

Research on Bus Priority Signal Planning Under Mixed Traffic Flow

Dan Yu, Juanjuan Cao, Yihu Wu and Minglei Song

School of Traffic Transportation Engineering, Changsha University of Science and Technology,
Changsha, 410076, China

Abstract: This study, based on principle of bus priority, uses complementary doctrines to create mixed traffic flow intersection signal-planning model under effective green time. By discussing the objective function of optimization model, we ensure and take advantage of nonlinear equivalence feature. After that, this study refers to Newton algorithm to solve this model. Considering the mixed characteristic of traffic flow, we put forward to adoption of traffic capacity, delay time and parking number together to evaluate operation effect. Through the example analysis, it was shown that this model and solution algorithm responded well and indications increased remarkably.

Keywords: Bus priority, mixed traffic flow, Newton algorithm, nonlinear complementarity, signal timing optimization

INTRODUCTION

Mixed traffic which consisted of vehicle, non-vehicle and pedestrian is a important characteristic of municipal traffic in our country. This is quite different from main application conditions that extensive adopts analysis of U. S. HCM manual. Control products which are popular aboard are lacking effective amendment design aimed at mixed traffic. This objectively makes the system which works well in foreign countries don't have desired control results after displaced to domestic. Especially in the peaking commuting, non-vehicle became the main factor to effect junction (Ma and Yang, 2010; Zhang *et al.*, 2004; Wang *et al.*, 2006). Then, it is not suitable to adopt F·Webster B·Curb for traffic signal timing directly in our country. Many domestic scholars propose a variety of intersection signal timing control scheme directed towards mixed traffic characteristic: Professor Wang Dianhai in JiLin University proposed: In city traffic management, effective organization of the vehicle, bicycle and pedestrian in urban intersection is key to traffic smooth. On the basis of traffic survey, using statistical technique to analyze the disipline of bicycle and pedestrian's arrival and release in urban intersection, proposed city intersection bicycle and pedestrian arrival model, demarcated the release saturate flow rate, release speed of bicycle in urban intersection and pedestrian walking speed, analyzed the formation reasons about traffic characteristics of bicycle flow and pedestrian in signalized intersection. Presentation of results which have application in control and management on urban signalized intersection mixed traffic. In addition, he analyzed the conflict situations about vehicle, bicycle and pedestrian under mixed traffic environment, studied

the effect on intersection traffic capacity, proposed set left-face bicycle phase and advanced end bicycle and pedestrian phase and other solutions. Those theoretical achievements can be used to solve vehicle, bicycle and pedestrian conflict problems under mixed traffic environment. Professor Yang Xiaoguang in TongJi University studied traffic design problem about road crossing under vehicle and non-vehicle fixed operation. Based on large number of measured data, analyzed bicycle flow running characteristics in signalized crossing, created bicycle traffic flow model, formed theory and method of road traffic design, which lay the foundation for solving urban mixed traffic problems, in vehicle and non-vehicle fixed operation crossing (Ma and Yang, 2007; Gao, 2010; Liu, 2005). This study, based on principle of bus priority, uses complementary doctrines to create mixed traffic flow intersection signal-planning model under effective green time. By discussing the objective function of optimization model, we ensure and take advantage of nonlinear equivalence feature. After that, this study refers to Newton algorithm to solve this model. Considering the mixed characteristic of traffic flow, we put forward to adoption of traffic capacity, delay time and number of stop together to evaluate operation effect. Through the example analysis, it was shown that the model and solution algorithm responded well and indications increased remarkably.

METHODOLOGY

Bus priority signal planning model under mixed traffic flow: At present, vehicle and non-vehicle mixed traffic flows in our country are in the majority. Even in the road with isolation bet, mixed traffic flows don't

have any effect on each other before the stabling siding. Once the vehicles enter internal part of the intersection, it would cause tremendous obstructions. Therefore, it is not enough to use delay time as the only indicator for evaluating intersection operation capacity. This study chooses traffic capacity together with delay time and parking number to evaluate intersection operation capacity, in order to achieve the goals that signal control can separate vehicle and pedestrian flow in different directions in terms of time, achieve the largest safety performance and raise the transportation efficiency of man and good in road network intersection.

