

Some Quantitative Issues in Pairs Trading

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Abstract: In this study, Pairs Trading (PT) is considered. An example of its application using real data is shown and a time series simulation of the Saudi stock market using the Vector Auto-Regressive model (VAR) is performed on two stocks. In addition, the influence of different simulation model parameters on the total profit is investigated. We conducted a quantitative analysis of different cross-correlations that occurred between simulated stocks, as well as of different values of linear trends in the auto-regressive model. Furthermore, the optimal conditions for the application of a PT strategy, as well as the optimal parameters used for this strategy were obtained. It was shown that parallel trends and a high correlation between considered stocks can lead to significant profits. In the case of highly correlated series with limited volatility, benchmarks should be used for opening and closing positions.

Keywords: Pairs trading, simulation, trading decision making, trends, vector auto-regressive model

INTRODUCTION

Pairs Trading (PT) is a popular quantitative method of statistical arbitrage that has been widely applied in the financial industry. This strategy was developed by Morgan Stanley in the 1980s and it involves choosing a pair of stocks that have moved together historically thus exploit statistical pair-trading profit opportunity (Gotev *et al.*, 2006a; Elliott *et al.*, 2005; Jurek and Yang, 2007; Do *et al.*, 2006; Whistler, 2004). In other words, their historic price changes are highly correlated. Sometimes the co-integration of the analyzed indices is also required. By applying a long-short strategy on this pair when they diverge, a profit will be made when the pair converges near the mean by unwinding the position.

The key idea is that the chosen indices are mean-reverting; hence, every movement of the ratio away from its historical, long-term average represents an opportunity to make profit. For example, when the spread between the first and second stock widens, the appropriate strategy is to sell the appreciating stock and buy the depreciating one (Gotev *et al.*, 2006a); this is called "opening a position." As the stocks become bound by a long-term relationship (reflected by robust correlations and co-integration), their ratios will start reaching a level that is indicative of long-term equilibrium at a given point. At this moment, the first stock should be bought and the second one should be sold. This is called "closing the position." To implement this strategy, precise rules of opening and closing positions are needed. The most common rule is to open positions when the ratio of stock 1/stock 2 hits bounds set as $m+k\sigma$ and $m-k\sigma$. In this way, m is the

mean value for the training data set, σ is the standard deviation and k is integer parameter which should be set based on quantitative analysis. Positions are closed when the ratio reverts to the mean value, indicating that it has reached its equilibrium point (Gotev *et al.*, 2006a). Quantitative trading often involves the utilization of diverse statistical models to describe and predict the direction of the stock market. The main objective of this study is to explore the main features and applications of quantitative techniques (mainly through application of the vector auto-regressive model) used in the process of modeling and analyzing such a trading strategy. In particular, data simulation will be performed. From this perspective, it will be possible to analyze and quantify the relationships between stock parameters and the obtained profit. Specifically, the linear trends and the cross-correlation of the stocks will be analyzed. The simulation results will help to set the conditions for profitable trading with real stocks.

PAIRS TRADING OVERVIEW

PT can be broadly categorized into three forms: fundamental, risk and statistical (Do *et al.*, 2006). The key distinction between these forms is related to the considered time period. A fundamental pair may be correlated over some historical period, usually over the course of several years or more, during which the pair displays a robust relationship that is reflective of long-term equilibrium (Do *et al.*, 2006). Conversely, risk pairs do not have to be fundamentally similar and their relationship exists over a shorter term (Do *et al.*, 2006).

A risk pair is obtained during mergers and acquisitions. One important concept that must be taken into account in successful pair trading is the similarity of the stocks (i.e., the stocks should belong to the same industrial sector, they should be of similar size and the financial standing of the companies should be similar). A statistical pair is usually not fundamentally linked, but it should be highly correlated over a very short period of time, such as over the course of several days. In such frequencies, the fundamental linkage is not of great importance; however, such short-term correlations can be used to obtain profits. In general, the statistical analysis of the historical price series for each stock can be useful in analyzing each PT form.

