

Nonlinear Arash Model in DEA

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Abstract: This study illustrates the nonlinear Arash Model (AM) in Data Envelopment Analysis (DEA) to distinguish between technical efficient DMUs and arrange both technical efficient and inefficient DMUs at the same time. It demonstrates how the proposed model it is able to eliminate the computational complexity of using most super-efficiency with selecting the variety of weights and scale. The nonlinear AM optimizes the efficiency score of linear AM, too. Some related propositions to the proposed model are proved which are also examined with a numerical example. The results clearly depict the differences between nonlinear and linear Arash Model and introduce the nonlinear AM as a valuable model in DEA.

Keywords: Arash method, data envelopment analysis, efficiency, technical efficiency

INTRODUCTION

In the last three decades Data Envelopment Analysis (DEA) which is a nonparametric method in operations research has rapidly applied in many branches of sciences to measure the efficiency of firms or Decision Making Units (DMUs). DEA estimates the performance evaluation of homogenous DMUs with multiple inputs and multiple outputs. It was proposed by Charnes *et al.* (1978) based on the earlier work of Farrell (1957). There are many methods in DEA which are made based on the conventional DEA techniques such as radial approaches and non-radial ones. However, those techniques are not able to distinguish between technical efficient DMUs and efficient ones where there are no any weights or cost information for input and output values of DMUs. Therefore, Khezrimotlagh *et al.* (2012a) recently proposed a new technique in DEA called Arash Method (AM) to characterize the efficient DMUs among the technical efficient ones with many other valuable properties such as arranging both technical efficient and inefficient DMUs at the same time which eliminates the computational complexities of using most super-efficiency models. They also proved that AM is able to measure cost-efficiency where the cost information is available (Khezrimotlagh *et al.*, 2012b). However, the efficiency score of linear AM may not measure the all inefficiencies that the model can identify. Although, Khezrimotlagh *et al.* (2012c) proposed a nonlinear AM which rectifies the shortcoming of linear AM to find better efficiency score for DMUs, it is not able to consider variety of weights as good as linear AM which is illustrated in this study, too. It is also not able to

assess the performance evaluation of DMUs where the cost information and unit price are available. Therefore, this study proposes a significant nonlinear Arash Model in order to comprehend all the DMUs' inefficiencies with diversity of selecting weights and scales. The nonlinear Arash Method is also examined with a numerical example of 18 DMUs with five inputs and two outputs and its capabilities in comparison with linear AM and Slack Based Measure (SBM) (Tone, 1997) is compared. The simulations are performed with Lingo11-win64 Software.

LITERATURE REVIEW

A technical efficient DMU is a DMU which the performances of other DMUs do not show that some of its inputs or outputs can be improved without worsening some of its other inputs or outputs (Charnes *et al.*, 1978). This definition is called Pareto-Koopmans definition in DEA and unfortunately it is wrongly interpreted as efficiency where there is no any cost information for DMUs. In fact, Khezrimotlagh *et al.* (2012a) proved that similar to economics the Pareto-Koopmans definition is only able to identify the technical efficient DMUs and the meaning of technical efficiency should not be wrongly interpreted as efficiency. In other words, they mentioned that each technical efficient DMU may only dominate some inefficient DMUs and there may be some inefficient DMUs which are not dominated with some technical efficient ones. Therefore, it is possible that an inefficient DMU be more efficient than a technical efficient one which does not dominate it. They proposed some counter examples to illustrate their

