attributes sampling constitutes one of the major areas of acceptance sampling sampling. There are several sampling plans available in the literature for the inspection attribute quality characteristics namely single sampling plan, double sampling plan, multiple sampling plan, chain sampling plan etc. Single sampling plan is one of the simplest attributes sampling plans which involves two parameters namely the sample size n and the acceptance number c. Double sampling plan can be used to minimize the producer's risk. In double sampling if the results of the first sample are not definitive in leading to acceptance or rejection of a lot, a second sample is taken which then leads to a decision on the disposition of the lot. This approach makes sense not only as a result of experience, but also in the mathematical properties of the procedure. It is also to be noted that the average sample of the double sampling plan will be lesser than the single sampling plan.

In all the attributes sampling plans, the basic assumption is that the lot or process fraction defective is constant, which indicates that the production process is stable. However, in practical situations, the lots formed from a process will have quality variations which are due to random fluctuations. These variations are classified into two, one is within-lot variation and the other is between-lot variation. When the second type variation is more than the first type variation, the fraction defective items in the lots will vary continuously. In such cases, the decision on the submitted lots should be made with the consideration of the second type variation so that the conventional sampling plans cannot be applied. It is very important to note that acceptance sampling plans based on Bayesian methodology can be used when the experimenter has the prior knowledge on the process variation to make a decision about the disposition of the lot. For further details about the prior distribution of the lot fraction defective, one can refer Hald (1981) and Calvin (1990). According to Vijayaraghavan *et al*. (2008) in the Bayesian theory, it is found that the gamma distribution is a natural conjugate prior for the sampling from a Poisson distribution. When the sample items are drawn randomly from a process, the number of defects in the sample is distributed according to Poisson law, and the gamma distribution is the conjugate prior to the average number of defects per item. Under these situations, Hald (1981) derived Operating Characteristic (OC) function of the single sampling plan based on gammaPoisson distribution. Vijayaraghavan *et al*. (2008) developed the tables for the selection of parameters of single sampling plan using the gamma-Poisson distribution. In the literature, there is not double sampling plan is available for gamma-Poisson situation. So this study attempts to develop a Bayesian

Corresponding Author: Saminathan Balamurali, Departments of Mathematics, Kalasalingam University, Krishnankoil 626190, TN, India

double sampling plan under gamma-Poisson model. The optimal parameters of the proposed plan can be determined plan for gamma prior and Poisson distribution under the specified requirements.

DOUBLE SAMPLING PLAN UNDER GAMMA-POISSON DISTRIBUTION

As mentioned earlier, the double sampling plan is an extension of single sampling plan. The double sampling plan was investigated by many authors under various situations. Aslam *et al.* (2009) investigated the double sampling plan for reliability tests under Weibull model. Aslam and Jun (2010) developed tables for the selection of a double sampling plan under generalized log logistic distribution. Balamurali and Kalyanasundaram (1999) developed a conditional double sampling procedure based on some switching rules. The operating procedure of the double sampling plan is as follows.

Step 1 : Select a random sample of size n_1 from the lot of size N and observe the number of nonconforming items in the sample, say d_1. If $d_1 \le c_1$, then accept the lot. If $d_1 > c_2$, then reject the lot, If $c_1 < d_1 \le c_2$, then go for second sampling as per step (2).

Step 2: Select a random sample of size n_2 from the same lot and observe the number of nonconforming items in the sample, say d_2. If the cumulative number of nonconforming items $d_1 + d_2 \le c_2$, then accept the lot. Otherwise reject the lot.

Thus a double sampling plan is completely specified by the parameters n_1, n_2, c_1 and c_2.

Gamma-Poisson distribution, which is also known as the negative binomial distribution, is one of the most widely used models in reliability analysis and also in the acceptance sampling plans. Recently, Vijayaraghavan *et al*. (2008) developed the selection of single sampling plan using this distribution. When the production process produces output in a continuous stream and observed number of defects (or nonconformities) in the sample drawn from this process is distributed as Poisson with parameter np, where n is the sample size and p is the average number of defects per unit (Hald, 1981). According to Schilling (1982), the Poisson distribution is an appropriate model for the number of non-conforming items in the sample when the ratio of sample size to the population size (n/N) is less than 10%, n is large and p<0.10 is small such that np<5. Under the conditions of applications of Poisson model, the probability of acceptance of the double sampling plan is given by:

