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An Amended Trinomial Tree Model Based on China Convertible Bonds Market

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Abstract: With the actual data in China convertible bonds market, the author tries to derive the new parameter relationship which reflects the law of price movement of the underlying stock in China and replaces the assumption of the traditional trinomial model and derive an amended trinomial tree model based on the new parameter relationship, promoting the development of pricing models as well as the convertible bond in China. The traditional trinomial tree model has a higher pricing efficiency than other traditional pricing models. However, its assumption on the movement law of the underlying stock price of convertible bonds is not suitable for China, which would loss its pricing efficiency in China convertible bonds market.

Keywords: Pricing efficiency, the amended trinomial tree model, the convertible bond, the traditional trinomial model

INTRODUCTION

The convertible bond is an extremely complex financial derivative and the pricing efficiency of the existed models is generally not high. The earliest scholars to study the pricing of convertible bonds are Ingersoll (1977) and Brennan and Schwartz (1977). Ingersoll (1977) assumed no cash dividends and the convertible bond was a discount bond which did not pay coupon interest. He developed and used the arbitrage theory to make sure the optimal redemption strategy and the optimal conversion strategy for investors and the issuers. Brennan and Schwartz (1977) assumed that the convertible bond value was a function of the company value and time, using the finite difference method to solve the Partial Differential Equations (PDE). McConnell and Schwartz (1986) assumed that the only variable which impacted the value of convertible bonds was company stock price and company stock price followed a geometric Brownian motion whose volatility was constant. They derived the partial differential equation satisfied by the convertible bond value in the framework of the Black-Scholes pricing model and obtained the convertible bond value by determining the boundary conditions of equation. Tsiveriotis and Fernandes (1998) divided innovatively convertible bonds value into the cash value and the equity value and used two equations to describe, whose discount rate was different. The model is referred to TF model (1998). Compared to the foreign theoretical models and innovation emerging continuously, domestic theoretical models and innovation are obviously insufficient. Zheng and Lin (2004a, b) used the basic principles and methods of financial engineering, getting the conclusion that there was a big difference between the issue price of first day and the theoretical value, which meant the value of the convertible bond was significantly undervalued. Lai *et al*. (2001) used the TF model (1998) combined with the binary tree model and MATLAB, getting the conclusion: The theoretical price of China convertible bonds was lower than the actual price and the convertible bond was overvalued, which was different from most of the existed conclusions.

On the pricing methodology, the pricing model is classified as analytical and numerical methods (Wilmott *et al*., 1995). The analytical method is the BS model and the numerical method is the Monte Carlo simulation, the tree graph models and the finite difference method (Kariya, 2000; Kwok and Lau, 2001). Existed literatures have proved the pricing efficiency of the traditional trigeminal tree model is higher than other traditional models (Sun and Zu, 2012; Zhang and Yang, 2007; He, 2007). However, its assumption (CRR parameter) on the movement law of the underlying stock price of convertible bonds is not suitable for China convertible bonds market, which would loss its pricing efficiency (Ding and Zeng, 2005; Guo and Zhang, 2003). This study improves this defect. CRR parameter refers to the parameter imposed by Cox, Ross, Rubinstein in the process of model derivation, which is $w = -v$. v is the upstream value of the logarithm of the underlying stock's closing price at each node and w is the downstream value of the logarithm of the underlying stock's closing price at each node, the same below.

THE DERIVATION OF RELATIONSHIP

The traditional trigeminal tree model uses the CRR parameter subjectively. This section will derive a new parameter relationship based on actual data to verify the suitability of CRR parameter for China convertible

Table 1: V and W summary table

Abbreviation	V	W	Abbreviation	V	W
XinGang	0.01575	-0.02046	Cheng Xing	0.02368	-0.03241
BoHui	0.01248	-0.01765	Zhong hang	0.00789	-0.00826
Shuang Lang	0.01419	-0.01914	Gong Hang	0.01192	-0.01159
Ge Hua	0.01316	-0.01845	Shen Ji	0.01043	-0.01177
HaiYun	0.01519	-0.01819	Tang Gang	0.01476	-0.01876
GuoTou	0.01247	-0.01408	Mei Feng	0.01616	-0.01986
Shi Hua	0.00997	-0.01032	Zhong Ding	0.01953	-0.02527
ChuanTou	0.01371	-0.01684	Yan Jing	0.01041	-0.01289
Zhong Hai	0.01255	-0.01612	JunLun2	0.01963	-0.02369
GuoDian	0.01245	-0.01257			

