

## Loss Allocation Using Game Theory in Pool Based Market

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**Abstract:** This study presents a new and practical way for the loss allocation in the restructuring systems problem. The restructured markets sell the electricity in two main categories; bilateral exchanges and pool based. The method which is used in this study investigates the loss allocation in pool based market. The deregulated systems are not under control of one person but there are other players such as generators and loads at which every one of such players has to pay the cost for some parts of network loss. The importance of this matter is that the loss ratio is a considerable part of the whole production. The method used in this study is to justify the loss allocation. This method is consisted of two different categories; finding the losses and the other is loss allocation using Game Theory. And to test this method, two systems of 4 and 14 IEEE bus is put in use. The results referring the generators show that the suggested method for the loss allocation to generators is close to the Pro Rata method and the results for the loads are something between the Proportion method and the ITL method.

**Keywords:** Game theory, loss allocation, pool based market, shapley value

### INTRODUCTION

Upon the deregulation practice in electricity market, many of its rules have changed. Such a change requires organizing and setting new rules by the ISO to maintain system stability, balance, economic operation and safety Lim *et al.* (2006). A problem that would be revealed after restructuring is the loss allocation Belati and da Costa (2008). Importance of this problem would be clearer considering to the fact that the range of these losses is expressed at 4 to 8% of the total product in different references, for instance for Brazil where such losses cost only half of a billion dollars Belati and da Costa (2008). Different methods have been used to investigate the loss allocation, which here are the most important ones:

- Pro Rata Method Lim *et al.* (2006)
- Proportional Sharing Belati and da Costa (2008)
- Z-bus Method Antonio *et al.* (2001)
- Modified Z-bus Method Parastar *et al.* (2011)
- Marginal Allocation Method: This method uses

ITL coefficients for the loss allocation in which the coefficients are equal to the change of the entire losses made by change of power injection to a specific bus Connejo *et al.* (2002). In this study the Theory of Game is used for loss allocation in pool based market. Although Game Theory does not have a long term history in

science, due to its high capability and applicability, the use of this theory is increasing in different branches. The applications of Game Theory can be investigated in two completely distinguished categories- anticipation, fair sharing and finding the shares of other players in the game. The first use of Game Theory is to assign the market price and suggested price for the generators (Zhenglin *et al.*, 2006; Yuan *et al.*, 2008; Yadollahi *et al.*, 2010), and the second use is to allocate transmission cost (Mepokee *et al.*, 2004; Filipe *et al.*, 2009). The principle of Game Theory use can be found in reference (Kattuman *et al.*, 1999) in which Shapley Value method is used to find power consumption. In this study the method for losses calculation, which is the same as AC Load Flow has been explained, then for the cooperation games Shapley Value method has been described. Finally, in simulation part, the results of proposed method, which is applied on two sample networks, 4 and 14 IEEE Standard Network, has been shown. Must be noted that loss allocation does not mean finding losses, but it is a mechanism for load flow Connejo *et al.* (2002).

### LOSS FINDING METHODOLOGY

To find losses, AC load flow and solving it by Newton-Raphson Method is used (Ahad, 1999). the matter of load flow or power flow means presenting a solution for finding voltages; power flow in lines;

generators reactive power, line losses and etc. which these calculations are done in steady state. To solve the load flow problem, the following equation must be done:

$$p_i - jq_i = v_i^* \sum_{j=1}^n y_{ij} v_j \quad (1)$$

By solving the load flow equation, the bus voltage could be found. Then using the following relations, we can find out the losses:

$$p_{ij} + jq_{ij} = v_i \left[ \frac{v_i - v_j}{z_k} + \frac{1}{2} y_k v_j \right]^* \quad (2)$$

$$p_{ji} + jq_{ij} = v_j \left[ \frac{v_j - v_i}{z_k} + \frac{1}{2} y_k v_i \right]^* \quad (3)$$

$$\Delta p_{ij} = \Delta p_{ji} = |p_{ij} + p_{ji}| \quad (4)$$

$$\Delta q_{ij} = \Delta q_{ji} = |q_{ij} + q_{ji}| \quad (5)$$

For every coalition, losses for every single line must be found and summed together in order to achieve a coalition from total loss.

**Using cooperation games theory:** Cooperation games theory is a method in which the specific share for every player could be found from one operator (Kattuman *et al.*, 1999). Game Theory also is used in power systems for the process of Transmission Cost Allocation (Mepokee *et al.*, 2004; Filipe *et al.*, 2009). The Game Theory by itself has a variety of branches and methods that we use the Shapley value method which is a method for cooperative games. First the players for this game must be specified. In bilateral exchanges markets, every exchange is called a player for this game. Since it is not specified which generator supplies which load, so afore mentioned assumption cannot be used. Here we are going to flow the load from the winner generators. To flow this load among generators, first we have to find out the producing ratio every generators to total production and multiply it to the load size:

$$x_i = \sum_{\forall s; i \in s} p_n(s) [v(s \cup \{i\}) - v(s)] \quad (6)$$

$$p_n(s) = (|s|!(n-|s|-1)!)/n! \quad (7)$$

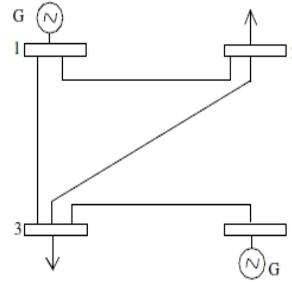


Fig. 1: Single line diagram of 4 bus network

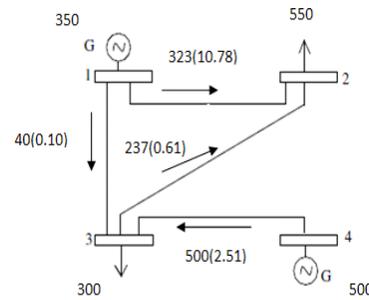


Fig. 2: Flow of 4 bus network

Table 1: Information about 4 bus network

Branch	R (Ω)	X (Ω)	b/2 (Ω)
1-2	0.01	0.04	0.0141
1-3	0.0012	0.05	0.0192
2-3	0.001	0.04	0.0153
4-3	0.001	0.03	0

Table 2: Generators suggested price

Genq	P <sub>Gen</sub> (MW)	Price (\$/MWh)
1	350	8
4	500	10
3	60	12

Table 3: Product and load for every bus after market clearing

Bus	P <sub>gen</sub> (MW)	P <sub>load</sub> (MW)
1	350	0
2	0	550
3	0	300
4	500	0

In which  $i$  = number of the players,  $S$  = coalition,  $|s|$  = number of the  $S$  coalition players,  $n$  = number of the total players,  $V(s)$  = loss in  $S$  coalition and  $V(s-\{i\})$  = loss in the  $S$  coalition without the  $i$  players

### CASE STUDIES

Two case studies have been used in results test. A 4 bus system is illustrated in Figure 1 and is mentioned in reference (Clodomiro *et al.*, 2004) with results comparison and the other one is a 14 bus system which is meant to be used for surveying the capability of this method at putting in practice in larger systems.

Table 4: Losses in coalition

Players	P loss (MW)
0	0
1	9.657
2	1.034
1,2	13.98

Table 5: Allocated loss to the players

Player	P <sub>loss</sub> (MW)
1	11.301
2	2.678

Table 6: Loss distribution among the generators and loads of players

Number of player	Number of load	Loss allocation to load (MW)	Number of gen	Loss allocation to gen (MW)
1	2	5.650	1	2.378
			4	3.265
2	3	1.339	1	0.563
			4	0.773

Table 7: Allocated loss to the loads and generators

Name	P loss (MW)
G1	2.941
D2	5.650
D3	1.339
G4	4.038
Total	13.968

**Case study for 4 bus system:** At first we have to specify the players for this game. We would flow the load among the succeeded generators in the market. In order to flow this load among the generators, first we have to find out the producing ratio of every generators from the total production and multiply it to the load size.

The specification of this system is as follows:

For example, for the top 4 bus system assuming the total load of 850 MW and generators price, we have according to the following Table 1.

According to the amount of 1 and 4 succeeded generator's load, the Market Clearing Price (MCP) would be 10 (\$/MWh). According to the following Table 2 which shows the product and load in whole system after Market-Clearing, we have:

According to the aloft Table 3 the share for the generator no. 1 is 350/850 and share for the generator no. 4 is 500/850, so in order to determine the players we have:

Player no 1	Player no 2
$P_{G1} = 550 \times 0.421 = 231.7$	$P_{G4} = 550 \times 0.578 = 318.30$
$P_{G1} = 300 \times 0.421 = 126.43$	$P_{G4} = 300 \times 0.578 = 173.64$
$P_{L2} = 550$	$P_{L2} = 0$
$P_{L3} = 0$	$P_{L3} = 300$

Upon using the MATPOWER (Zimmerman *et al.*, 2006) to find losses we have Table 4. Now using the Shapley value method we have Table 5. In order to find loss we must act this way: half of the loss for every player

Table 8: Comparing results

Bus number	P <sub>loss</sub> <sup>b</sup>	Pro rata	ITL	SV
1	4.54	2.87	4.48	2.941
2	7.02	4.51	6.90	5.650
3	0.20	2.46	0.09	1.339
4	2.22	4.10	2.51	4.038
Total	13.98	13.968	13.98	13.968

Table 9: Market clearing in 14 bus network

Bus	P gen (MW)	P load (MW)
1	16	0
2	42	0
3	50	0
4	0	50
5	0	0
6	50	0
7	0	0
8	0	0
9	0	0
10	0	0
11	0	0
12	50	50
13	0	0
14	50	50

Table 10: Generation's coefficient

Gen number	P gen (MW)	Generator coefficient
1	16	0.08
2	42	0.21
3	50	0.25
6	50	0.25
8	42	0.21
Total	200	1

is for the load and half related to the generator. We distribute the related loss to the generators according to the supply submultiples according to Table 6. Therefore by summing related loss to generator, every generator and every load we can find out the load loss and generators loss according to Table 7.