Without considering the case of initial queuing, bus priority signal planning model under mixed traffic flow can be described as follows:

$$\begin{aligned} \min f(x) &= \sum_i (h_i^1 D_i + h_i^2 H_i + h_i^3 Q_i) \\ \text{s.t.} \sum_i (x_i + l_i) &\leq c, \forall i \\ x_i &\geq g_i, \forall i \\ x_i &\geq \frac{C y_i}{0.95}, \forall i \end{aligned} \tag{1}$$

In which,

- $f(x)$ = Function about $R^n \rightarrow R, x \in R^n$
- n = Intersection phase number
- Q_i = Pedestrian volume in phase i , pcu/h
- D_i = Average delaying time in phase i , $D_i = d_u d_r$, among, d_u is Uniform delay from vehicle to pedestrian, d_r is Random delay from vehicle to pedestrian
- H_i = Average parking number in phase i
- O_i = Traffic capacity in phase i
- X_i = Phase effective green time
- S_{ij} = Total saturation flow in phase i lane j
- g_i = Minimum green time in phase i
- λ_i = Split in phase i
- y_{ij} = Total saturation flow rate in phase i lane j
- C = Duration, s
- L = Total lost time in intersection, s
- Y = The sum of each Max Y from all signal phase

which makes up the cycle:

$$Y = \sum_i \max[y_i, y_i', \dots]$$

The objective function of this model makes total intersection delay time to be minimum, stop time to be least and traffic capacity maximum. Constraint condition Two describes green time and total sum of lost time in all phases which is equivalent to the duration. Constraint condition 3 indicates that effective green time in phase i is not less than minimum green

time in the same phase. In traffic ordinary hours, the goal of single timing is to reduce buses' delay and stop in intersection as far as possible. However, in traffic peak hours, improving intersection traffic capacity should be emphasized. Therefore, the ratio of weighting coefficient of stop, delay and weighting coefficient of traffic capacity should decrease along with intersection flow rate's growth. They should satisfy the following relations:

$$h_i^1 = 2(1-Y)\sqrt[3]{s_i}, h_i^2 = \sqrt[3]{s_i} \times \frac{1-Y}{0.9}, h_i^3 = 2Y \frac{C}{3600} \tag{2}$$

Therefore, the computation formula of flow rate can be described as follows:

$$\begin{aligned} s &= s_0 N f_w f_{HV} f_g f_p f_{bb} f_a f_{LT} f_{RT} f_{Lpb} f_{Rpb} \\ \min f(x) &= \sum_i (k_i^1 D_i + k_i^2 H_i + k_i^3 Q_i) \\ x_i &\geq a_i \end{aligned} \tag{3}$$

In which,

- S = Basic saturation traffic flow rate in the road under discussion, it states the sum of the traffic flow in all lanes of lane group, vehicle/green hour
- N = Total lanes (depend on the situation) in lane group
- f_w = Lane width correction factor, $f_w = 1 + \frac{w-3.6}{9}$ w is lane width
- f_{HV} = Correction index of heavy-duty vehicle in traffic flow, $f_{HV} = \frac{1}{1 + P_{HV}(PCE_{HV} - 1)}$ PCE_{HV} is heavy vehicle reduction coefficient in lane groups, PHV is the rate of heavy vehicle flow in total traffic flow
- f_g = Ramp gradient correction index, $f_p = 1 - G/2$ G is ramp gradient, generally $G \in [-0.06, 0.10]$
- f_p = Stop situation in contiguous lane groups and stopping time correction index, $f_p = \frac{N - 0.1 - \frac{18N_m}{3600}}{N}$ N_m is the frequency of stopping in one hour, when $N_m = 0$, then $f_p = 1$
- f_{bb} = Blocked correction index when local bus stop in the intersection area, $f_p = \frac{N - \frac{14.4N_b}{3600}}{N}$ N_m is stopping frequency of local bus in 1 h, $0 \leq N_b \leq 250, f_{bb} \geq 0.050$

f_a = Region type correction index, business district $f_a = 0.90$, others $f_a = 1$

f_{LT} = Correction index of left-turn vehicles in lane group, dedicated lanes for left-turn vehicles

$$f_{LT} = 0.95 \text{ , public lane } f_{LT} = \frac{1}{1.0+0.05P_{LT}} \text{ ,}$$

