

The Effects of Van Hiele's Phase-Based Instruction Using the Geometer's Sketchpad (GSP) on Students' Levels of Geometric Thinking

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Abstract: The purpose of this study was to test the effectiveness of Van Hiele's phases of learning geometry using the Geometer's Sketchpad (GSP) on students' levels of geometric thinking. This quasi-experiment involved 94 students and two teachers. A total of 47 students were in the control group and the rest were in the treatment group. The students in the treatment group learned Form Two's Transformation topic through the Van Hiele's phases of learning using the GSP, while the students in the control group learned the same topic conventionally. Before the study started, students from both groups were given Van Hiele's Geometry Test (VHGT) to identify their initial levels of geometric thinking. The experiment took place for 6 weeks. At the end of the study, the students in both groups were given the VHGT for the second round to analyse their final levels of geometric thinking. Wilcoxon on-t test for the design of repeated measurement was used for the data analysis. The results found that the students in both groups showed increment in their post-VHGT as compared to the pre-VHGT. However, the students in the treatment group achieved better levels of geometric thinking compared to the students in control group ($t = 34.50$, $p < 0.05$). Thus, the Van Hiele's phases of learning geometry can be applied in classrooms in order to help students achieve better level of geometric thinking.

Keywords: Geometer's Sketchpad (GSP), students' levels of geometric thinking, Van Hiele's phases of learning geometry

INTRODUCTION

Thinking skills are the main goal of the Mathematics syllabus for secondary schools in Malaysia, which aim to produce students who are able to think mathematically and to apply mathematical knowledge effectively in solving problem and making decision (Curriculum Development Centre, 2002). In geometry, students' geometric thinking level is best described by Van Hiele's model (Battista, 2002; Noraini, 2007). There are five levels in the model: visualisation, analysis, informal deduction, deduction and rigor. However, the practice in the geometry curriculum in the school nowadays does not encourage students' thinking processes. This is due to the fact that it focuses only on recognising and naming geometric shapes and learning to write symbols for simple geometrical concepts. Reports by Trends in International Mathematics and Science Study (TIMSS) in 1999, 2003 and 2007 for the Malaysian educational system showed that most students reported that half or more of the lessons are spent memorising formulas and procedures (Mullis *et al.*, 2000, 2004, 2008). This is in

line with a study by Kouba *et al.* (1988) that showed 80% of lower secondary students opined that geometry learning was based on rules and 50% assumed that they learned geometry only by memorising. These findings concerned Battista (2002) who stated that traditional methods in learning geometry topics focused only on the need for students to memorise the definitions and characteristics of shapes. The practices in the process of teaching and learning geometry today have caused students to think less and made it difficult for them to achieve the levels of geometric thinking Abdul and Mohini (2008).

A number of studies in Malaysia have shown that the level of geometric thinking of secondary school students in Malaysia is still low. Chong (2001) studied the level of geometric thinking of Form Two students after they learned the Circle topic using traditional methods. The research found that that most students were only able to achieve level 1 of Van Hiele's geometric thinking. Rafidah (2003) conducted a study to identify the level of geometric thinking of 268 students, who were in Form One, Form Three and Form Four at a secondary school. She found out that, overall,

the students' level of geometric thinking was low, which was at level 1. This achievement is not in accordance with the duration the students have taken in learning mathematics. Tay (2003) studied the level of geometric thinking in geometry topics among Form One students after they learned the topics using traditional methods. The study showed that most of the students were still at level 1 of Van Hiele's geometric thinking. Hong (2005) studied Van Hiele's geometric thinking level among Form Six science students and also assessed their achievements in writing geometric proofs. The samples of the study consisted of 39 male and 38 female upper Form Six students of a secondary school. His study found that most of the upper Form Six students were at level 2 of Van Hiele's geometric thinking with 70% of the total samples scoring less than level 3. Razananahidah (2006) conducted a qualitative study to identify the level of Van Hiele's geometric thinking of Form Two students based on their study and explanation in solving problems related to triangles and quadrilaterals. Out of the four study samples, she found that two of them achieved level 1 while the other two achieved level 2 of Van Hiele's geometric thinking. In addition, Kor (1995) stated that the Van Hiele's geometric thinking levels in the Malaysian textbooks need to be revised. He found that in the Quadrilateral subtopic in the Form Two Mathematics textbook, the definition and characteristics of the shape are presented in tabular form to ease memorisation. Learning through memorisation without understanding is considered not achieving the levels of Van Hiele's model. Therefore, the teaching of geometry should be done systematically to help students move from one level to another. Furthermore, the presence of various educational technologies can facilitate the process of teaching and learning geometry in the classroom.