claims and suggested a new method called Arash Method (AM) to examine the Farrell frontier and arrange technical efficient DMUs. They also proved that a technical efficient DMU may even be less efficient than an inefficient one and defined that an efficient DMU is a technical efficient DMU which the ratio of its output to its input (i.e., output/input) does not much change where a little error happens in its data. This definition avoids the need for unit price, cost information or other assumption of weights for DMUs' data and when they are available it is the same as the definition of efficiency in economics which shows the validities of the proposed definition. Moreover, there are many measures of technical efficiency in data envelopment analysis that the most traditional ones is Charnes, Cooper and Rhods (CCR), a radial model, which is proposed by Charnes *et al.* (1978) based on the work of Farrell (1957). Charnes *et al.* (1985) also proposed a non-radial model, additive model (ADD), which is considered the possibility of simultaneous each input decreases and each output increases. Since both CCR and ADD are not able to arrange technical efficient DMUs, Anderson and Petersen (1993) developed a modified version of DEA model called AP to compare technical efficient DMUs. DEA has been constructed on the previous techniques and increasingly developed in many various fields in the last three decades. Some similar common DEA models to CCR can be recognized as BCC (Banker *et al.*, 1984) SBM (Tone, 1997) and ERM (Pastor *et al.*, 1999) and some analogous super-efficiency models to AP can be acknowledged as MAJ (Mehrabian *et al.*, 1999), SBM (Tone, 2002) and other models by Jahanshahloo *et al.* (2008). Recently, Khezrimotlagh *et al.* (2012a) with some counter examples proved that the results of AP model and other super-efficiency models based on that technique may not be significant and proposed Arash Method (AM) to eliminate the previous shortcomings in super-efficiency and conventional DEA models. Khezrimotlagh *et al.* (2012b) also proved that AM is able to measure cost-efficiency of DMUs which depicts that the model not only is independence of the unit price but also it can consider the cost information.

In order to illustrate the models, let us assume that there are n DMUs (DMU_i, i = 1, 2, ..., n) with m nonnegative inputs (x_{ij}, j = 1, 2, ..., m) and p nonnegative outputs (y_{ik}, k = 1, 2, ..., p) for each DMU which at least one of its inputs and one of its outputs are not zero. Assume that DMU_i (i = 1, 2, ..., n) is evaluated, w_j⁻ and w_k⁺ are the user specified weights obtained through values judgment, ε = (ε₁, ε₂, ..., ε_m), ε_j ≥ 0, s_j⁻ and s_k⁺ are nonnegative slacks, for j = 1, 2, ..., m and k = 1, 2, ..., p. Moreover, if the weights w_j⁻ and w_k⁺ are unknown they are defined as 1/x_{ij} and 1/y_{ik} where x_{ij} ≠ 0 and y_{ik} ≠ 0, respectively and N_j and M_k where x_{ij} = 0 and y_{ik} = 0 respectively, for j = 1, 2, ..., m

and k = 1, 2, ..., p. The N_j and M_k can be nonnegative real numbers regarding to the goals of each DMU. The Slack-Based Measure (SBM) and linear ε-AM, are as following:

ε -AM:

$$\max \sum_{j=1}^m w_j^- s_j^- + \sum_{k=1}^p w_k^+ s_k^+$$

Subject to

$$\sum_{i=1}^n \lambda_i x_{ij} + s_j^- = x_{lj} + \varepsilon_j / w_j^-, \forall j$$

$$\sum_{i=1}^n \lambda_i y_{ik} - s_k^+ = y_{lk}, \forall k$$

$$\lambda_i \geq 0, \forall i$$

$$s_j^- \geq 0, \forall j$$

$$s_k^+ \geq 0, \forall k$$

$$\text{Targets: } \begin{cases} x_{lj}^* = x_{lj} + \varepsilon_j / w_j^- - s_j^{-*}, \forall j, \\ y_{lk}^* = y_{lk} + s_k^{+*}, \forall k, \end{cases}$$

$$\text{Score: } A^* = \frac{\sum_{k=1}^p w_k^+ y_{lk} / \sum_{j=1}^m w_j^- x_{lj}}{\sum_{k=1}^p w_k^+ y_{lk}^* / \sum_{j=1}^m w_j^- x_{lj}^*}$$

SBM:

$$\min \frac{1 - (1/m) \sum_{j=1}^m s_j^- / x_{lj}}{1 + (1/p) \sum_{k=1}^p s_k^+ / y_{lk}}$$

Subject to

$$\sum_{i=1}^n \lambda_i x_{ij} + s_j^- = x_{lj}, \forall j$$

$$\sum_{i=1}^n \lambda_i y_{ik} - s_k^+ = y_{lk}, \forall k$$

$$\lambda_i \geq 0, \forall i$$

$$s_j^- \geq 0, \forall j$$

$$s_k^+ \geq 0, \forall k$$

$$\text{Targets: } \begin{cases} x_{lj}^* = x_{lj} - s_j^{-*}, \forall j, \\ y_{lk}^* = y_{lk} + s_k^{+*}, \forall k, \end{cases}$$

$$\text{Score: } \rho^* = \frac{1 - (1/m) \sum_{j=1}^m s_j^{-*} / x_{lj}}{1 + (1/p) \sum_{k=1}^p s_k^{+*} / y_{lk}}$$