PLT is the rate of left-turn vehicles in total traffic flow from the entrance lane

f_{RT} = Correction index of right-turn vehicles in lane group, dedicated lanes for right turn vehicles $f_{RT} = 0.85$, public lane $f_{RT} = 1.0 - 0.15 P_{RT}$, P_{RT} is the rate of right-turn vehicles in total traffic flow from the entrance lane

f_{Lpb} = Left-turn pedestrians correction index and

$$f_{Lpb} = 10. - P_{LT} (1 - A_{pbT}) (1 - P_{LTA})$$

f_{Rpb} = Right-turn pedestrians and bicycles correction index and

$$f_{Rpb} = 10. - P_{RT} (1 - A_{pbT}) (1 - P_{RTA})$$

Among, P_{LT} is the rate of left-turn pedestrians in total pedestrians from entrance lane, P_{RT} is the rate of right-turn pedestrians in total pedestrians from entrance lane, A_{pbT} permissible is adjustment coefficient at the present stage, P_{LTA} is the rate of left-turn green time for pedestrian in total green time, P_{RTA} is the rate of right-turn green time for pedestrian in total green time.

As a matter of convenience, we take $a_i = \max \{g_i, C_{vi}\}$, so the model described as follows:

$$(P) \quad \min f(x) = \sum_i (k_i^1 D_i + k_i^2 H_i + k_i^3 Q_i)$$

$$\text{s. t. } \sum_i (x_i + l_i) \leq b_i \quad \forall i \quad (4)$$

Complementary theory introduction: Complementary problem consists of two group of decision variables which satisfy a kind of 'complementary relationship'. Complementary relationship is a basic relationship widely existed. According to the differences on conditions which variables satisfied in the problem and different types of complementary relationship, complementary problem can be divided into amount of types. Next, we cite their mathematical forms. We define: R^n states n-dimensional dimensional spatial, R_+^m states nonnegative quadrant in R^n , all vectors in R^n are column vectors, $x^T Y$ states inner product between x^T and y , T states transposition of vector sum matrix.

Let $M \in R^{n \times n}$ is a $n \times n$ real matrix, $q \in R^n$ is a n-dimensional vector. Linearity complementary problem is: finding $x \in R^n$, satisfy:

$$x \geq 0, Mx + q \geq 0, x^T (Mx + q) = 0$$

linearity complementary problem can be remar LCP (M, q). If induct vector $y \in R^n, y = Mx + q$, then variable x_i and y_i satisfy $x_i \geq 0, y_i \geq 0, x_i y_i = 0, i = 0, 1, 2, \dots, n$. That indicated that variable $\{x_i\}$ and $\{y_i\}$ satisfied complementation relationship. The basic source of linearity complementary problem is linear programming and quadratic programming. Considering Quadratic Programming (QP):

$$\min Q(x) = c^T x + \frac{1}{2} x^T Q x$$

$$\text{s. t. } Ax \geq b, x \geq 0 \quad (5)$$

Among, $Q \in R^{n \times n}$ is symmetric matrix, $A \in R^{m \times n}$. If $Q = C$, quadratic programming degenerate s into linear programming. Given x is a locally optimal solution for quadratic programming, according to Karush-Kohn Tucker optimality theory, there is Lagrange multiplier vector satisfy K-K-T condition:

$$\begin{cases} u = c + Qx - A^T y \geq 0, x \geq 0, x^T u = 0 \\ v = -b + Ax \geq 0, y \geq 0, y^T v = 0 \end{cases} \quad (6)$$