VAN HIELE'S MODEL

Van Hiele's model emphasises students' levels of geometric thinking. It was inspired by Pierre Van Hiele and Dina Van Hiele-Geldolf from the University of Utrecht, The Netherlands. Van Hiele's model has become a subject of ongoing academic research in the field of geometry and has been applied in various researches in the field of geometry. Many researchers have recognised Van Hiele's levels of geometric thinking. Battista (2002) stated that students' thinking pattern towards two-dimensional geometry is clear and best described using Van Hiele's model.

Van Hiele's levels of geometric thinking: There are five levels of geometric thinking in Van Hiele's model:

- **Level 1 is visualisation:** At this level, students can identify the geometry figures based on the overall entity and not from their components or characteristics Crowley (1987).
- **Level 2 is analysis:** Whereby students can identify the characteristics and concepts of geometry Crowley (1987). For example, a square has sides of same length, parallel and all four angles are 90° .
- **Level 3 is informal deduction:** With students able to recognise the relationship between figures and derive the relationship they can relate their existing knowledge and develop arguments to come with correct generalisation Crowley (1987).
- **Level 4 is deduction:** Students understand the meaning and significance of deduction and role of postulates, theorems and proof. They can write proof with their own understanding Crowley (1987).
- **Level 5 is rigor:** Whereby students understand how to study in an axiom system Crowley (1987).

They can create more abstract deductions. However, lower secondary students normally only achieve level 3 of Van Hiele's model, which is informal deduction. The characteristics of Van Hiele's model include students having to go through the levels in the model sequentially, students going through all the levels without missing any levels and instructions being given at each level to ensure that learning take place. If instructions are given at a higher level of students' capability, they would have difficulty following the thinking process. Several previous studies showed a strong correlation between students' levels of geometric thinking and their geometry achievement (Usiskin, 1982; Frykholm, 1994; Mohammad A. Yazdani, 2007).

Phases of learning geometry: Liu (2005) stated that learning method affects how students learn geometry. In addition, according to Noraini (2005), an effective process of learning geometry is not the same as the teaching and learning process of other topics in mathematics such as numbers, algebra and probability. Thus, in order to provide a guideline for teachers to help students learn geometry, the Van Hiele model suggested phases in learning geometry. According to Serow (2008) geometric learning activities will be more structured in Van Hiele's learning phase framework. Van Hiele's phases give attention to concepts and students' learning will be easier and more systematic with appropriate guidance from teacher. Students and teachers have a chance to discuss certain concepts and this will gradually help students to enhance their language use to be more technical. These learning phases can assist in students' mental development through a shift from a level of Van Hiele geometric thinking to a higher level. According to Teppo (1991) students' advancement from one level to another is a result of learning activities that are organised into five learning phases that emphasise exploration, discussion and integration activities. NCTM (1989, 2000) emphasised that the Van Hiele model can be utilised to teach geometry effectively. NCTM stressed the

importance of learning as organised as that proposed in the Van Hiele model. Five phases in the learning process can help students to move from one thinking level to a higher thinking level.

The phases are information, guided orientation, explanation, free orientation and integration. In the first phase, which is information, teachers and students use the question and answer approach with regards to the objects learnt Crowley (1987). Teachers pose questions to the students while making the observation Noraini (2005). In the guided orientation phase, students learn geometry through exploration Noraini (2005). In this phase, students learn the topics deeper through teaching aids provided by the teachers. Students explore through the carefully planned activities so that the characteristics and attributes of a certain level of thinking is exposed gradually (Crowley, 1987; Serow, 2008). In the explanation phase, students' new knowledge is formed through past experience and knowledge. They explain and state their views about the geometry structure they observe. Students will explain their observation towards activities carried out before Crowley (1987). In the fourth phase, free orientation, students can solve complex tasks that involve many steps and that can be solved in various ways Crowley (1987). In the final phase, called integration, students review and summarise what they have learnt in order to make a novel overall view about network of objects and their relationship (Crowley, 1987; Serow, 2008).