In addition, it is generally defined that ε = (ε₁, ε₂, ..., ε_m) and when ε > 0 and A_ε^{*} < 1 for a DMU, ε-AM proposes the DMU to change its data to the new ε-AM target and otherwise i.e., when A_ε^{*} ≥ 1, ε-AM warns that the DMU should not change its data, because it may decrease its efficiency score. Khezrimotlagh *et al.* (2012c) also proposed the following definition for efficiency in DEA:

Definition: A technical efficient DMU is efficient with ε -degree of freedom in inputs if $A^*_0 - A^*_\varepsilon \leq \delta$. Otherwise, it is inefficient with ε -degree of freedom in inputs. The proposed amount for δ is $10^{-1}\varepsilon$ or ε/m .

THE EXTENDED NONLINEAR ARASH MODEL (AM)

The nonlinear ε -AM was formulated as following where DMU_l ($l = 1, 2, \dots, n$) is evaluated (Khezrimotlagh *et al.*, 2012d):

Nonlinear ε -AM:

$$A^*_\varepsilon = \min \frac{1+(1/m)\sum_{j=1}^m(\varepsilon_j - s_j^-/x_{lj})}{1+(1/p)\sum_{k=1}^p s_k^+/y_{lk}}$$

Subject to

$$\sum_{i=1}^n \lambda_i x_{ij} + s_j^- = x_{lj} + \varepsilon_j x_{lj}, \forall j$$

$$\sum_{i=1}^n \lambda_i y_{ik} - s_k^+ = y_{lk}, \forall k$$

$$\lambda_i \geq 0, \forall i$$

$$s_j^- \geq 0, \forall j$$

$$s_k^+ \geq 0, \forall k$$

$$\text{Targets: } \begin{cases} x_{ij}^* = x_{lj} + \varepsilon_j x_{lj} - s_j^-, \forall j, \\ y_{ik}^* = y_{lk} + s_k^+, \forall k, \end{cases}$$

$$\text{Score: } A^*_\varepsilon = \frac{1+(1/m)\sum_{j=1}^m(\varepsilon_j - s_j^-/x_{lj})}{1+(1/p)\sum_{k=1}^p s_k^+/y_{lk}}$$

If there is an $x_{lj} = 0$ ($y_{lk} = 0$), similar to SBM the term s_j^-/x_{lj} (s_k^+/y_{lk}) is deleted in the objective of nonlinear ε -AM and m (p) should be reduced to $m-1$ ($p-1$). From the above model, the following propositions are proposed.

Proposition 1: The nonlinear 0-AM is SBM model:

Proof: Let $\varepsilon = 0$. Clearly, the objective, constraints and targets of ε -AM are the same as the SBM objective, constraints and targets and the proof is completed.

Proposition 2: The score of nonlinear ε -AM is always less or equal than the score of linear ε -AM where the weights are unknown.

Proof: First, suppose that $x_j \neq 0$ and $y_k \neq 0$. Since the weights are unknown they are defined $w_j^- = 1/x_j$ and $w_k^+ = 1/y_k$. From the score and targets of nonlinear ε -AM ($\varepsilon \geq 0$) the following equations are yielded.

Table 1: The example of six DMUs

DMU	Input1	Input2	Output	Linear 0.001-AM	Nonlinear 0.001-AM
A	1	12	1	0.99977	1.00000
B	2	8	1	0.99980	1.00000
C	3	5	1	0.99987	0.99990
D	4	3	1	1.00000	1.00000
E	6	2	1	0.99987	1.00000
F	9	1	1	0.99980	1.00000