$$L(p) = \sum_{d_1=0}^{c_1} \frac{e^{-n_1 p} (n_1 p)^{d_1}}{d_1!} + \sum_{d_1=c_1+1}^{c_2} \frac{e^{-n_1 p} (n_1 p)^{d1}}{d_1!} \left[\sum_{d_2=0}^{c_2-d_1} \frac{e^{-n_2 p} (n_2 p)^{d_2}}{d_2!} \right] \qquad (1)$$

where c_1 and c_2 are the acceptance numbers of the double sampling plan. sampling from the Poisson distribution, the density function of prior distribution of p is given by (Vijayaraghavan When p varies from lot-to-lot at random and is distributed as gamma distribution which is the natural conjugate prior for *et al*., 2008):

$$f(p/a,m) = \frac{e^{-ap} a^m p^{m-1}}{\sqrt{m}}, \; 0 \le p < \infty, \; a,m > 0 \qquad (2)$$

where a is the scale parameter and m is the shape parameter. If $E(p) = \overline{p}$ is given then the scale parameter is obtained by $a = m/\overline{p}$. Here m is either specified or estimated using the Maximum Likelihood Estimate (MLE) using prior information about the production process. The posterior distribution of the number of nonconformities is reduced to the gamma-Poisson distribution. When the production is unstable, both the number of nonconforming items in the sample, d and the average number of defects p are independently distributed. So, according to Hald (1981), the sampling distribution of d, under the conditions that the process average $\overline{p} < 0.1$, $\frac{\overline{p}}{m} < 0.2$ is given by:

$$P(d; n\overline{p}, m) = \frac{(m+d-1)!}{d!(m-1\overline{p})!} \left(\frac{n\overline{p}}{m+n\overline{p}} \right)^d \left(\frac{m}{m+n\overline{p}} \right)^m, \; d = 0, 1, 2, \ldots \qquad (3)$$

Based on this, the probability of acceptance of the double sampling plan under gamma-Poisson model is given by:

$$P_a(\overline{p}) = \sum_{d_1=0}^{c_1} \frac{(m+d_1-1)}{d_1!(m-1)!} \left(\frac{n_1\overline{p}}{m+n_1\overline{p}} \right)^{d_1} \left(\frac{m}{m+n_1\overline{p}} \right)^m$$

$$+ \sum_{d_1=c_1+1}^{c_2} \frac{(m+d_1-1)!}{d_1!(m-1)!} \left(\frac{n_1\overline{p}}{m+n_1\overline{p}} \right)^{d_1} \left(\frac{m}{m+n_1\overline{p}} \right)^m \qquad (4)$$

$$\left[\sum_{d_2=0}^{c_2-d_1} \frac{(m+d_2-1)!}{d_2!(m-1)!} \left(\frac{n_2\overline{p}}{m+n_2} \right)^{d_2} \left(\frac{m}{m+n_2\overline{p}} \right)^m \right]$$

DESIGNING OF GAMMA-POISSON DOUBLE SAMPLING PLAN

The ultimate aim of a sampling plan is to minimize the producer's and consumer's risks. To design the double

Table 1: Optimal gamma-Poisson double sampling plan for given p_1, p_2, a = 5% and β = 10% for m = 5

p_1	p_2 0.05	0.06	0.07	0.08	0.09	0.10
0.005	74; 0. 2 (96.453)	62; 0.2 (80.688)	53; 0.2 (69.028)	40; 0.1 (46.579)	36; 0.1 (41.822)	32; 0.1 (37.263)
0.010	117; 0.5 (175.842)	85; 0.4 (123.937)	63; 0.3 (87.656)	55; 0.3 (76.585)	49; 0.3 (68.177)	37; 0.2 (48.227)
0.015	278; 0.15 (456.166)	151; 0.9 (239.931)	95; 0.6 (145.608)	74; 0.5 (110.699)	57; 0.4 (82.930)	51; 0.4 (74.362)
0.020	***	366; 0.25 (613.732)	175; 0.13 (285.305)	113; 0.9 (179.722)	83; 0.7 (128.998)	59; 0.5 (88.377)
0.025	***	***	532; 0.44 (906.268)	204; 0.18 (337.675)	128; 0.12 (207.099)	91; 0.9 (144.320)