Previous studies on the implementation of phases of geometry learning: Tay (2003) studied the effectiveness of the implementation of phases of geometry learning using manipulative materials to give students opportunities to explore and investigate the properties of geometric shapes. Shi-Pui and Ka-Luen (2009) also implemented phases of geometry learning using manipulative materials in the solid geometry topic. Liu (2005) studied the effectiveness of Van Hiele's phases of learning geometry in the Circle topic. He used worksheet to implement the phases. These studies (Liu, 2005; Shi-Pui and Ka-Luen, 2009; Tay, 2003) found that students in the treatment group who were exposed to Van Hiele's learning phases achieved a better Van Hiele's level of geometric thinking than the students in the control group who were exposed to the same learning topic but used traditional approaches. However, according to Tay (2003), dynamic geometry software can be used to replace manipulative materials to give students the opportunity to explore the concepts of geometry. Selecting the appropriate and suitable technology would help students develop the ability to understand concepts of mathematics better and faster Curriculum Development Centre (2002). One of them is dynamic geometry software that gives opportunity to students to explore geometry shapes intuitively and inductively Serow (2008).

Choi-Koh (2000) developed activities based on Van Hiele's phases of learning geometry using Geometer's Sketchpad (GSP) software. The activities were conducted by students with the assistance of GSP software and covered the topic of variety of the triangle. Serow (2008) also implemented a project that used the approach of these phases of learning by including the elements of technology to assist the process of teaching and learning geometry in Mathematics classes. Topics included in this study were space and geometry, which included the subtopics of classification, construction and identification of the properties of triangle and quadrilateral and proof of properties of the quadrilaterals. Chew (2009) conducted a research about the geometry of solids learning among Form One students in the learning environment based on Van Hiele's phases of geometric thinking using GSP software. The objectives of his study were to determine the initial Van Hiele's levels of Van Hiele's model on cubes and cuboids and how students' Van Hiele's level of Van Hiele's model changed after the teaching based on the phases by using GSP. He found that the students' initial Van Hiele's levels of geometric thinking varied between level 1 and 2. After the teaching based on the phases by using GSP, students' Van Hiele's levels of geometric thinking either increased or remained at the same level.

RESEARCH OBJECTIVE

Based on the literature studies discussed earlier, the researchers have developed activities based on Van Hiele's phases of learning geometry using Geometer's Sketchpad (GSP) software as the medium of implementation. The subtopics involved are the Concepts of Transformation (Translation, Reflection and Rotation) and Quadrilaterals. The activities were arranged based on Van Hiele's phases of learning geometry, which are information, guided orientation, exploitation, free orientation and integration, with assistance from GSP software in order to assist students to move from one level of geometric thinking to a more advanced level of geometric thinking, which is from information level to rigor level. Student must go through all phases to achieve each level of geometric thinking. That means students must go through the phases of information, guided orientation, exploitation, free orientation and integration to achieve one level and must go through the same phases of learning geometry to achieve the next higher level (Choi-Koh, 2000; Serow, 2008). According to Van Hiele (1986) and Halat and Peker (2008), lower secondary school students normally achieve up to third level, which is informal deduction. Therefore, the researchers developed activities based on the phases of learning geometry for two learning sessions for each subtopic. The framework of the activities developed is as shown in Fig. 1 (a) and