There are three main PT methods: the distance method (Gotev *et al.*, 2006a), the co-integration method (Vidyamurthy, 2004) and the stochastic spread approach (Elliott *et al.*, 2005). The distance method is related to the analysis of the sum of squared differences between the two normalized time series. The value of this sum is the basis of PT. We will use a similar approach in our research, but in our case, the stock ratio is used rather than the sum of squared differences; it should be noted that the idea remains the same. This method is quite a simple and efficient approach (Gotev *et al.*, 2006a). The co-integration approach is based on the analysis of time series, which is obtained as the linear combination of the two considered stocks (Vidyamurthy, 2004). The distance and co-integration approaches are widely adopted for practical applications (Do *et al.*, 2006). As for the stochastic spread approach, it considers the differences between the stocks as a random process. The main benefit of this method is that it can offer approximate forecasting of the time it takes for the value of the stock to converge to the mean (Elliott *et al.*, 2005).

In real trading, the number of pairs monitored per trader can range from a few to a thousand or more; however, the performance of PT method is shown to be profitable only in the case of careful analysis and only when right decisions are made in the market. Profitability and performance of the PT strategy have been studied by many scholars and have many different applications in different markets. As has been the case for several Asian indices, PT has yielded profits as high as 33% annually (Hong and Susmel, 2003). This applies even to conservative trading strategies and long holding periods. The analysis of PT as has been applied to the Istanbul Stock Exchange has shown that it can bring positive abnormal profits for both developed and developing markets. This method has been found to be especially profitable in the short-term (Yuksel *et al.*, 2010). Evidence from Finnish stocks has shown that PT strategies that are regularly implemented can bring about annual profits as high as 15%; it should be known that profits are not related to market risk (Broussard and Vaihekoski, 2010). Also, the application of PT on the

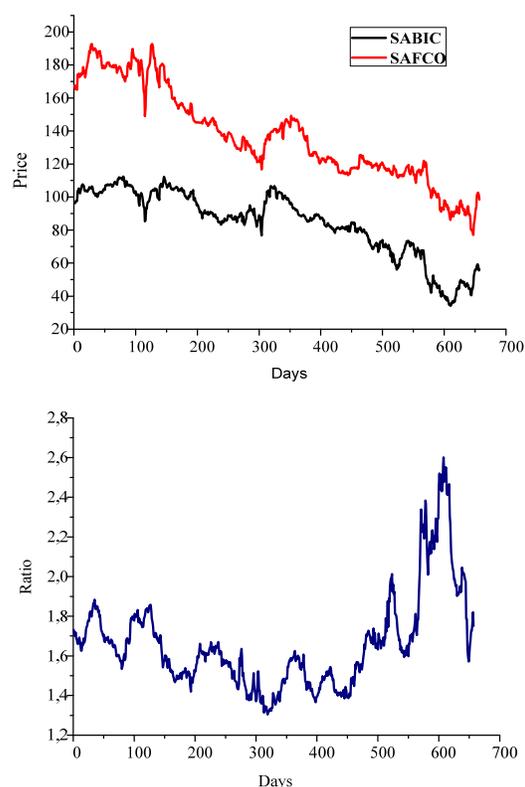


Fig. 1: Price time series and price ratio

American stock exchange has yielded substantial profits (Gotev *et al.*, 2006b). Similar results have been achieved when the PT strategy has been applied on the Brazilian market (Perlin, 2009). In the next section, a real example using correlated pairs is illustrated.

Pairs trading in the Saudi stock market: In this study, we will analyze a data sample taken from two large petrochemical companies that are traded in the Saudi stock market, SABIC and SAFCO. Both companies are among the largest in the world in their respective fields.

The data corresponds to a 650-day period. The time series and the ratio of the two stocks being considered are shown in Fig. 1.

Figure 1 clearly highlights that both stocks are highly correlated. The correlation coefficient for these data is 0.885, which indicates a highly significant correlation (t -statistic = 44.4, p -value < 0.01). The ratio of these stocks fluctuates around a stable value. One can divide the whole data into “training” and “test” parts. The ratio of the two stocks and their values ($m \pm k\sigma$) are shown in Fig. 2.

Let us explain the principles of PT. We start with point 1, which is below the mean value. At this point, SAFCO is valued at 121.25 and SABIC is valued at 87.75. Thus, we will buy $1/121.25 = 0.00824$ units of SAFCO and sell $1/87.75 = 0.01134$ units of SABIC. Now we will wait until the ratio reverts to the mean,