Model solution: According to the introduction about complementary theory, we can find that bus priority signal planning model under mixed traffic flow which the author established fits equivalence theorem well. That is to say, the effective green time in phase i optimal timing of single point intersection signal control satisfies constraint condition two or three, so it satisfies nonlinearity complementary problem-NCP (G, F) based on effective green time. In other words, we need to find vector:

$$(x^*, u^*) \in \Omega \times R$$

$$G(x^*, u^*) \geq 0, F(x^*, u^*) \geq 0, G^T F = 0, \forall x \in \Omega \quad (7)$$

Therefore, intersection timing model created in this study has the feature of nonlinear complementary. So we can be transferred it into unconstrained optimization problem or nonlinear equations to seek the answer.

Table 1: Traffic volume and saturation flow in every entrance lane

Project	North approach	South approach	East approach	West approach
Traffic volume q	620	720	390	440
Saturation flow rate s	2400	2400	1000	1000
Flow ratio y	0.26	0.30	0.39	0.44
Max [y, y']	0.30		0.44	

Table 2: Traffic volume and saturation flow in each approach

Project	North approach	South approach	East approach	West approach
Traffic volume q	380.000	400.000	576.000	500.000
Saturation flow rate s	2150.204	2131.416	2009.394	2244.922
Flow ratio y	0.177	0.188	0.287	0.227
Max [y, y']	0.188		0.287	

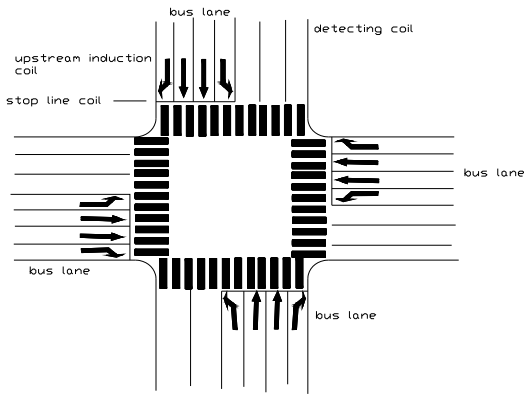


Fig. 1: Bus priority intersection geometric model

Inducting nonlinear complementary functions $\phi(a,b,u) = a + b - \sqrt{a^2 + b^2 + u^2}$, we transfer NCP (F) into nonlinear complementary functions $T(t) = 0$ to seek the answer. Choosing initial point freely, calculation procedures are as follows:

Step 1: Initialization: Giving initial timing freely x^0 , C^0 , Lagrangian multiplier u^0 and corresponding parameter $\delta, y, \sigma, \bar{u} = (\mu_0, 0)$ evaluation initially. $\rho_k = \text{ymin} (1 \| T(t^k) \|)$, command $k = 0$, turn Step 2.

Step 2: Finish judgment: If $\|T(t^k)\| \leq \epsilon$, the finish; otherwise turn Step 3.

Step 3: Newton step: Progressing iteration, correct effective green time and duration:

- Solve a generalized Newton's equation, define descent direction t_k .
- According to one step Armijo line-search, define step length, command l_k satisfies the minimum nonnegative integers of the following line-search:

$$\|T(t^k + \delta^l)\|^2 \leq [1 - 2\sigma(1 - \gamma\bar{\mu})\delta^l] \|T(t^k)\|^2 \quad (8)$$

Command $a_k = \delta^{l_k}$.

- Iteration, command $t^{k+1} = t_k + a_k t_k, k = k+1$
- There are three terminal conditions in this arithmetic:
 - $\|T(t^k)\| \leq \epsilon$, given $\epsilon = 1.0 \times 10^{-5}$
 - $\|x_{k+1} - x_k\| \leq 0.1$, for considering the practical application and accuracy of the optimum relation (Normally, practical data can't get the accuracy of numerical algorithm). When calculate signal time and distribute effective green time to every phase, we only reach second.
 - Saturation is equal to or more than 0.98, this is because the model established in this study mainly in connection with intersection timing under unsaturated state. In this situation, the accuracy of the arithmetic is worse than the former two situations.