$$\begin{aligned} A^*_\varepsilon &= \frac{1+(\frac{1}{m})\sum_{j=1}^m(\varepsilon_j - s_j^-/x_{lj})}{1+(\frac{1}{p})\sum_{k=1}^p \frac{s_k^+}{y_{lk}}} \\ &= \frac{p}{m} \cdot \frac{m+\sum_{j=1}^m(\varepsilon_j - s_j^-/x_{lj})}{p+\sum_{k=1}^p \frac{s_k^+}{y_{lk}}} \\ &= \frac{p}{m} \cdot \frac{\sum_{j=1}^m(x_{lj} + \varepsilon_j x_{lj} - s_j^-)/x_{lj}}{\sum_{k=1}^p (y_{lk} + s_k^+)/y_{lk}} = \frac{p}{m} \cdot \frac{\sum_{j=1}^m x_{lj}^*/x_{lj}}{\sum_{k=1}^p y_{lk}^*/y_{lk}} \\ &= \frac{p/m}{\sum_{k=1}^p (y_{lk}^*/y_{lk}) / \sum_{j=1}^m (x_{lj}^*/x_{lj})} \\ &= \frac{\sum_{k=1}^p y_{lk}/y_{lk} / \sum_{j=1}^m x_{lj}/x_{lj}}{\sum_{k=1}^p (y_{lk}^*/y_{lk}) / \sum_{j=1}^m (x_{lj}^*/x_{lj})} \end{aligned}$$

The last equation is similar to the efficiency score of linear ε -AM when the weights are unknown. Now, if the optimum slacks (or x_{li}^* and y_{lk}^*) are the same by both models, there is no any differences between those scores. Otherwise, since nonlinear ε -AM minimizes the above equations, then A^*_ε in nonlinear ε -AM is less than A^*_ε in linear ε -AM. It is analogously deduced for when $x_j = 0$ and $y_k = 0$ for some $j = 1, 2, \dots, m$ and $k = 1, 2, \dots, p$ and the proof is completed.

On the other hand, when the weights are available for input and output values of DMUs, it is not suggested to apply the above nonlinear ε -AM, to assess the performance evaluation of those DMUs, because in this case, the nonlinear ε -AM may not have any effects to arrange technical efficient DMUs in comparison with linear ε -AM. For instance, let us suppose the six DMUs in Table 1 with two inputs and one single constant output which have the same weights i.e., $w_1 = w_2 = w^+$. In this example all six DMUs A, B, C, D, E and F are technical efficient and their efficiencies are $1/(1+12)=1/13$, $1/10$, $1/8$, $1/7, 1/8$ and $1/10$, respectively, which show the following ranking. $C = E > B = F > A$. From the table, the results of linear 0.001-AM are significant whereas the results by nonlinear 0.001-AM are almost completely the same as technical efficiency.

To rectify the shortcoming, let us suppose that the units' weights are available. The following equations deduce from the score of linear ε -AM and its targets:

$$A^*_\varepsilon = \frac{\sum_{k=1}^p w_k^+ y_{lk}}{\sum_{j=1}^m w_j^- x_{lj}} = \frac{\sum_{k=1}^p w_k^+ y_{lk}}{\sum_{j=1}^m w_j^- x_{lj}}$$

$$\begin{aligned}
 &= \frac{\sum_{k=1}^p w_k^+ y_{lk}}{\sum_{j=1}^m w_j^- x_{lj}} \times \frac{\sum_{j=1}^m w_j^- x_{lj}^*}{\sum_{k=1}^p w_k^+ y_{lk}^*} \\
 &= \frac{\sum_{k=1}^p w_k^+ y_{lk}}{\sum_{j=1}^m w_j^- x_{lj}} \times \frac{\sum_{j=1}^m (w_j^- x_{lj} + \varepsilon_j - w_j^- s_j^-)}{\sum_{k=1}^p (w_k^+ y_{lk} + w_k^+ s_k^+)} \\
 &= \frac{\sum_{k=1}^p w_k^+ y_{lk}}{\sum_{j=1}^m w_j^- x_{lj}} \times \frac{\sum_{j=1}^m w_j^- x_{lj} + \sum_{j=1}^m (\varepsilon_j - w_j^- s_j^-)}{\sum_{k=1}^p w_k^+ y_{lk} + \sum_{k=1}^p w_k^+ s_k^+} \\
 &= \frac{1 + \sum_{j=1}^m (w_j^- / A) (\varepsilon_j / w_j^- - s_j^-)}{1 + \sum_{k=1}^p (w_k^+ / B) s_k^+}
 \end{aligned}$$