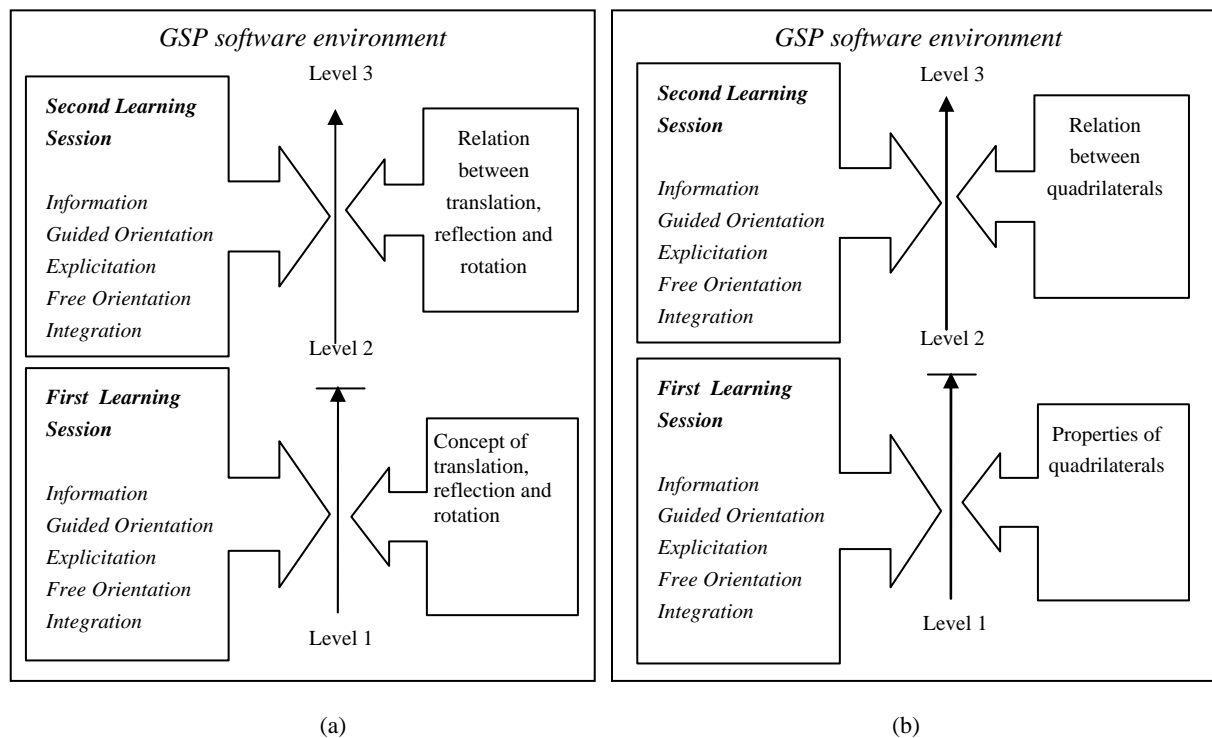


Fig. 1: (a) The framework of the activities developed for concepts of transformation, (b) the framework of the activities developed for quadrilaterals

(b). The activities developed were initially given to six experts comprising four content experts, two technical experts and two language experts. The researchers then tested the effectiveness of the activities developed on students' level of geometric thinking. These quantitative data were obtained from the test that will be explained in the research instrument part.

RESEARCH METHODOLOGY

Research sample: A total of 94 Form Two students were involved in this quasi-experiment study. They consisted of 47 students in the treatment group, who are those learning the Form Two's Transformation topic by implementing the activities developed based on Van Hiele's phases of learning geometry with the assistance of the GSP and 47 students in the control group who learned the same topic conventionally. The detailed information is shown in Table 1.

Research instrument: The students' levels of geometric thinking were measured using Van Hiele's Geometry Test (VHGT), which was developed by the Cognitive Development and Achievement in Secondary School Geometry (CDASSG) group from the University of Chicago (Usiskin, 1982). However, the Malay version of VHGT was obtained from Tay (2003). Table 2 concludes the distribution of questions contained in the VHGT.

Table 1: Sample profile

| | Group | |
|---|---------|-----------|
| | Control | Treatment |
| Gender | | |
| Male | 26 | 29 |
| Female | 21 | 18 |
| Mathematics grade obtained in the Primary School assessment test (UPSR) | | |
| A | 47 | 47 |
| B | | |

Table 2: Distribution of questions in VHGT

| Van Hiele's levels of geometric thinking | Question number |
|--|-----------------|
| Level 1: Visualisation | 1-5 |
| Level 2: Analysis | 6-10 |
| Level 3: Informal deduction | 11-15 |
| Level 4: Deduction | 16-20 |
| Level 5: Rigor | 21-25 |

Table 3: Marking criteria in VHGT

| Mark | Criteria of the items to be fulfilled | Van hiele's levels of geometric thinking |
|------|---------------------------------------|--|
| 1 | 1-5 | 1 |
| 2 | 6-10 | 2 |
| 4 | 11-15 | 3 |
| 8 | 16-20 | 4 |
| 16 | 21-25 | 5 |