***: Plan does not exist (Value given in bracket is the ASN of the plan at LQL)

Table 2: Optimal gamma-Poisson double sampling plan for given p_1, p_2, a = 5% and β = 10% for m =10

p_1	p_2 0.05	0.06	0.07	0.08	0.09	0.10
0.005	66; 0.2 (88.359)	55; 0.2 (73.633)	41; 0.1 (48.333)	36; 0.1 (42.406)	32; 0.1 (37.695)	29; 0.1 (34.109)
0.010	91; 0.4 (138.231)	65; 0.3 (93.957)	56; 0.3 (80.789)	41; 0.2 (54.999)	37; 0.2 (49.388)	33; 0.2 (44.180)
0.015	161; 0.9 (270.664)	99; 0.6 (159.136)	75; 0.5 (117.622)	57; 0.4 (86.506)	44; 0.3 (63.228)	39; 0.3 (56.374)
0.020	330; 0.21 (590.870)	158; 0.11 (270.264)	105; 0.8 (174.386)	75; 0.6 (120.013)	59; 0.5 (92.069)	46; 0.4 (69.55)
0.025	***	333; 0.26 (603.088)	166; 0.14 (289.173)	110; 0.10 (186.325)	74; 0.7 (121.005)	60; 0.6 (96.011)

***: Plan does not exist (Value given in bracket is the ASN of the plan at LQL)

sampling plan, we use two points on the OC curve approach. The plan parameters are determined satisfying the following inequalities:

$$P_a(p_1) \ge 1 - a \quad (5)$$

$$p_a(p_2) \le \beta \quad (6)$$

Here p_1 is the quality level corresponding to the producer's risk, which is called Acceptable Quality Level (AQL). On the other hand, p_2 is the quality level corresponding to the consumer's risk which is also called Limiting Quality Level (LQL). It is important to note that there may exist multiple solutions, so we may determine these parameters to minimize the Average Sample Number (ASN) at LQL. The ASN of the double sampling plan is given by:

$$ASN(p) = n_1 P_1 + (n_1 + n_2)(1 - p_1) \quad (7)$$

where P_1 is the probability that the decision was made by the first sample and based on gamma-Poisson model which is given by:

$$P_1 = 1 - \sum_{d_1 = c_1 + 1}^{c_2} \frac{(m + d_1 - 1)!}{d_1!(m-1)!} \left(\frac{n_1 p}{m + n_1 p} \right)^{d1} \left(\frac{m}{m + n_1 p} \right)^m$$

Hence to develop tables for designing an optimal double sampling plan, we use the following nonlinear programming problem:

Minimize: $ASN(p_2)$ (8a)

Subject to: $P_a(p_1) \ge 1 - a$ (8b)

$$p_a(p_2) \le \beta \quad (8c)$$

$$n_1 \ge 1; n_2 \ge 1; c_1 \ge 0; c_2 > c_1 \quad (8d)$$

The design parameters such as the sample sizes n_1, n_2 and the acceptance numbers c_1 and c_2 are determined for various values of p_1, p_2, a, β and m. To reduce the number of parameters, in this paper we consider n_2=n_1. The optimal parameters of the double sampling plan under gamma-Poisson model are tabulated in Table 1-6 for the shape parameters m=5, 10, 25, 50, 100 and 150, respectively.

In this study, we have assumed that the shape parameter of the gamma-Poisson distribution is known. The proposed plan can also be used for the situation where the shape parameter is unknown. Normally, producers keep the record of the estimated shape parameter value for their product or it can be estimated from the available data. From these tables we can observe interesting trends in the parameter values. For

Table 3: Optimal gamma-Poisson double sampling plan for given p_1, p_2, α = 5% and β = 10% for m = 25