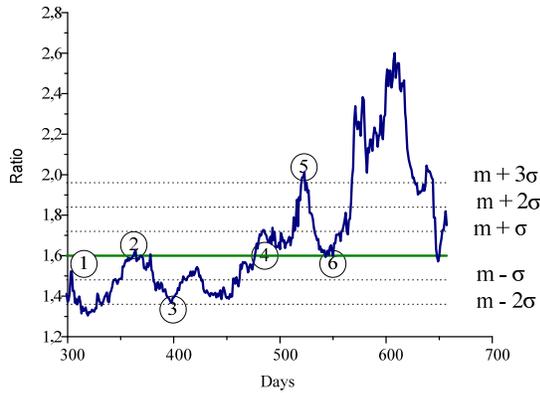


Fig. 2: Ratio of SAFCO and SABIC stocks

which happens at point 2. At this point, SAFCO is valued at 140 Saudi Riyals (SR) and SABIC is valued at 89 SR. We close the position, which means that we will sell 0.00824 units of SAFCO and buy 0.01134 units of SABIC. The total profit is $(1.15-1) \text{ SR} + (1-1.01) \text{ SR} = 0.14 \text{ SR}$. Hence, money is lost on SABIC but earned on SAFCO; PT often works in this way. We can see from Fig. 2 that there is an opportunity to make money at point 3 and 5.

The key question in this strategy relates to the choice of the optimal value of k . This parameter is often chosen to be 2. However, its choice should be analyzed in detail as it might lead to a significant loss of money. For example, if k is too large, the ratio will not converge close to the mean value within the given time interval (i.e., the deviations that are chosen are too large and the indices will need too much time to converge to the equilibrium). Moreover, when the value of k is too large, many possible starting points can be missed. On the other hand, if k is too small, positions are opened too early, resulting in values diverging from the mean at insignificant levels.

The part of the data which we called the “training” part has been used to determine the optimal value of k . Later in this report, the choice of this value will be tested on the testing set to verify if the choice and the methods used are correct and yield satisfactory profits.

Before conducting the simulation experiment, it is worth considering several important issues. First, when we divide the data set into the training and the test parts, we assume that the examined time series are stationary (i.e., they have constant moments). In real life trading, some tests of stationary should be performed, such as the Dickey-Fuller unit root test (Halim, 2007) and so on; however, our goal here is to introduce the main ideas behind the PT approach.

The specific ideas behind PT and the detailed analysis of this strategy are described in the next section. Trade tests using varying parameters will be performed on the simulation data.

SIMULATION OF STATISTICAL DATA

We used the vector auto-regressive model for the simulation of the time series of two stocks. Each stock is defined as the sequence of the points:

$$\begin{aligned}
 Y_t^{(1)} &= \beta_0^{(1)} + \beta_1^{(1)}t + X_t^{(1)} \\
 Y_t^{(2)} &= \beta_0^{(2)} + \beta_1^{(2)}t + X_t^{(2)}
 \end{aligned}
 \tag{1}$$

where,

$\beta_0^1 \beta_0^2$ = Denote the intercepts (baseline)

$\beta_1^1 \beta_1^2$ = Denote the parameters of linear trends

$X_t^{(1)}$ and $X_t^{(2)}$ are random components of the data. These components account for the cross-correlation and time correlation of the series. We define them as:

$$\begin{aligned}
 X_t^{(1)} &= \rho X_{t-1}^{(1)} + \psi(1-\rho)X_{t-1}^{(2)} + \varepsilon_t \\
 X_t^{(2)} &= \rho X_{t-1}^{(2)} + \psi(1-\rho)X_{t-1}^{(1)} + v_t
 \end{aligned}
 \tag{2}$$

Hence, parameter ρ stands for the time correlation of the data (in other words, the correlation of values of a given series with its own values from the precedent period). Parameter ψ stands for the cross-correlation of the data (i.e., the correlation of values of a given series with lagged values taken from other series). In order to exclude exploding series and high volatility, we used parameters which fulfill:

$$0 \leq \rho, \psi \leq 1$$

Furthermore, we added a random component to the data. Components that are added to the series follow the normal distribution:

$$\begin{aligned}
 \varepsilon_t^{(i)} &\sim N(0, \sigma_i^2) \\
 v_t^{(i)} &\sim N(0, \sigma_i^2)
 \end{aligned}
 \tag{3}$$

Consequently, the whole series follows the normal distribution:

$$X_1^{(i)} \sim N(0, \sigma_i^2)$$

Our goal is to investigate the influence of linear trends $\beta_1^{(i)}$ and the cross-correlation coefficient ψ on the overall PT strategy; however, other parameters are not of interest and they have been kept constant:

$$\beta_1^{(1)} = \beta_1^{(2)} = 100, \rho = 0.99, \sigma_i = 1$$

In order to analyze the different variations associated with stock co-movements, we have decided