Primary data: DaZhiHui software

Table 2: Regression analysis table

Dependent variable: W
Method: Least squares
Date: 06/16/12/time: 20:43
Sample: 1 19
Included observations: 19

Variable	Coefficient	S.E.	t-Statistic	Prob.
C	0.003446	0.001333	2.585425	0.0193
V	-1.479200	0.091997	-16.07877	0.0000
R-squared	0.938300	Mean dependent var		-0.017279
Adjusted R^2	0.934671	S.D. dependent var		0.005793
S.E. of regression	0.001481	Akaike info criterion		-10.09333
Sum squared reside	3.73E-05	Schwarz criterion		-9.993915
Log likelihood	97.88663	F-statistic		258.5268
Durbin-Watson stat	1.719261	Prob. (F-statistic)		0.000000

bonds market and pave the way for the following derivation of the amended trinomial tree model.

The sample and data processing:

- **The Sample:** the daily closing price of the underlying stock of 19 currently listed convertible bonds in Chinese convertible bonds market from January 5, 2011 to December 15, 2011, a total of 235 trading days. The closing price is the one of the previous day if there is no trading in the trading day.
- **Data Processing:** take the logarithm of closing price of each trading day and calculate *v* and *w* by the logarithm of the closing price of the underlying stock minus the one of the previous day. Aggregate *v* and *w* and calculate the average respectively. The average of the upstream value is *V* and the average of the downstream value is *W*. Deduce the parameter relationship $W = f(V)$ by regression analysis and test the statistical significance of the regression equation.

The derivation of relationship: By collecting data and calculating, we can get Table 1. Based on the data, the scatter plot can be gotten.

In Table 2, the coefficient of determination is 0.938300 and the F-distribution value is 258.5268 which is significantly greater than the critical value $F_{0.05}$ (1, 17) = 4.45, which mean the overall regression equation is a linear significance. The t-distribution value is -16.07877 whose absolute value is significantly greater than the critical value $t_{0.025}(17) = 2.11$, which means the coefficients of regression equation are significant. Next, test whether residuals meet the regression assumptions. First, test residuals independence with D-W statistic. In Table 2, DW statistic is 1.719261 and du = 1.28, 4-d_u = 2.72 so residuals are independent. Second, test heteroscedasticity with white statistic.

Table 3: White heteroskedasticity test

F-statistic	0.44948	Probability	0.64576
Obs*R^2	1.01074	Probability	0.60328

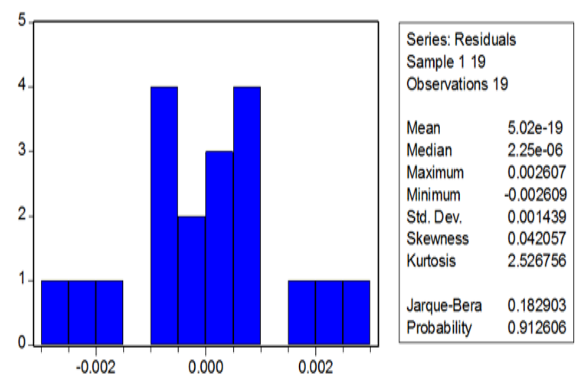


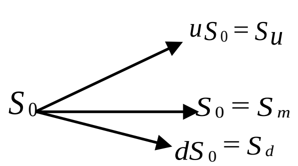
Fig. 1: Residuals normality test

From Table 3, we can see the accompanying probability of white statistic is 0.60328, in the 5% significance level, accept the null hypothesis: the same variance. Last, test residuals normality with residuals histogram and Jarque-Bera statistic.