	P_2					
P_1	0.05	0.06	0.07	0.08	0.09	0.10
0.005	62; 0.2 (84.396)	51; 0.2 (69.744)	38; 0.1 (45.294)	33; 0.1 (39.406)	30; 0.1 (35.629)	27; 0.1 (32.066)
0.010	85; 0.4 (132.976)	61; 0.3 (90.229)	52; 0.3 (77.090)	39; 0.2 (52.966)	34; 0.2 (46.496)	31; 0.2 (42.198)
0.015	124; 0.7 (210.900)	82; 0.5 (132.681)	61; 0.4 (95.232)	54; 0.4 (83.867)	41; 0.3 (60.441)	37; 0.3 (54.483)
0.020	201; 0.13 (366.954)	125; 0.9 (219.298)	80; 0.6 (132.668)	62; 0.5 (99.945)	48; 0.4 (74.549)	43; 0.4 (66.922)
0.025	389; 0.28 (750.254)	189; 0.15 (349.325)	117; 0.10 (207.239)	86; 0.8 (148.480)	62; 0.6 (102.990)	49; 0.5 (79.433)
0.03	***	324; 0.28	171; 0.16 (624.973)	110; 0.11 (317.824)	84; 0.9 (197.224)	62; 0.7 (146.837) (105.450)
0.035		***	***	278; 0.28 (536.100)	158; 0.17 (294.783)	105; 0.1282; 0.10 (189.884) (145.165)
0.04		***	***	***	243; 0.28 (468.730)	147; 0.18101; 0.13 (275.789) (184.019)
0.045		***	***	***	443; 0.54 (873.000)	216; 0.28139; 0.19 (416.649) (261.559)
0.05		***	***	***	*** (721.913)	367; 0.50201; 0.29 (388.161)

***: Plan does not exist (Value given in bracket is the ASN of the plan at LQL)

Table 4: Optimal gamma-Poisson double sampling plan for given p_1, p_2, α = 5% and β = 10% for m = 50

	p_2					
p_1	0.05	0.06	0.07	0.08	0.09	0.10
0.005	60; 0.2 (82.525)	50; 0.2 (68.771)	37; 0.1 (44.293)	32; 0.1 (38.419)	29; 0.1 (34.650)	26; 0.1 (31.095)
0.010	84; 0.4 (131.326)	60; 0.3 (89.324)	51; 0.3 (76.195)	38; 0.2 (52.014)	34; 0.2 (46.426)	30; 0.2 (41.263)
0.015	121; 0.7 (208.918)	80; 0.5 (131.148)	60; 0.4 (94.425)	45; 0.3 (66.993)	40; 0.3 (59.549)	36; 0.3 (53.594)
0.020	183; 0.12 (337.913)	112; 0.8 (196.379)	78; 0.6 (131.241)	60; 0.5 (98.361)	47; 0.4 (73.726)	42; 0.4 (66.097)
0.025	304; 0.22 (590.005)	163; 0.13 (303.281)	105; 0.9 (186.924)	76; 0.7 (130.924)	61; 0.6 (102.372)	48; 0.5 (78.689)
0.030	621; 0.49 (1236.403)	254; 0.22 (492.669)	149; 0.14 (278.885)	99; 0.10 (179.067)	75; 0.8 (131.237)	61; 0.7 (104.924)
0.035	***	460; 0.43 (913.999)	217; 0.22 (421.217)	138; 0.15 (259.973)	95; 0.11 (173.638)	73; 0.9 (130.344)
0.04	***	***	352; 0.38 (697.799)	190; 0.22 (368.753)	122;0.15 (230.247)	92; 0.12 (169.525)
0.045	***	***	***	286; 0.35 (565.832)	169; 0.22 (327.947)	116; 0.16 (220.066)
0.050	***	***	***	497; 0.64 (991.880)	248; 0.34 (490.193)	152; 0.22 (295.002)

***: Plan does not exist (Value given in bracket is the ASN of the plan at LQL)

the same values of p_1, p_2, α, β as the value of shape parameter increases, there is a drastic reduction in the sample size when compared to the single sampling plan based on gamma-Poisson distribution. Also when p_2 increases, the sample size decreases for other fixed values and at the same time, as p_1 increases, the sample size also increases.

Examples: Suppose that an experimenter wants to run an experiment to make a decision on a product whether to accept or reject it. Assuming that the life time of the product follows the gamma-Poisson distribution with shape parameter m =10. Suppose one wants to find the parameters of a double sampling scheme for specified AQL (p_1) = 0.015, LQL (p_2) = 0.07 with α = 5% and β = 10%. Then from Table 2, we can find the values of the parameters as $n_1 = n_2 = 75$, $c_1 = 0$ and c = 5 with ASN = 117.622. This double sampling plan is operated as follows:

Step 1: Select a random sample 75 from the lot and observe the number of nonconforming items. If there is no nonconforming item is observed, then accept the lot. If more than 5 nonconforming items are observed, then

