

## Suppress the Finger Reflection Error of Littlewood-pelay Wavelet Transformation Device of Surface Acoustic Wave

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**Abstract:** In this study, a Wavelet Transformation (WT) device of Surface Acoustic Wave (SAW) technology is developed on the basis of acoustics, electronics, wavelet theory, applied mathematics and semiconductor planar technology. The Finger Reflection (FR) error is the primary reason for this kind of device. To solve the problem, a mathematic model of Littlewood-pelay wavelet was established first, which is matched with the model of SAW. Using the methods of split finger and fake finger to design IDT of Littlewood-pelay WT device of SAW with L-edit software, the FR error can be reduced and the equivalent construction of IDT is simulated.

**Keywords:** Finger Reflection (FR), Interdigital Transducer (IDT), l-edit software, littlewood-paley WT, Surface Acoustic Wave (SAW)

### INTRODUCTION

The SAW is one kind of new science and technology since 1960s and it is the union of acoustics and electronics. The SAW is a kind of elastic wave that transmits into the surface of the object. The SAW was explained by Rayleigh who studied earthquake waves (Rayleigh, 1885). The first application of SAW can be traced from the invention of the piezoelectric transducer in the form of an interdigital comb (White and Voltmer, 1965). SAW devices accelerate growth because of their broad uses and their performances are raised constantly, but different errors have made some weakness during SAW device's application (Sankaranarayanan and Bhethanabotla, 2008). Through researching and calculating in the error of Finger Reflection (RF), the study uses different methods to resolve this problem. The analytic conclusion has provided a more stable academic fundamental for manufacture and application.

This paper studied on the theory and structure of WT device of SAW. After calculating the function of Littlewood-paley WT, drawing the envelope curve and constantly changing the width and the height between transducers, we draw the conclusion that the overlap envelope curve is equal to the impulse response curve of Littlewood-paley WT device of SAW. This study also analysis the Finger Reflection (FR) error, which is the primary reason for this kind of device. To

suppressing the error, a mathematic model of Littlewood-pelay wavelet was established first that is matched with the model of SAW and then the methods of split finger and fake finger were used to design IDT of Littlewood-pelay WT device of SAW with L-edit software. Therefore, the FR error can be reduced and the equivalent construction of IDT is simulated.

### STRUCTURE OF INTERDIGITAL TRANSDUCER

The components electrode of SAW has piezoelectric properties, named Interdigital Transducer (IDT), which is polished and produced in the substrate material surface. The structure of IDT is shown in Fig. 1. The principle of SAW is the input of IDT transform electrical signals into acoustic signals through inverse piezoelectric effect and the output of IDT converts acoustic signals to electrical signals (Lu *et al.*, 2005).

The impulse response of WT device of SAW refers to the output signal that is produced by IDT (Lu *et al.*, 2003). In the simple construction, it is very easy to identify parameters of the impulse response. In filtering theory, we know that the impulse response  $\Psi(t)$  and the frequency response  $\Psi(\omega)$  are Fourier and Fourier inverse transform:

$$\Psi(\omega) = \int_{-\infty}^{\infty} \Psi(t)e^{-j\omega t} dt \quad (1)$$

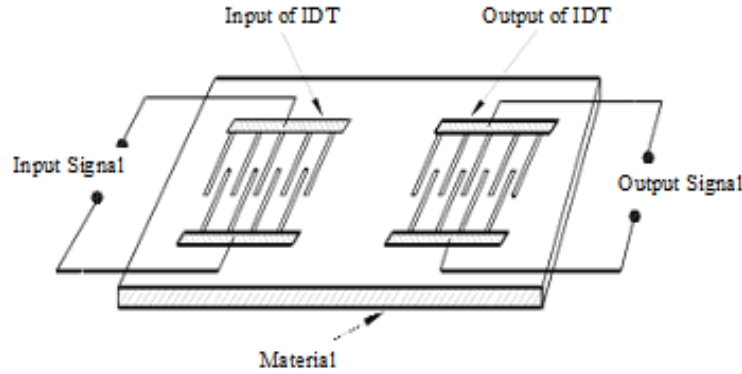


Fig. 1: The structure of IDT

$$\Psi(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \Psi(\omega) e^{j\omega t} d\omega \quad (2)$$

There is one-to-one relationship between the impulse response and the frequency response from the Eq. (1) and (2). The WT device of SAW uses these performances.