The marking is as follows: A student was considered to have achieved a certain level in VHGT when he or she answered three out of five questions correctly. For example, as shown in Table 3, a student would be given one mark when he or she could answer at least three out of five questions correctly for questions 1-5, two marks for answering any three out of five questions from

Table 4: The weighted sum score for forced Van Hiele level

| Forced Van Hiele level | Weighted sum score |
|------------------------|------------------------------|
| 0 | 0, 16, 2, 4, 8, 18, 20 or 24 |
| 1 | 1, 17, 5, 9, 21 or 25 |
| 2 | 3, 19, 11, or 27 |
| 3 | 7, 23, 22 or 6 |
| 4 | 15, 31, 29, 13, 14 or 30 |
| No fit | 10, 12, 26 or 28 |

Usiskin (1982)

questions 6-10 correctly and so on. The student's total mark in the VHGT was then summed up in order to determine the level of Van Hiele's geometric thinking possessed by the student. Forced Van Hiele level table was used as a reference to determine the student's level of geometric thinking.

Based on Table 4, a high mark does not mean that the student's level of geometric thinking is high as well. This is because, based on the characteristics of Van Hiele's model, students must go through the levels in the model sequentially and they must go through all levels in this model without leaving any levels. For example, if the student can fulfil the criteria in level 1 and 2, he or she will get 3 marks (1+2). If the student meets the criteria in level 1, 2 and 4, he or she will get 11 marks (1+2+8). However, based on the table, the student only achieves up to level 2 because he or she fulfils the criteria in level 1 and 2 sequentially and skips the level 3 even though he or she fulfils the criteria in level 4.

Research procedure: For this study, permission was first obtained from the Educational Planning and Research Division, Ministry of Education (MOE) and the State Education Department (JPN). The letter produced by JPN was given to the principal of a school in Negeri Sembilan, Malaysia. Two teachers were selected and before the study was conducted, the teachers were briefed and taught by the researcher on how to use the GSP software and how to conduct the activities developed for two weeks. The briefing was held in the school's computer lab. The GSP software

was first installed in the computers to be used. Before the research started, the students in both the control and treatment groups were given VHGT to measure their initial levels of geometric thinking. Students in the treatment group were then briefed on the use of GSP software because, according to Nik (2008) the use of technology in the process of teaching and learning has a precondition, that students must have accessed the required technological tool, understand the tool and mastered the basics on how to use it. The study was performed in six weeks. Students in the treatment group learned the Form Two's Transformation topic by implementing the activities developed based on Van Hiele's phases of geometric learning with the assistance of the GSP software, while students in the control group were those who learned the same topic conventionally. After the research ended, the students were given VHGT again in order to identify their final Van Hiele's levels of geometric thinking.

RESEARCH FINDINGS AND DISCUSSION

Initial levels of students' geometric thinking: The students' initial levels of Van Hiele's geometric thinking in the control group are shown in Table 5. Based on the table, a majority of the students achieved level 1, which is visualisation. Three (6.4%) students failed to achieve any level. A total of 33 (70.2%) students were at level 1. Of those, 30 students obtained a 1 mark by answering at least three out of five level 1 questions correctly and 3 students obtained 5 marks by answering at least three out of five level 1 and 3 questions correctly. However these three students skipped level 2. Based on the forced Van Hiele level table, they only achieved level 1. Eleven (23.4%) students were at level 2 which is analysis. All of them obtained 3 marks by answering at least three out of five level 1 and 2 questions correctly.

Table 5: Initial levels of students' geometric thinking for control group

| Van Hiele's level | Sum of scores | Level | | | | Answer 3 out of 5 questions correctly | Total (%) |
|-------------------|---------------|-------|---|---|---|---------------------------------------|-----------|
| | | 1 | 2 | 3 | 4 | | |
| 0 | 0 | | | | | 3 | 3 (6.4) |
| 1 | 1 | x | | | | 30 | 33 (70.2) |
| | 5 | x | | x | | 3 | |
| 2 | 3 | x | x | | | 11 | 11 (23.4) |
| Total | | | | | | 47 | 47 (100) |