This study chose measured data of a single intersection in city X for example analysis. Table 1 states saturation flow rate in every entrance lane. Lost time in every phase is 6 sec, maximum duration is 130 sec. Though the road grade in this intersection are quite different, traffic flow in each lane basically the same. Table 2 shows the traffic volume and saturation flow in each approach. Figure 1 gives the bus priority intersection geometric model.

Example analysis: Table 3 gave traffic flow and saturation flow rate of one intersection, east and west belong to primary route and relatively speaking, the passing vehicles are more than other directors. However, road in the two phases are equal, the width of entrance lane from all directions are 3.5 m. Suppose the bicycles and pedestrians from every entrance direction estimate 150 and 300 veh/h, respectively. Given vehicle desired saturation flow rate are 1800 veh/h, maximum duration is 120 sec, lost time in every phase is 4 sec. we can work out the coefficients of saturation flow rate. Coefficients of saturation flow rate are demonstrated as Table 4, given other coefficients 1.

Table 3: Performance index comparison of Newton algorithm and other algorithm

Algorithm	Cycle/s	Effective green time/s		Total delay time/s	Number of stops	Capacity (pcu/h)
		Phase 1	Phase 2			
Webster algorithm	100	34	52	5844390	1778.10	2312
Ant algorithm	94	41	39	4746154	1745.00	2923
Newton algorithm	135.377	60.6027	60.7742	3591410	1742.28	3046.79

Table 4: The related correction coefficient of saturation flow rate in each approach

Project	North approach	South approach	East approach	West approach
Correction coefficient of lane width fw	0.990	0.990	0.990	0.990
Correction coefficient of right-turn in lane systems fRT	0.987	0.973	0.980	0.976
Correction coefficient of left-turn in lane systems fLT	0.990	0.995	0.998	0.990
Pedestrian correction coefficient of left-turn fLpb	1.150	1.075	1.037	1.150
Pedestrian-bicycle correction coefficient of right-turn fRpb	1.075	1.150	1.112	1.135

From the results in the tables, it is obviously that if we established intersection timing model based on complementary theory and used Newton arithmetic to solve, the results is better than it get from other arithmetic.

CONCLUSION

This study, based on principle of bus priority, uses complementary doctrines to create mixed traffic flow intersection signal-planning model under effective green time. By discussing the objective function of optimization model, we ensure and take advantage of nonlinear equivalence feature. This study referred to the Newton algorithm to solve this model. Considering the mixed characteristic of traffic flow, we put forward to adoption of traffic capacity, delay time and parking number together to evaluate operation effect. Through the example analysis, it showed that this model and solution algorithm responded well and indications increased remarkably.

ACKNOWLEDGMENT

The project was supported by Open Fund of Engineering Research Center of Catastrophic Prophylaxis and Treatment of Road & Traffic Safety (Changsha University of Science & Technology), Ministry of Education (kfj00307), the ministry of

communication P.R. China (No. 09C070) and Hunan natural science foundation (No. 10JJ6072).

REFERENCES

- Gao, K.H., 2010. Research on Bus Priority Signal Coordination Control and Simulation in Urban Road Intersections [D]. Beijing Jiaotong University, China.
- Liu, Z.P., 2005. Research on Bus Priority Intelligent Signal Control System at an Intersection [D]. Wuhan University of Science and Technology, China.
- Ma, W.J. and X.G. Yang, 2007. Transit passive priority control method based on isolated intersection of optimization of time-space [J]. China J. Highway Transp., DOI: CNKI:SUN:ZGGL.0.2007-03-015.
- Ma, W.J. and X.G. Yang, 2010. Lane-based optimization model of passive bus priority control for isolated intersection [J]. China J. Highway Transp., DOI: CNKI:SUN:ZGGL.0.2010-05-014.
- Wang, Q.P., X.L. Tan and S.R. Zhang, 2006. Signal timing optimization of urban single-point intersections [J]. J. Traffic Transport. Eng.
- Zhang, W.H., H.P. Lu, Q. Shi and Q. Liu, 2004. Optimal signal-planning method of intersections based on bus priority [J]. J. Traffic Transport. Eng.