Subject to

$$\sum_{i=1}^n \lambda_i x_{ij} + s_j^- = x_{ij} + E_j, \forall j$$

$$\sum_{i=1}^n \lambda_i y_{ik} - s_k^+ = y_{lk}, \forall k$$

$$\lambda_i \geq 0, \forall i$$

$$s_j^- \geq 0, \forall j$$

$$s_k^+ \geq 0, \forall k$$

In the above equation, A and B are $\sum_{j=1}^m w_j^- x_{lj}$ and $\sum_{k=1}^p w_k^+ y_{lk}$, respectively. Let us define $W_j = w_j^- / A$, $W_k^+ = w_k^+ / B$ and $E_j = \varepsilon_j / w_j^-$. Therefore, the extended nonlinear ε -AM is proposed as following:

$$\text{Targets: } \begin{cases} x_{ij}^* = x_{ij} + E_j - s_j^-, \forall j, \\ y_{lk}^* = y_{lk} + s_k^+, \forall k, \end{cases}$$

$$A_\varepsilon^* = \min \frac{1 + \sum_{j=1}^m W_j (E_j - s_j^-)}{1 + \sum_{k=1}^p W_k^+ s_k^+}$$

$$\text{Score: } A_\varepsilon^* = \frac{1 + \sum_{j=1}^m W_j (E_j - s_j^-)}{1 + \sum_{k=1}^p W_k^+ s_k^+}$$

Table 2: The example of 18 DMUs

DMU	Input 1	Input 2	Input 3	Input 4	Input 5	Output 1	Output 2
A01	6	7	275	19	158	307	92
A02	4	6	200	13	108	151	94
A03	32	58	325	18	471	442	75
A04	48	49	225	10	96	36	77
A05	7	7	275	13	171	292	91
A06	6	5	200	16	76	106	89
A07	76	55	200	37	86	148	86
A08	6	8	200	16	79	124	87
A09	8	7	275	16	80	153	90
A10	7	7	200	19	83	154	83
A11	13	8	275	11	174	288	89
A12	5	8	275	16	171	285	88
A13	5	8	275	19	148	260	89
A14	12	10	275	21	230	378	84
A15	38	24	300	21	301	473	86
A16	5	8	275	16	158	261	90
A17	9	9	275	21	193	340	86
A18	7	8	200	19	89	154	85

Table 3: The linear AM scores

DMU	0-AM	0.0001-AM	0.001-AM	0.01-AM	0.1-AM
A01	1.0000000	1.0000000	1.0000000	1.0000000	1.0000000
A02	1.0000000	1.0000000	1.0000000	1.0000000	1.0000000
A03	1.0000000	0.9987119	0.9871861	0.8781629	0.5892823
A04	1.0000000	0.9955830	0.9571936	0.6729322	0.2368148
A05	1.0000000	1.0000000	1.0000000	1.0000000	1.0000000
A06	1.0000000	0.9999396	0.9993965	0.9939923	0.9425213
A07	1.0000000	0.9915010	0.9150104	0.5387195	0.5291152
A08	1.0000000	0.9996989	0.9969888	0.9698883	0.8867237
A09	1.0000000	0.9999486	0.9994859	0.9948593	0.9485933
A10	1.0000000	0.9999085	0.9990847	0.9908467	0.9096913
A11	1.0000000	0.9999389	0.9993892	0.9938935	0.9390748
A12	1.0000000	0.9999804	0.9998035	0.9980355	0.9803545
A13	1.0000000	0.9995185	0.9951853	0.9518529	0.9238886
A14	1.0000000	0.9998960	0.9989607	0.9896769	0.9032795
A15	1.0000000	0.9998533	0.9985338	0.9854359	0.8634762
A16	0.9672552	0.9672338	0.9670412	0.9651160	0.9520869
A17	0.9573612	0.9572487	0.9562362	0.9461837	0.8523980
A18	0.9296110	0.9295174	0.9286753	0.9202537	0.8435230