### ULTIMATE PRINCIPLE OF WAVELET TRANSFORMATION

**Principle of dyadic wavelet transformation:** The formula of dyadic wavelet transformation is as follows (Lu *et al.*, 2011):

$$\Psi_{2^k, \tau}(t) = 2^{-k/2} \Psi\left(\frac{t-\tau}{2^k}\right) \quad (3)$$

- K : The int number from  $-\infty$  to  $+\infty$
- $2^k$  : The scale factor
- $\tau$  : The displacement
- t : The continuous variable

The wavelet function is  $\Psi(t)$  and the inverse transform is  $\Psi(\omega)$ , if there are two  $\omega$  constants have the limits in  $0 < A \leq B < \infty$ , then:

$$A \leq \sum_{k \in \mathbb{Z}} |\Psi(2^k \omega)|^2 \leq B \quad (4)$$

We define the dyadic wavelet function of  $f \in L^2(\mathbb{R})$  in type (5):

$$WT_{2^k}(\tau) = f(t) * \Psi_{2^k}(t) = 2^{-k/2} \int_{\mathbb{R}} f(t) \Psi\left(\frac{\tau-t}{2^k}\right) dt \quad (5)$$

From the convolution theorem, we know the wavelet transform of  $WT_{2^k}(\tau)$  is:

$$WT_{2^k}(\omega), WT_{2^k}(\omega) = F(\omega) 2^{-k/2} e^{-j\omega\tau} \Psi(2^k \omega)$$

We get the conclusion that:

$$A \|f\|^2 \leq \sum \|WT_{2^k}(\tau)\|^2 \leq B \|f\|^2 \quad (6)$$

The formula (6) explains the  $L_2(\mathbb{R})$  forms the framework of dyadic wavelets  $\Psi_{2^k, \tau}^{(t)}$ .

**Little-wood paley wavelet:** The time domain equation of Little-wood pelay wavelet is defined as:

$$\Psi(t) = \frac{1}{\pi t} (\sin 2\pi t - \sin \pi t) \quad (7)$$

$$\Psi(\omega) = \begin{cases} 1, & \pi \leq |\omega| \leq 2\pi \\ 0, & \text{others} \end{cases} \quad (8)$$

The wave of frequency and time domain are shown in Fig. 2.

We can get:

$$\begin{aligned} \Psi(t) &= \frac{1}{2\pi t} [\sin 3\pi t + \sin \pi t - \sin 2\pi t + i \\ &(\cos 3\pi t - \cos \pi t - \cos 2\pi t + 1)] e^{j\omega_0 t} \quad (9) \\ &= A(t) e^{j\omega_0 t} \end{aligned}$$

Therefore,

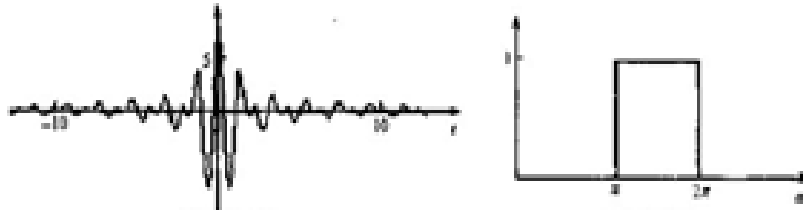


Fig. 2: Time and frequency domain characteristics of Littlewood-pelay wavelet

$$A(t) = \frac{1}{2\pi t} [\sin 3\pi t + \sin \pi t - \sin 2\pi t + i(\cos 3\pi t - \cos \pi t - \cos 2\pi t + 1)]$$

If  $t$  approaches to 0,  $\Psi(t)$  tends to 1  
 If  $t$  approaches to  $\infty$ ,  $\Psi(t)$  tends to 0  
 From the Eq. (9), we know that Littlewood-pelay WT has the similar mathematic model.

### THE FR ERROR OF WT DEVICE OF SAW

The wave will be reflected in the place of an impedance discontinuity through the transmission line theory. In the surface of IDT, wave is transmitted in the internal and the overlap place between some interdigital. This kind of reflection named Finger Reflection (FR), it affects the frequency response of IDT and produces fluctuations in the band pass. There are two reasons to generate the TR error of IDT.

**The electromechanical coupling factor:** When acoustic waves spread through metal fingers, the speed of acoustic waves will decrease and the changing of the acoustic impedance can be explained in the following equation:

$$\frac{\Delta Z}{Z_0} = \left| \frac{\Delta V_s}{V_s} \right| \approx \frac{1}{2} K^2 \quad (10)$$

where,  $\frac{\Delta v_s}{v_s}$  is the normalized frequency shift of wave speed and  $K^2$  is the electromechanical coupling factor of Substrate materials.

From the above equation, it is very clear that lower electromechanical coupling factor of Substrate materials can suppressed the FR error.