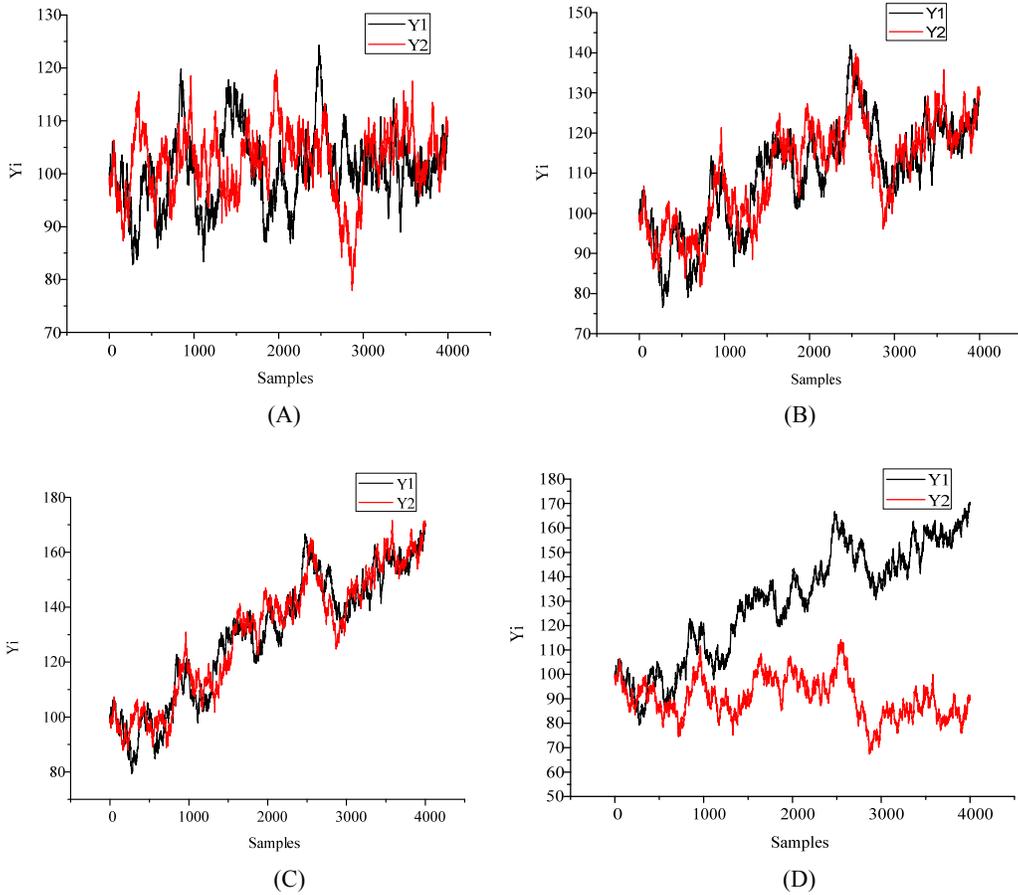


Fig. 3: Simulation of data: A) no correlation, no trends; B) cross-correlation, no trends; C) cross-correlation with positive trends; D) cross-correlation with opposite trends

to simulate four sets of data following different assumptions:

- No cross-correlation, no trends: $\psi = 0, \beta_1^{(i)} = 0$ (case A)
- Cross-correlation, no trends: $\psi = 0.99, \beta_1^{(i)} = 0$ (case B)
- Cross-correlation, both indices having a positive linear trend: $\psi = 0.9, \beta_1^{(i)} = 0.01$ (case C)
- Cross-correlation, indices exhibiting opposite trends (positive for the first, negative for the second): $\psi = 0.9, \beta_1^{(1)} = 0.01, \beta_1^{(2)} = -0.01$ (case D)

We have combined all four cases with general formulas (2) to obtain four sets of stocks. The results are displayed in the form of graph A, B, C and D in Fig. 3.