Table 5: Optimal gamma-Poisson double sampling plan for given p_1, p_2, α = 5% and β = 10% for m = 100

p_1	p_2: 0.05	0.06	0.07	0.08	0.09	0.10
0.005	60; 0.2 (82.468)	50; 0.2 (68.723)	36; 0.1 (43.346)	32; 0.1 (38.377)	28; 0.1 (33.713)	26; 0.1 (31.059)
0.010	83; 0.4 (131.392)	59; 0.3 (88.453)	51; 0.3 (76.183)	37; 0.2 (51.108)	33; 0.2 (45.526)	30; 0.2 (41.234)
0.015	107; 0.6 (182.053)	79; 0.5 (130.408)	59; 0.4 (93.604)	44; 0.3 (66.124)	39; 0.3 (58.680)	36; 0.3 (53.581)
0.020	169; 0.11 (311.640)	110; 0.8 (194.982)	77; 0.6 (130.558)	59; 0.5 (97.583)	46; 0.4 (72.900)	42; 0.4 (66.121)
0.025	276; 0,20 (537.262)	151; 0.12 (281.137)	103; 0.9 (185.511)	75; 0.7 (130.284)	60; 0.6 (101.645)	48; 0.5 (78.770)
0.030	506; 0.40 (1008.665)	230; 0.20 (447.719)	138; 0.13 (259.147)	98; 0.10 (178.683)	74; 0.8 (130.643)	54; 0.6 (91.481)
0.035	***	374; 0.35 (744.011)	197; 0.20 (383.540)	128; 0.14 (242.295)	87; 0.10 (158.708)	66; 0.8 (116.989)
0.04	***	***	296; 0.32 (587.735)	172; 0.20 (335.019)	114; 0.14 (215.659)	85; 0.11 (156.384)
0.045	***	***	499; 0.57 (997.260)	252; 0.31 (499.945)	153; 0.20 (297.966)	109; 0.15 (207.281)
0.050	***	***	***	380; 0.49 (758.900)	211; 0.29 (417.866)	138; 0.20 (268.631)

***: Plan does not exis (Value given in bracket is the ASN of the plan at LQL)

Table 6: Optimal gamma-Poisson double sampling plan for given p_1, p_2, α =5% and β = 10% for m = 150

p_1	p_2: 0.05	0.06	0.07	0.08	0.09	0.10
0.005	59; 0.2 (81.581)	49; 0.2 (67.838)	36; 0.1 (43.331)	32; 0.1 (38.363)	28; 0.1 (33.702)	26; 0.1 (31.047)
0.010	82; 0.4 (130.550)	59; 0.3 (88.450)	42; 0.2 (58.147)	37; 0.2 (51.097)	33; 0.2 (45.516)	30; 0.2 (41.224)
0.015	107; 0.6 (182.166)	79; 0.5 (130.460)	59; 0.4 (93.621)	44; 0.3 (66.123)	39; 0.3 (58.680)	35; 0.3 (52.726)
0.020	168; 0,11 (310.964)	110; 0.8 (195.182)	77; 0.6 (130.636)	59; 0.5 (97.623)	46; 0.4 (72.912)	41; 0.4 (65.275)
0.025	262; 0.190 (510.203)	150; 0.12 (280.394)	103; 0.9 (185.735)	75; 0.7 (130.390)	60; 0.6 (101.705)	48; 0.5 (78.797)
0.030	479; 0.38 (955.364)	229; 0.20 (446.985)	137; 0.13 (258.339)	98; 0.10 (178.927)	73; 0.8 (129.789)	54; 0.6 (91.535)
0.035	***	362; 0.34 (720.804)	196; 0.20 (382.682)	120; 0.13 (226.204)	87; 0.10 (158.925)	66; 0.8 (117.109)
0.04	***	***	286; 0.31 (568.422)	164; 0.19 (319.262)	113; 0.14 (214.738)	84; 0.11 (155.482)
0.045	***	***	463; 0.53 (925.475)	236; 0.29 (468.267)	153; 0.20 (298.511)	108; 0.15 (206.323)
0.050	***	***	***	349; 0.45 (697.093)	204; 0.28 (404.256)	131; 0.19 (255.101)

***: Plan does not exist (Value given in bracket is the ASN of the plan at LQL)

immediately reject the lot. If the number of nonconforming items is in between 0 and 6 (i.e., 1 to 5) then go for second sampling as per step (2).

Step 2: Select another random sample of size 75 from the same lot and observe the number of nonconforming items. If the cumulative number nonconforming tems from the first and second sample is less than or equal to 5 then the lot is accepted, otherwise the lot is rejected.