Table 6: Initial levels of students' geometric thinking for treatment group

| Van Hiele's level | Sum of scores | Level | | | | Answer 3 out of 5 questions correctly | Total (%) |
|-------------------|---------------|-------|---|---|---|---------------------------------------|-----------|
| | | 1 | 2 | 3 | 4 | | |
| 0 | 0 | | | | | 0 | 0 (0) |
| 1 | 1 | x | | | | 32 | |
| | 5 | x | | x | | 2 | |
| | 9 | x | | | x | 2 | 36 (76.6) |
| 2 | 3 | x | x | | | 11 | 11 (23.4) |
| Total | | | | | | 47 | 47 (100) |

The students' initial levels of Van Hiele's geometric thinking in the treatment group are shown in Table 6. Based on the table, a majority of the students achieved level 1, which is visualisation. None of the students failed to achieve any level. A total of 36 (76.6%) students achieved level 1. Out of that, 32 students obtained 1 mark by answering at least three out of five level 1 questions correctly; two students obtained 5 marks by answering at least three out of five level 1 and 3 questions correctly; and two students obtained 9 marks by answering at least three out of five level 1 and 4 questions correctly. However, based on the forced Van Hiele table, the students who obtained 5 marks only achieved level 1 because they skipped level 2. Similarly, the students who obtained 9 marks also achieved level 1 because they skipped level 2 and 3. Totally 11 (23.4%) students were at level 2 which is analysis. All of them obtained 3 marks by answering at least three out of five level 1 and 2 questions correctly.

Final levels of students' geometric thinking: The students' final levels of Van Hiele's geometric thinking in the control group are shown in Table 7. Based on the table, a majority of the students achieved level 1, which is visualisation. None of the students failed to achieve any level. Twenty four (51.1%) students achieved level 1. Out of that, 20 students obtained a 1 mark by answering at least three out of five level 1 questions correctly and 2 students obtained 5 marks by answering at least three out of five level 1 and 3 questions correctly. However, these two students skipped level 2. Based on the forced Van Hiele level table, they only achieved level 1. Apart from that, 2 students obtained 9 marks by answering at least three out of five level 1 and 4 questions correctly. However, these two students skipped level 2 and 3. Totally 22 (46.8%) students were at level 2, which is analysis. From the total, 21 students obtained 3 marks by answering at least three out of five

level 1 and 2 questions correctly. One student obtained 11 marks by answering at least three out of five levels, level 1, 2 and 4 questions correctly. However, this student skipped level 3. Only one (2.1%) student achieved level 3. The student obtained 7 marks by answering at least three out of five levels 1, 2 and 3 questions correctly.

The students' final levels of Van Hiele's geometric thinking in the treatment group are shown in Table 8. Based on the table, a majority of the students achieved level 2, which is analysis. None of the students failed to achieve any level. Seven (14.9%) students achieved level 1. Of those, 5 students obtained a 1 mark by answering at least three out of five level 1 questions correctly and two students obtained 5 marks by answering at least three out of five level 1 and 3 questions correctly. However, these two students skipped level 2. Based on the forced Van Hiele level table, they only achieved level 1. A total of 22 (46.8%) students were at level 2, which is analysis. From the total, 21 students obtained 3 marks by answering at least three out of five level 1 and 2 questions correctly. One student obtained 11 marks by answering at least three out of five levels 1, 2 and 4 questions correctly. However, this student skipped level 3. Totally 18 (38.3%) students were at level 3, which is informal deduction. All of them obtained 7 marks by answering at least three out of five levels 1, 2 and 3 questions correctly.

The effectiveness of Van Hiele's phases of learning geometry in the dynamic geometry software environment towards the students' level of geometric thinking:

H₀₁: There is no significant difference between the initial levels of geometric thinking and the final levels of students' geometric thinking in the control group.