**The changing of impedance:** The impedance changed through concentrating masses on metal fingers, which is:

$$\frac{\Delta Z}{Z_0} = \frac{2 \rho_c h}{3 \rho_j \lambda} \quad (11)$$

where,

- $\rho_c$  : The volume density of metal finger
- $\rho_j$  : The volume density of IDT
- $h$  : The thickness of interdigital
- $\lambda$  : The wavelength of acoustic wave

From the Eq. (11), we can draw a conclusion that more thinner number of interdigital and lighter quality of metal, the inflection from IDT can be ignored. The insertion loss uses for this function is:

$$IL = 10 \lg \left| 4 \left( \frac{G}{G_a} + 1 \right)^2 \right| = 10 \lg \left| \frac{\pi}{4K^2} \left( \frac{\Delta f}{f_0} \right)_{\max}^4 \right| \quad (12)$$

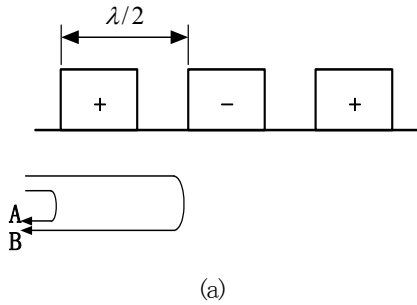
- $G$  : The electrode resistance loss
- $G_a$  : The acoustic radiation transducer conductivity
- $\hat{G}_a$  : The transducer acoustic radiation conductivity of center frequency  $f_0$
- $K^2$  : The electromechanical coupling coefficient
- $\left( \frac{\Delta f}{f_0} \right)_{\max}$  : The maximum relative bandwidth

From (12), it is apparently that higher electromechanical coupling factor could decrease the insertion loss of IDT, which is against the conclusion from Eq. (10). To solve this problem, we use divided finger to reduce the FR error of IDT.

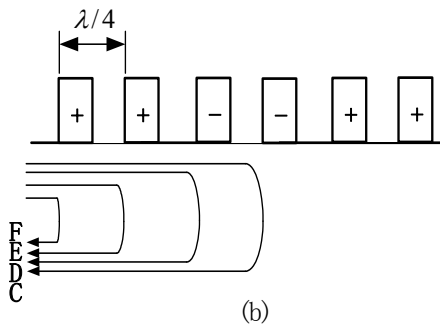
### SOLUTION TO THE PROBLEM OF FINGER REFLECTION ERROR OF SAW

**Using lower electromechanical coupling factor of substrate material:** From formula (10), we choose low electromechanical coupling coefficient of substrate material, which is aroused by the short circuit of metal finger.

So we use the material of X-112°C, Y-litao<sub>3</sub> to design the WT device of SAW, which reduce the FR error of WT device of SAW and get the conclusion that the smaller center frequency of WT device of SAW, the smaller the FR error is.



(a)



(b)

Fig. 3: The sketches of single and split finger edge reflection of IDT (a) Single finger reflection of IDT (b) Split finger reflection of IDT

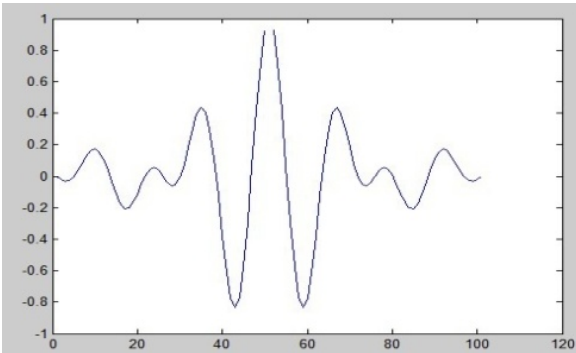


Fig. 4: Simulation of littlewood-pelay wavelet

**Reconstruction of IDT:**

**Split finger of IDT:** In Fig. 3a, the period of single finger reflection of IDT is  $\lambda/2$ . The transmission reflection distance of acoustic wave B is  $\lambda/2 + \lambda/2 = \lambda$ , and its phase difference is as same as wave A that is  $2\pi$ . The sum of acoustic wave A and B will enhance the edge reflection of IDT.

From Fig. 3b we know that the period of split finger reflection of IDT is  $\lambda/4$ . The transmission reflection distance of acoustic wave E is  $\lambda/4 + \lambda/4 = \lambda/2$ , and its phase difference is in contrast to wave F that is  $\pi$ . The wave D and C can be cancelled each other out.