Comparison: In this section, we will compare the results of the proposed plan with the gamma-Poisson single sampling plan of Vijayaraghavan *et al*. (2008) by considering only two values of the shape parameter namely 5 and 150 for different combinations of AQL and LQL values. We have tabulated the sample sizes and the acceptance numbers along with the average sample number of the proposed plan as well as of the existing single sampling plan based on Gamma-Poisson distribution in Table 7. From this table, it is easily observed that the average sample number of the

Table 7: Average sample number of the proposed plan and gamma-Poisson single sampling plan for specified p_1, p_2, α = 5% and β -= 10%

		Sampling Plan with ASN at p_2			
		Gamma-Poisson (m = 5)		Gamma-Poisson (m = 150)	
p_1	p_2	SSP	DSP	SSP	DSP
0.005	0.06	124.2 (124)	62; 0.2 (80.687)	66.1 (66)	49; 0.2 (67.838)
0.005	0.08	66.1 (66)	40; 0.1 (46.579)	50.1 (50)	32; 0.1 (38.363)
0.010	0.08	119.3 (119)	55; 0.3 (76.585)	68.2 (68)	37; 0.2 (51.097)
0.010	0.09	106.3 (106)	49; 0.3 (68.177)	60.2 (60)	33; 0.2 (45.516)
0.015	0.09	176.6 (176)	57; 0.4 (82.930)	76.3 (76)	39; 0.3 (58.680)
0.015	0.10	138.5 (138)	51; 0.4 (74.362)	54.2 (54)	35; 0.3 (52.726)
0.020	0.09	451.18 (451)	83; 0.7(128.998)	91.4 (91)	46; 0.4 (72.912)
0.020	0.10	262.11 (262)	59; 0.5(88.377)	82.4 (82)	41; 0.4 (65.275)
0.025	0.09	***	128;0.12(207.099)	119.6 (119)	60;0.6(101.705)
0.025	0.10	***	91;0.9(144.320)	95.5 (95)	48; 0.5 (78.797)

***: Plan does not exist (Value given in bracket is the ASN of the plan at LQL)

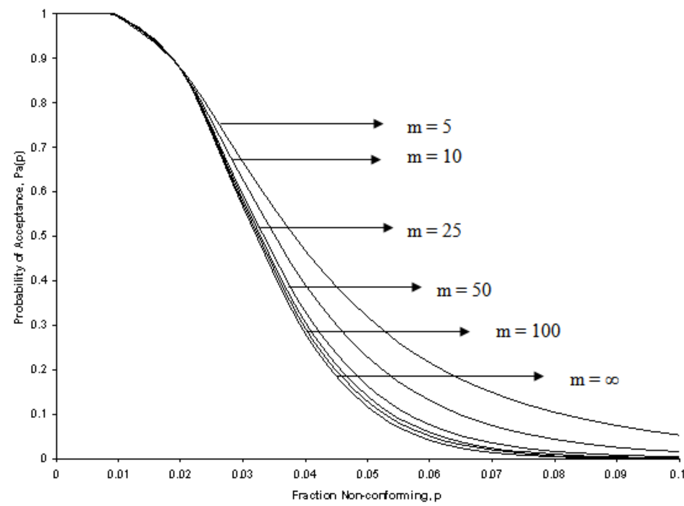

Fig. 1: Operating characteristic curves of Poisson and gamma-Poisson doublesampling plans with $n_1 = n_2 = 150$, $c_1 = 1$ and $c_2 = 3$ for different shape parameters