It was mentioned above that the parameter ψ controls the cross-correlation between two stocks. The calculated cross-correlated coefficients for the simulated data are: $\rho_{cross} = -0.048, \rho_{cross_B} = 0.839, \rho_{cross_C} = 0.950, \rho_{cross_D} = -0.034$. We can see that the opposite trends strongly affect the experimental cross-correlation coefficient. Even the

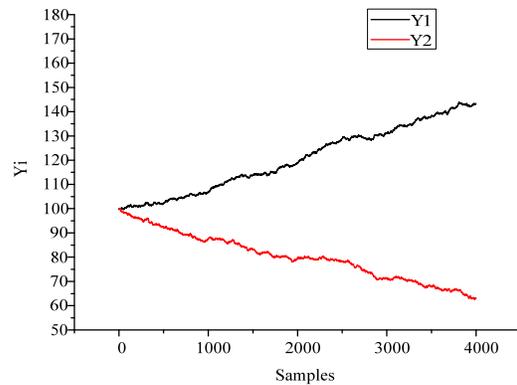


Fig. 4: Averaged time series over 100 experiments for $\psi = 0.9$ and $\beta_1^{(1)} = 0.01, \beta_1^{(2)} = -0.01$

definition $\psi = 0.9$ leads to a small cross-correlation. It is necessary to examine this feature in more detail. The averaged time series are shown in Fig. 4. The number of independent experiments here is 100.

We can see that the noise fluctuations have been smoothed and that the opposite trends are clearly seen; however, even for $\psi = 0.9$ we can see an experimental

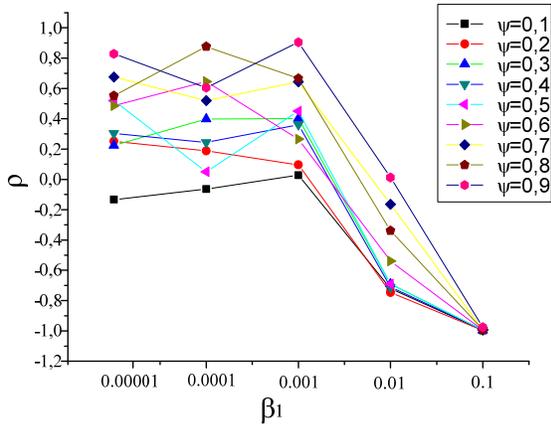


Fig. 5: Cross correlation coefficient for different ψ and $|\beta_1|$, opposite trends case: $\beta_1^{(1)} = |\beta_1|, \beta_1^{(2)} = -|\beta_1|$

correlation that nears zero. This is related to the large value of the linear trend coefficient, β_1 . Let us examine the dependence of the experimental cross-correlation on both β_1 and ψ . For now, we will consider the case of the opposite trends.

We can see from Fig. 5 that for large values of β_1 , a cross-correlation is not noted. The two main conditions for cross-correlation are a large value of ψ and a small value of β_1 . These results must be taken into account in the simulation study.

ANALYSIS AND DISCUSSION

Our goal in this section is to investigate the performance of PT strategy across different combinations of parameters. We study the effects of varying:

- The correlation parameter ψ
- Linear trends $\beta_1^{(1)}$ and $\beta_1^{(2)}$
- Choice of k

We chose the following values for the simulation:

$$0 < \psi < 1, d\psi = 0.1$$

$$1) \beta_1^{(1)} = 0, 2) \beta_1^{(1)} = 0.01, 3) \beta_1^{(1)} = -0.01, 4) \beta_1^{(1)} = 0.01, \beta_1^{(2)} = -0.01$$