Table 4: The SBM and nonlinear AM scores

DMU	SBM	0-AM	0.0001-AM	0.001-AM	0.01-AM	0.1-AM
A01	1.000000	1.000000	0.9999910	0.9999096	0.9991023	0.9915691
A02	1.000000	1.000000	0.9999784	0.9997840	0.9978599	0.9803684
A03	1.000000	1.000000	0.9987116	0.9871861	0.8781629	0.5892823
A04	1.000000	1.000000	0.9955829	0.9571936	0.6729322	0.2325592
A05	1.000000	1.000000	0.9999964	0.9999637	0.9996389	0.9965477
A06	1.000000	1.000000	0.9999396	0.9993965	0.9939923	0.9425213
A07	1.000000	1.000000	0.9915010	0.9150104	0.5383086	0.5291152
A08	1.000000	1.000000	0.9996989	0.9969888	0.9698883	0.8867237
A09	1.000000	1.000000	0.9999486	0.9994859	0.9948593	0.9485933
A10	1.000000	1.000000	0.9999085	0.9990847	0.9908467	0.9096913
A11	1.000000	1.000000	0.9999389	0.9993892	0.9938935	0.9390748
A12	1.000000	1.000000	0.9999800	0.9997996	0.9980086	0.9803545
A13	1.000000	1.000000	0.9995185	0.9951853	0.9518529	0.9204999
A14	1.000000	1.000000	0.9998960	0.9989607	0.9989679	0.9032795
A15	1.000000	1.000000	0.9998533	0.9985335	0.9854359	0.8634762
A16	0.9672552	0.9672552	0.9672338	0.9670412	0.9651160	0.9492975
A17	0.9573612	0.9573612	0.9572487	0.9562362	0.9461837	0.8523980
A18	0.9296110	0.9296110	0.9295174	0.9286753	0.9202537	0.8435230

From the above illustrations the following proposition is proved. Indeed, if the weights are available, the extended nonlinear ϵ -AM minimizes the score of linear ϵ -AM. Moreover, when they are not available it is quite the same as the previous nonlinear ϵ -AM and the proof follows from Proposition 2.

Proposition 3: The score of extended nonlinear ϵ -AM is always less or equal than the score of linear ϵ -AM.

A NUMERICAL EXAMPLE TO EXAMINE LINEAR AND NONLINEAR AM

Let us consider the 18 DMUs in Table 2 with five inputs and two outputs.

The scores of linear AM and nonlinear AM are represented in Tables 3 and 4. The Tables clearly depict that only one ten thousandth errors (one hundredth percentage) in each input are enough that 0.0001-AM arranges those DMUs together and distinguishes between technical efficient DMUs simultaneously. Moreover, linear ϵ -AM illustrates that the performance of DMUs A01, A02 and A05 are similar to each other and suggests them as efficient DMUs.

On the other hand, from Table 3 raising the amounts of epsilons in linear ϵ -AM has no any affect to distinguish between efficient DMUs A01, A02 and A05, whereas nonlinear 0.0001-AM strongly suggests that DMU A05 is the best performer among them, even 10 percentage errors happen in its inputs.

In addition, the proposed score for technical efficient DMU A07 by linear and nonlinear 0.001-AM is 0.9150104 which shows that if one thousandth errors happen in its input values, its score is quite less than those inefficient DMUs A16, A17 and A18. This phenomenon illustrates that there may some technical efficient DMUs which have the worst performance among all DMUs and there may some inefficient DMUs which have better performance in comparison

with some technical efficient ones which do not dominated them.

Furthermore, there is no significant difference between linear and non-linear AM to arrange DMUs in this example. For instance, both of them rank A04 to 14th when $\epsilon = 0.0001$ and rank it to 18th when $\epsilon = 0.1$. However, the proposed efficiency scores may rarely be different by those models, because from Propositions 2 and 3, the score of nonlinear ϵ -AM is non-greater than the score of linear ϵ -AM. For example, the score of nonlinear ϵ -AM is 0.5383086 for A07 whereas it is 0.5387195 by linear ϵ -AM.

CONCLUSION

This study extends nonlinear Arash Model to be a fair benchmarking model in DEA. The extended Model is able to arrange both technical efficient and inefficient DMUs together significantly, find efficient DMUs among the technical efficient ones, consider the weights either they are available or unknown and comprehend all the inefficiency that the model can identify. Moreover, three propositions are proved to identify the validities of the model. It is exactly suggested to apply the Arash Method for assessing the performance evaluation of decision making units.

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