From the Fig. 1, we can see although residuals histogram is not very standard for the bell-shaped, the accompanying probability of Jarque-Bera statistic is 0.912606, in the 5% significance level, accept the null hypothesis: residuals fit the normal distribution. Sum to up, the relationship is W = -1.4792V+0.003446.

THE DERIVATION OF THE AMENDED TRINOMIAL TREE MODEL

The eigenvalues of the trinomial tree model include size and probability of price changing. Focus on the logarithm price of the underlying asset at the first phase of the ternary tree, which is ln $S_{\Delta t}$. ln $S_{\Delta t}$ fits the normal distribution whose mean is ln $S_0 + \mu\Delta_t$ and variance is $\sigma^2\Delta t$. Δt is the length of a time period; S_0 is the initial price of the underlying stock; $S_{\Delta t}$ is the price of the underlying asset at the first phase of the ternary tree; μ is the expectation of Logarithmic of return rate in a time period; σ is the standard deviation of Logarithmic of return rate in a time period, the same below. The one phase trigeminal tree model is:



$$\ln S_u = \ln u + \ln S_0 = v + \ln S_0; \ln S_d = \ln d + \ln S_0 = w + \ln S_0$$

Thus, we can get the following equations:
Mean constraint:

$$E(\ln S\Delta_t) = \ln S_o + \mu\Delta_t = p_u \ln S_u + P_m \ln S_m + p_d \ln S_d \text{ simplified to } P_u v + P_d w = \mu\,\Delta t \qquad (1)$$

Variance constraint:

$$p_u v^2 + p_d w^2 = \sigma^2\Delta t \qquad (2)$$

Deduced by the following formula:

$$D(\ln S_{\Delta t}) = \sigma^2\Delta t = p_u[v + \ln S_0 - E(\ln S_{\Delta t})]^2 + p_m[\ln S_0 - E(\ln S_{\Delta t})]^2 + p_d[w + \ln S_0 - E(\ln S_{\Delta t})]^2$$

P_u = The probability of the logarithm price or price increasing at each node
p_m = The probability of the logarithm price or price not changing at each node
p_d = The probability of the logarithm price or price declining at each node
u = The rate of the price increasing at each node
d = The rate of the price declining at each node

The third constraint is taken from the research of Boyle and Kamrad and Ritchken (referred to as KR parameters): $v = \lambda\sigma\sqrt{\Delta t}$ (3), $\lambda \geq 1$. The fourth constraint for the traditional trinomial tree model is CRR parameter: $w = -v$ (4), which is not suitable for convertible bonds market in China; for the amended trinomial tree model is the new parameter relationship: W = -1.4792V+0.003446 (5), which has been deduced in the study .With the (1) (2) (3) (4), we can deduce the traditional trinomial tree model:

$$p_u = \frac{1}{2\lambda^2} + \frac{\mu\Delta t}{2\lambda\sigma\sqrt{\Delta t}},\ p_m = 1 - \frac{1}{\lambda^2},\ p_d = \frac{1}{2\lambda^2} - \frac{\mu\Delta t}{2\lambda\sigma\sqrt{\Delta t}},$$
$$u = e^v = e^{\lambda\sigma\sqrt{\Delta t}},\ d = e^w = e^{-\lambda\sigma\sqrt{\Delta t}},\ v = \lambda\sigma\sqrt{\Delta t};\ w = -\lambda\sigma\sqrt{\Delta t}$$

With the (1) (2) (3) (5), we can deduce the amended trinomial tree model:

$$P_u = \frac{9}{10}A,\quad P_m = 1 - \frac{1}{\lambda^2} - \frac{1}{2}A,\quad P_d = \frac{1}{\lambda^2} - \frac{2}{5}A,\quad A = \frac{\lambda\mu\sqrt{\Delta t} + \sigma}{\lambda^2\sigma}$$
$$u = e^{\frac{2}{3}\lambda\sigma\sqrt{\Delta t}},\quad d = e^w = e^{-\lambda\sigma\sqrt{\Delta t}},\quad v = \frac{2}{3}\lambda\sigma\sqrt{\Delta t},\quad w = -\lambda\sigma\sqrt{\Delta t}$$