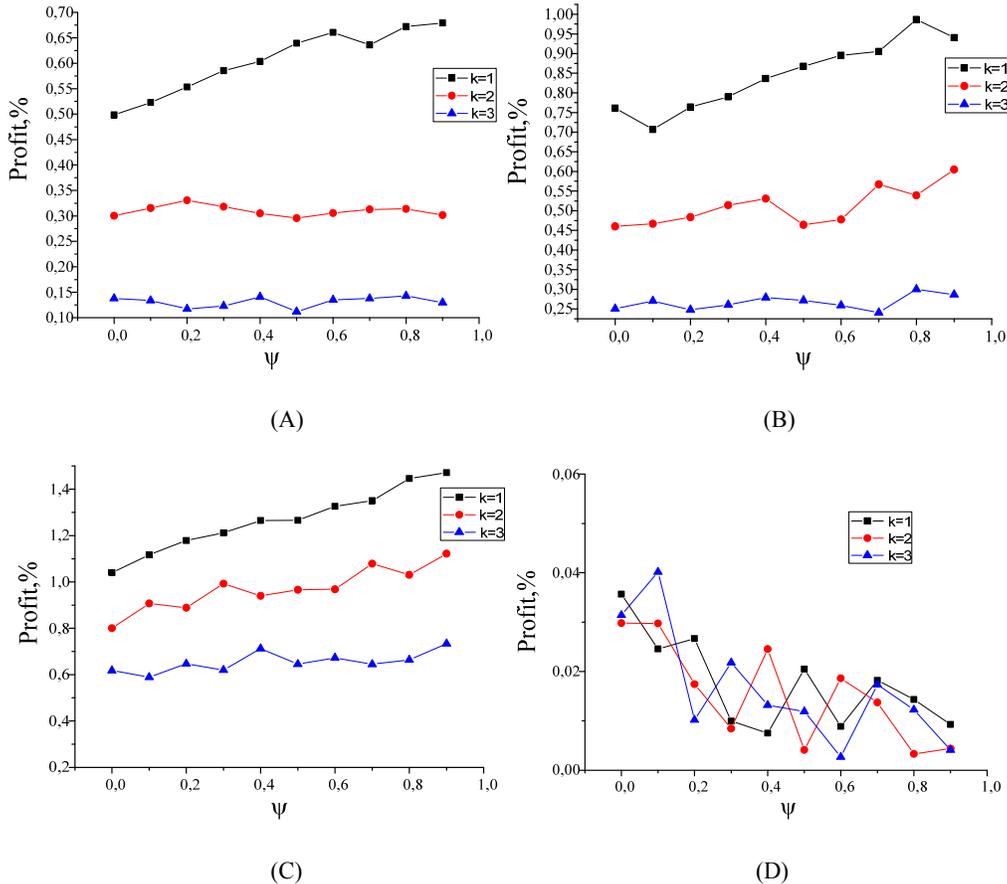


Fig. 6: Average profit for different stock trends and cross-correlation parameters where: A) represents no trends, $\beta_1^{(1)} = 0$, B) represents both increasing, $\beta_1^{(1)} = 0.01$, C) represents both decreasing, $\beta_1^{(1)} = -0.01$, D) represents opposite trends, $\beta_1^{(1)} = 0.01, \beta_1^{(2)} = -0.01$

$$1 \leq k \leq 3$$

Analysis was carried out for all combinations of the given parameters. The approach here was to divide the simulated data by two parts as described in Section 2. The value of interest is the average profit. For each parameter combination, we performed independent simulations and acquired the total profit. This procedure was repeated 100 times and we obtained the total averaged profit. The transaction cost was chosen to be equal to 0.01%.

In the results of this analysis, we will see how the average profit depends on $(\psi, \beta_1^{(i)}, k)$.

It is necessary to examine the results obtained after the simulation. The average profit depends on the cross-correlation, ψ . In the case where $k = 1, 2, 3$ for different slopes $\beta_1^{(i)}$ the parameters are shown in Fig. 6:

Figure 6 A illustrates the ways in which the average profit depends on ψ for zero linear trends and different values of k . First, one can see that the profit is almost linearly related to the cross-correlation factor ψ . Second, it is also possible to make trades with small values of k . The explanation of the first result is straightforward, given that the implementation of the PT strategy is closely related to the cross-correlation of two considered stocks. The second result is related to the given simulation model. We used the equal baselines β_0 and standard deviation σ in (2). Thus, it is logical to expect that for the given simulation parameters and size of the test data, the number of possibilities required to open the trade is small with larger value of k . One can confirm this using the results obtained from the simulation.

Figure 6B and C show that the average profit also grows gradually with increasing of cross-correlation factor ψ . One more interesting feature of the results from this simulation is that the average profit for the equal and negative linear trends was larger than expected for the equal positive linear trends.

Figure 6D shows that the opposite linear trends strongly affect the obtained profit; this is a situation that is not preferred when using the PT strategy.

CONCLUSION

The principles of PT strategy were examined. In particular, examples using a simulated data set as well as a real data set were shown. The vector autoregression model was used in the experiment for data simulation. The detailed analysis of the different combinations of stock parameters was conducted, showing that the PT strategy yields the best performance for highly correlated series with similar trends. By applying the PT strategy to the simulated

data, it was demonstrated that PT can be an effective instrument for profitable trading. Specifically, the most favorable conditions for opening the position were analyzed. It became evident that for highly cross-correlated data, the k parameter (which determines the opening and closing point of trading positions) can be relatively low. The obtained conclusions can be effectively used in real life for actual trading. It has been shown that three main conditions have to be fulfilled in order to successfully implement PT: a large value of cross-correlation factor ψ , parallel linear trends β_1 of stock prices and the use of small values of k .

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