As it is clear from the digits of Table 7 most allocated loss is referred to the load 2 which its proportion is 550 MW and it's relating line to the closer generator (generator 1) has got the most resistance. The least allocated loss goes to the load 3 which has relationship with both generators and its power is 300 MW and almost is located at the center of the generators. Though is the suggested method both network specification factors and power proportion have come to attention. Comparing the achieved results with this method to the previous methods is illustrated in Table 8.

As it is mentioned in the Table 8 the order for the loss allocation for all three methods is somehow the same, but the amount of loss allocation is different. Most difference goes to the generator 4 and the least loss to the load 3. In order to survey the results of the following figure we consider the items in which bus powers, line loss and crossing power through the lines are specified.

The digits on the arrow show the line transmitting power and the digits in bracket are the line loss. The most losses to line are between buses of 1 and 2 and the least

Table 11: Player introduction

Bus number	Player 1	
	P load (MW)	P gen (MW)
1	0	50×0.08
2	0	50×0.21
3	0	50×0.25
4	50	0
5	0	0
6	0	50×0.25
7	0	0
8	0	50×0.21
9	0	0
10	0	0
11	0	0
12	0	0
13	0	0
14	0	0
Total	50	50

Table 12: Characteristic function for all coalition of players

Number	P loss	Player
1	0	0
2	1	0.723
3	2	0.765
4	3	3.105
5	4	2.657
6	1,2	1.100
7	1,3	3.420
8	1,4	3.179
9	2,3	3.640
10	2,4	3.165
11	3,4	5.660
12	1,2,3	4.081
13	1,2,4	3.814
14	1,3,4	6.288
15	2,3,4	6.456
16	1,2,3,4	7.214

Table 13: Allocated loss to the players

Player	P loss(MW)
1	0.611
2	0.715
3	3.122
4	2.764

Table 14: Loss allocation flow to the generators and loads

Player	P loss (MW)	P loss allocated load (MW)	P loss allocated gen (MW)
1	0.611	P14 = 0.3055	P gen 1 = 0.024 P gen 2 = 0.064 P gen 3 = 0.076 P gen 6 = 0.076 P gen 8 = 0.064
2	0.715	P15 = 0.3575	P gen 1 = 0.028 P gen 2 = 0.074 P gen 3 = 0.087 P gen 6 = 0.087 P gen 8 = 0.074
3	3.122	P112 = 1.561	P gen 1 = 0.124 P gen 2 = 0.327 P gen 3 = 0.390 P gen 6 = 0.390 P gen 8 = 0.327
4	2.764	P114 = 1.382	P gen 1 = 0.110 P gen 2 = 0.290 P gen 3 = 0.345 P gen 6 = 0.345 P gen 8 = 0.290

Table 15: Allocated loss to the generators and loads

Bus	P loss (MW)
1	0.286
2	0.755
3	0.898
4	0.3055
5	0.3575
6	0.98
8	0.755
12	1.561
14	1.382
Total	7.28

losses are between buses of 1 and 3 which connect 3 MW power to the bus 3. So 40 MW of bus 3 is supplied by the generator 1 and 260 MW is supplied by the generator 4 and the remainder of the bus 4 which is 240 MW will be sent to the bus 1 to get supplied. More Loss allocation takes system to a modified point. (Closing the loads and generators to lessen the loss) which is purpose among the loss allocation. There are other advantages of this method rather than the previous method such as: no need to calculate reverse matrix so the calculations are easier. In this method we can dedicate all the losses to the generators or only to the loads or according to the market type distribute them within the loads and generators equally.

**Case study for 14 bus IEEE network:** The specification of the IEEE 14 bus system is mentioned in reference (DU *et al.*, 2006). Considering the Market Clearing Price (MCP) we have according to Table 9.

In order to introduce players, the share for every generator from the total product must be found which we call this coefficient, the power supply coefficient and we have Table 10.

Players are equal to the load or 4, which we can see player 1 in the following Table 11 so do other players.

According to the players' determination, different coalitions must be formed which according to 4 players now we have 16 coalitions, so by the following Table 12 we form the coalition and find the loss at any situation. Now upon the Shapley method we can find the loss allocated power to the players according to Table 13.

According the every load and generator, we have Table 14.

According the players and sum of related loss to the generators and specific loads, we have Table 15.

## CONCLUSION

According to the change in electricity market from traditional to deregulation, loss allocation is necessarily unavoidable. The importance comes from the matter that the nonlinear functional loss of the power, therefore a method should be used which considers both the players (generators and loads) and network's features. The method has been used in this study is based on cooperating Game Theory. This method has been applied

on two systems of 4 and 14 buses. The advantage of this approach rather than the previous ones is that, this method does not need the inverted matrix, which also includes the active and reactive losses. Both factors consider the network's feature and support decreasing losses.

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