CONCLUSION

With the actual data in China convertible bonds market, the Table 1: V and W Summary table can be get and base Table 1, we have derived the parameters relationship: W = -1.4792V+0.003446 which reflects the law of price movement of the underlying stock in China convertible bonds market and replaces the one of the traditional trinomial model. From Fig. 2: Regression scatter plot, Fig. 1: Residuals normality test and Table 3 White heteroskedasticity test, we can know that the new

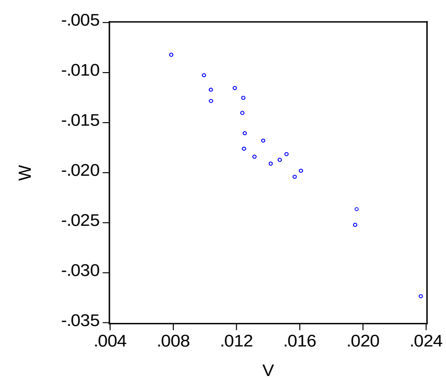


Fig. 2: Regression scatter plot

parameters relationship is established. We have derived an amendments trinomial tree model based on the new parameter relationship. At present, the pricing efficiency of China convertible bonds pricing models is not high, which is uncoordinated with the rapid and stable development of China convertible bonds in recent years. Design pricing model with higher pricing efficiency based on the convertible bonds market is imperative. The amended trinomial tree model provides a more accurate model for China convertible bonds and also provides a feasible idea for convertible bonds pricing model. It is conducive to the faster and better development of China convertible bonds.

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REFERENCES

Brennan, M.J. and E.S. Schwartz, 1977. Convertible bonds: Valuation and optimal strategies for call and conversion [J]. J. Financ., 32: 1699-1715.

Ding, Z.Z. and H. Zeng, 2005. The traditional trinomial model of real options [J]. Stat. Res., 11: 25-28.

Guo, Z.J. and C.Q. Zhang, 2003. The option pricing under multiplicative triple tree model [J]. School Mag. South China Agric. Univ. Soc. Sci., 2: 61-65.

He, Y.Y., 2007. The compare about the convergence speed of the binary tree model and the traditional trinomial model in American option [J]. School Mag. Hangzhou Normal Univ. Nat. Sci., 6: 424-429.

Ingersoll, J., 1977. A contingent claims valuation of convertible securities [J]. J. Financ. Econ., 4: 289-322.

Kariya, T., 2000. CB-time dependent markov model for pricing convertible bonds [J]. Asia-Pacific Financ. Market., 4: 239-259.

Kwok, Y.K. and K.W. Lau, 2001. Pricing algorithms for options with exotic path dependence [J]. J. Deriv., 9: 28-38.

Lai, Q.N., Z.W. Yao and C.H. Wang, 2005. Empirical research on the pricing of convertible bonds in China [J]. Financial Res., 9:105-121.

McConnell, J.J. and E.S. Schwartz, 1986. LYON taming [J]. J. Financ., 41(3): 561-576.

Sun, Q.D. and H.Y. Zu, 2012. The application of the traditional trinomial model in pricing convertible bonds [J]. Sci. Technol. Eng., 9: 2267-2270.

Tsiveriotis, K. and C. Fernandes, 1998. Valuing convertible bonds with credit risk [J]. J. Fixed Income, 3: 95-102.

Wilmott, P. and S. Howison and J. Dewynne, 1995. The Mathematics of Financial Derivatives [M]. Cambridge University Press, Cambridge.

Zhang, J.H. and S.C. Yang, 2007. The traditional trinomial model in pricing convertible bonds [J]. School Mag. Taiyuan Normal Univ. Nat. Sci., 6: 8-11.

Zheng, Z.L. and H. Lin, 2004a. On the pricing of convertible bonds in China [J]. J. Xiamen Univ., 2: 93-99.

Zheng, Z.L. and H. Lin, 2004b. The stochastic dominance test of Chinese convertible bond market efficiency [J]. Contemp. Financ., 3: 44-46.