Table 7: Final levels of students' geometric thinking for control group

| Van Hiele's level | Sum of scores | Level | | | | Answer 3 out of 5 questions correctly | Total (%) |
|-------------------|---------------|-------|---|---|---|---------------------------------------|-----------|
| | | 1 | 2 | 3 | 4 | | |
| 0 | 0 | | | | | 0 | 0 (0) |
| 1 | 1 | x | | | | 20 | |
| | 5 | x | | x | | 2 | |
| 2 | 9 | x | | | x | 2 | 24 (51.1) |
| | 3 | x | x | | | 21 | |
| 3 | 11 | x | x | | x | 1 | 22 (46.8) |
| | 7 | x | x | x | | 1 | 1 (2.1) |
| Total | | | | | | 47 | 47 (100) |

Table 8: Final levels of students' geometric thinking for treatment group

| Van Hiele's level | Sum of scores | Level | | | | Answer 3 out of 5 questions correctly | Total (%) |
|-------------------|---------------|-------|---|---|---|---------------------------------------|-----------|
| | | 1 | 2 | 3 | 4 | | |
| 0 | 0 | | | | | 0 | 0 (0) |
| 1 | 1 | x | | | | 5 | |
| | 5 | x | | x | | 2 | 7 (14.9) |
| 2 | 3 | x | x | | | 21 | |
| | 11 | x | x | | x | 1 | 22 (46.8) |
| 3 | 7 | x | x | x | | 18 | 18 (38.3) |
| Total | | | | | | 47 | 47 (100) |

Table 9: Wilcoxon on-t statistics test

| | Before-after |
|------------------------|--------------|
| Z | -3.557 (a) |
| Asymp. sig. (2-tailed) | 0.000 |

Table 10: Descriptive statistics of the levels of geometric thinking for the control group

| | N | Mean | S.D. | Min. | Max. |
|--------|----|------|-------|------|------|
| Before | 47 | 1.17 | 0.524 | 0 | 2 |
| After | 47 | 1.51 | 0.547 | 1 | 3 |

S.D.: Standard deviation; Min.: Minimum; Max.: Maximum

Table 11: Wilcoxon on-t statistics test

| | Before-after |
|------------------------|--------------|
| Z | -5.475 (a) |
| Asymp. sig. (2-tailed) | 0.000 |

Table 12: Descriptive statistics of the levels of geometric thinking for the treatment group

| | N | Mean | S.D. | Min. | Max. |
|--------|----|------|-------|------|------|
| Before | 47 | 1.23 | 0.428 | 1 | 2 |
| After | 47 | 2.23 | 0.698 | 1 | 3 |

S.D.: Standard deviation; Min.: Minimum; Max.: Maximum

Table 13: Wilcoxon on-t statistics test

| | Treatment group-control group |
|------------------------|-------------------------------|
| Z | -4.388 (a) |
| Asymp. sig. (2-tailed) | 0.000 |

Table 14: Descriptive statistics of final levels of geometric thinking for control and treatment groups

| | N | Mean | S.D. | Min. | Max. |
|-----------------|----|------|-------|------|------|
| Treatment group | 47 | 1.51 | 0.547 | 1 | 3 |
| Control group | 47 | 2.23 | 0.698 | 1 | 3 |

S.D.: Standard deviation; Min.: Minimum; Max.: Maximum

As the data related to the students' levels of geometric thinking were ordinal scale data, Wilcoxon on-t test for the design of repeated measurement was used in order to test the above hypothesis. This Wilcoxon on-t test has the same function as the t-test to do repeated

measurement where the difference between these two tests is that the t-test is applied in repeated measurements to analyse two interval scale data groups or ratio scale, while Wilcoxon on-t test analyses two ordinal data scale groups.

Based on Table 9, the significant value 0.00 is less than 0.05. The result of Wilcoxon on-t test is significant ($t = 0.00, p < 0.05$), which shows that geometric thinking levels of students in the control group have showed a significant improvement after the learning process. The descriptive statistics of the levels of geometric thinking for the control group are shown in Table 10.

H₀₂: There is no significant difference between the initial levels of geometric thinking and the final levels of students' geometric thinking in the treatment group.

As the data related to the students' levels of geometric thinking were ordinal scale data, Wilcoxon on-t test for the design of repeated measurement was used in order to test the above hypothesis.

Based on Table 11, the significant value 0.00 is less than 0.05. The result of Wilcoxon on-t test is significant ($t = 0.00, p < 0.05$), which shows that geometric thinking levels of students in the treatment group have showed a significant improvement after experiencing the activities based on the Van Hiele's phases of learning geometry using the Geometer's Sketchpad (GSP) computer software. The descriptive statistics of the levels of geometric thinking for the treatment group are shown in Table 12.