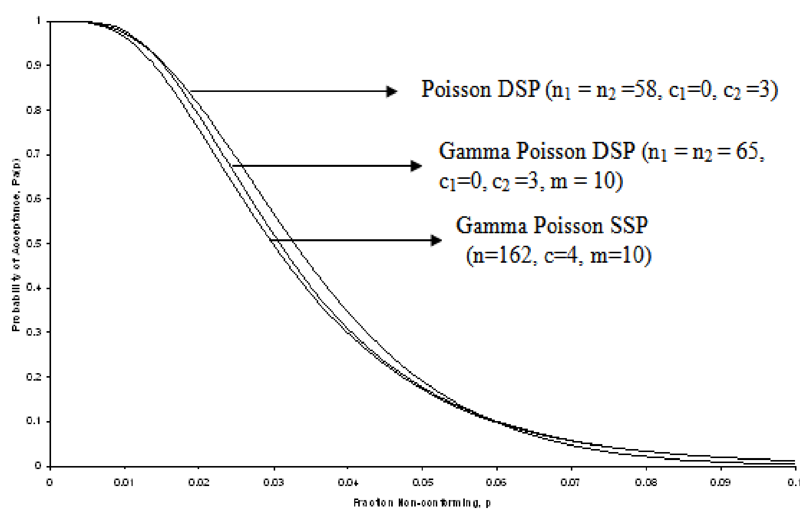

Fig. 2: Operating characteristic curves of Poisson double sampling plan, gamma-Poisson single sampling plan and gamma Poisson double sampling plans corresponding to $p_1 = 0.01$ (α= 0.05) and p_2 =0.06 (β= 0.10)

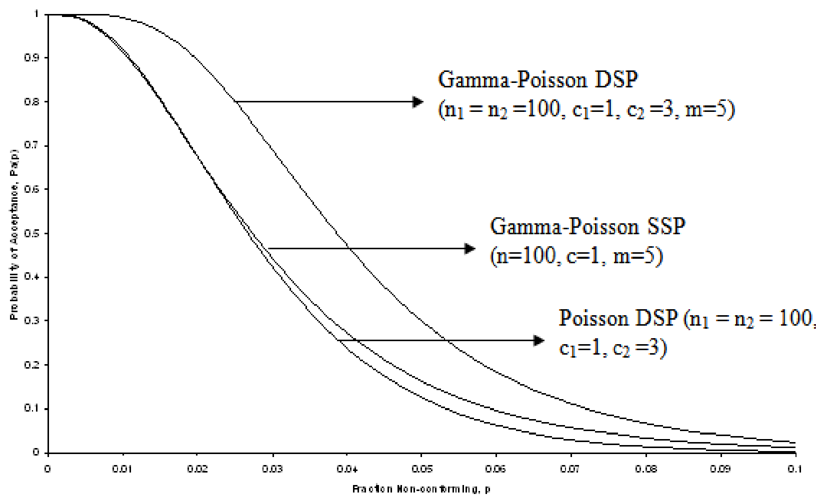

Fig. 3: Operating characteristic curves of Poisson double sampling plan, gamma-Poisson single sampling plan and gamma Poisson double plans with same sample size

proposed plan is lesser than the ASN of the gamma-Poisson single sampling plan for all the combinations of the AQL and LQL. Table 7 is around here.

For further comparison, three OC curves are presented. Figure 1 gives the OC curve of gamma-Poisson double sampling plan for different shape parameters. From this figure, it is understood that for small fraction non-conforming, almost all the curves have equal probability of acceptance but at high fraction non-conforming, the OC curve of m = 5 has more probability of acceptable but when the values of m increases the probability of acceptance reduces. Figure 2 provides the OC curves of gamma-Poisson double sampling plan along with the gamma Poisson single sampling plan and conventional Poisson double sampling plans, all having same AQL and LQL. From this figure, it can be observed that the OC curve of gamma-Poisson double sampling plan has desirable shape as a composite OC curve. For good quality, i.e. for smaller values of fraction nonconforming, the OC curve of the gamma-Poisson double sampling plan coincides with the OC curve of the conventional Poisson double sampling plan. As quality deteriorates, the OC curve of the gamma-Poisson double sampling plan moves toward that for the gamma-Poisson single sampling plan and comes close to it beyond the indifference quality level. It indicates that all the gamma-Poisson double sampling schemes, in general, protect the producer's interest against good quality levels and at the same time safeguard the consumer's interest against poor quality levels. In general, gamma-Poisson double sampling plan has more probability of acceptance than the other existing gamma-Poisson single sampling plan and the conventional double sampling plan for fixed sample size and this can be easily observed from Fig. 3.

CONCLUSION

In this study, we have developed a Bayesian double sampling plan under the gamma-Poisson distribution. The optimal design parameters of the proposed plan are determined using the two points on the OC curve approach. The proposed plan is better than the single sampling plan for the gamma-Poisson distribution in terms of minimum ASN. The proposed plan provides the lesser ASN than the existing sampling plans. So, the double sampling plan performs better than the conventional single, double sampling plan and gamma-Poisson single sampling plan and the proposed plan can be easily applied for the industrial use.

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