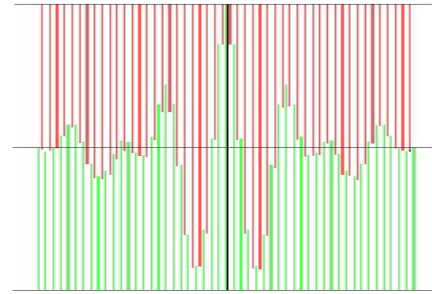


Fig. 5: Single finger diagram of IDT

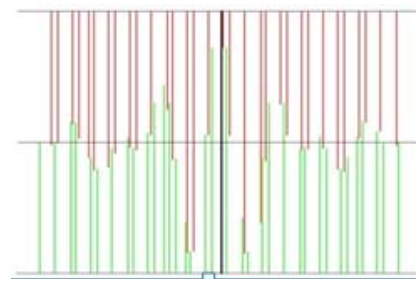


Fig. 6: Split finger diagram of IDT

Comparative of the above analysis, we can see that using split fingers can eliminate the FR error. We design the IDT by L-edit software and draw coordinates by Matlab, the program is as follows, the graph is in Fig. 4:

```
t = -4: 0.08: 4
for i = 1: length(t)
y(i) = (sin(2*pi*t(i)) - sin(pi*t(i)))/(pi*t(i))
End
plot(y)
```

Single finger diagram of IDT is drawing in Fig. 5, where,  $a_1 = b_1$  and  $\lambda/2 = 2a_1 = 2b_1$ ,  $a_1$  and  $b_1$  are the width and space of IDT, the unit is  $\mu\text{m}$ ;  $\lambda$  is the wavelength of littlewood-pelay wavelet.

Therefore,  $a_1 = b_1$ ,  $\lambda/4 = v_s / 4f_0$ ,  $f_0$  is the center frequency, which is 10 MHz-3 GHz and  $v_s$  is the transmission speed of wavelet, the unit is m/s. Among them  $a_1 = b_1 = 40 \mu\text{m}$  and  $f_0 = 10.3 \text{ MHz}$ .

The split finger diagram shows that  $\lambda/4 = 2a_2 = 2b_2$  and  $a_2 = b_2$ ,  $\lambda/8 = v_s / 8f_0$ , parameters of  $a_2, b_2, \lambda, f_0, v_s$  are in accord with the single finger of IDT.

In Fig. 6 the split finger diagram of IDT is designed by L-edit, which  $a_1$  equals to  $b_1$ , which is 40  $\mu\text{m}$  and  $f_0$  equals to 10.3 MHz.

**Fake electrodes of IDT:** For long overlap weighted fingers, the metal electrodes reflection will produce wave front distortion. Each pair of transducer simulates

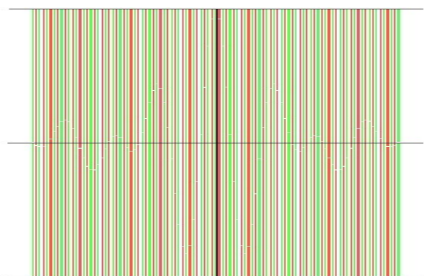


Fig. 7: Fake electrodes of IDT

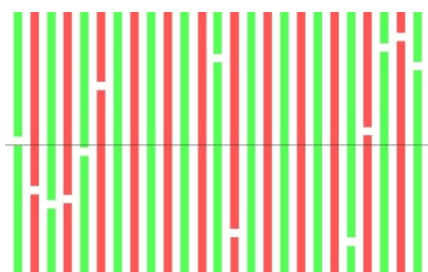


Fig. 8: Detailed graph of fake electrodes of IDT

waves, which transform through many rest electrodes. If we divide the pair of transducer into several small fingers, the numbers of electrode fingers transmission from acoustic waves that generated by each small finger are different. Acoustic waves located in the middle section produce the maximum number of electrodes, which are on the contrary to the least number of electrodes on the side section. For the different speed of acoustic waves with metal electrodes area and no metal electrodes area, the phase difference will be inevitably produced between the middle and side sections, thus makes the wave distortion that is obviously not allowed.

In order to solve this problem, we add fake electrodes to the long overlapping weighted section. These fake fingers connect to the opposite of the confluence of IDT that can reduce the potential phase difference as shown in Fig. 7. Figure 8 refers to the detailed picture of the graph 7.

The method to draw with L-edit is as same as the content in 4.2.1, we still have got 100 coordinate points and made  $a = b = 40 \mu\text{m}$  and  $f_0 = 10.3 \text{ MHz}$ . In this way, fake fingers will compensate the phase.

### CONCLUSION

This study studied on the theory and structure of WT device of SAW. After calculating the function of

Littlewood-paley WT, drawing the envelope curve and constantly changing the width and the height between transducers, we draw the conclusion that the overlap envelope curve is equal to the impulse response curve of Littlewood-paley WT device of SAW. This study also analysis the Finger Reflection (FR) error, which is the primary reason for this kind of device. To suppressing the error, a mathematic model of Littlewood-paley wavelet was established first that is matched with the model of SAW and then the methods of split finger and fake finger were used to design IDT of Littlewood-paley WT device of SAW with L-edit software. Therefore, the FR error can be reduced and the equivalent construction of IDT is simulated.

### ACKNOWLEDGMENT

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