H₀₃: There is no significant difference between the student's final levels of geometric thinking in the treatment group and control group.

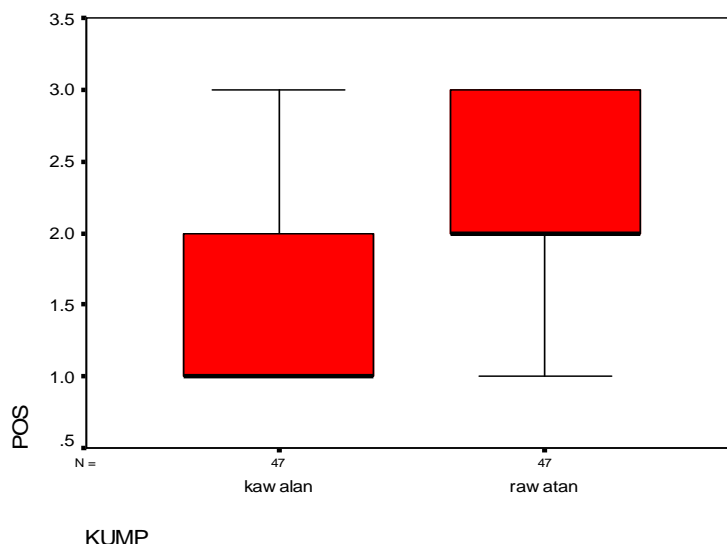


Fig. 2: Box plot graph of the final levels of geometric thinking for control and treatment groups

To test the above hypothesis, the Wilcoxon on-t test for the design of matching samples to make comparison between two matching group samples in two different situations was used.

Based on Table 13 above, the significant value 0.00 is less than 0.05. The result of Wilcoxon on-t test is significant ($t = 34.50$, $p < 0.05$), which shows that there is a significant difference between the final levels of geometric thinking of the two groups. The descriptive statistics of final levels of geometric thinking for control and treatment groups are shown in Table 14. This result is supported by the Box plot graph median value for both the ordinal scores of the two groups Fig. 2 which clearly shows the treatment group's final levels of geometric thinking is higher than the control group's final levels of geometric thinking.

DISCUSSION AND CONCLUSION

The aim of this study was to identify the effectiveness of Van Hiele's phases of learning geometry in the learning of Form Two's Transformation topic that consists of the Concepts of Transformation (Translation, Reflection and Rotation) and Quadrilaterals subtopics in order to assist the students in enhancing their levels of thinking to higher levels. The phases involved were information, guided orientation, exploitation, free orientation and integration. Geometer's Sketchpad (GSP) software was used as a medium to implement the activities developed. The students in the treatment group learned Transformation topic based on the activities developed from the Van Hiele's phases of learning geometry by using GSP software as a medium. Meanwhile, the students in the control group learned the same topic by using the traditional approach. Based on the previous discussions, most of the students' initial levels of geometric thinking were at the first level, which is visualisation. This is parallel to the finding by Razananahidah (2006) that most students only achieved the first level at the beginning of their school education. The findings revealed that both groups showed enhancement in students' final levels of geometric thinking compared to their initial levels of geometric thinking. However, the findings also revealed that the final students' levels of geometric thinking in the treatment group were better than the levels of geometric thinking for students from the control group. Therefore, this means that the implementation of Van Hiele's phases of learning geometry with assistance from GSP software assisted students in achieving better levels of geometric thinking as compared to those students who learned the topics in geometry traditionally. In a Malaysian context, these findings are in line with the studies conducted by Tay (2003) who focused on application of manipulative materials in the phase-based activities and Chew (2009) who focused on other

geometry topics. Therefore, in accordance with the national education transformation concept as stressed by Malaysia's Ministry of Education (MOE), teachers should introduce new approaches in their teaching and learning practices in the topics of geometry. One new approach that can be implemented is delivering the contents of geometry topics based on Van Hiele's phases of learning geometry. The GSP software, the license of which has been bought by the MOE to be used in schools, can be beneficial, besides its advantages in the teaching and learning process that have certainly been proven by previous studies. This is important in the context of learning geometry in Malaysia because the geometry topics comprise about 40% of the Mathematics topics taught in secondary schools.

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