

Most Satisfactory Camping: Multi-objective Optimization Based on Composite Index

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Abstract: Given the rise in popularity of river rafting, we need to schedule an optimal mix of trips of varying duration and propulsion to meet the needs of travelers and improve the company's interests. To provide a wilderness experience for travelers, we should try to reduce the contacts during the traveling. We firstly analyze the traveling process. And one contact (on-river) is counted. We apply step-to-step method to this study. We firstly study this relationship when there is only one kind of travel days. Then we add another kind of travel days to draw the universal rule and finally we derive the relationship when there are many kinds of trips. In order to combine the contacts and company's interests, we define two satisfaction degree functions by using fuzzy logic. We define a satisfaction degree function to describe the contacts. Then we define another function to describe the company's interests. Lastly, we define a composite index to combine the two functions by giving them different weight. We base this model on maximizing this composite index and obtain the optimal schedule. When determining the carrying capacity of the river, we take contacts, company's interests and river's resources into account. In the same way, we define a satisfaction degree function for river's resources. We finally define a composite index to combine the three factors by giving them different weights. We base this model on maximizing this composite index and derive the river's carrying capacity.

Keywords: Carrying capacity, composite index, contacts, step-to-step method, satisfaction degree

INTRODUCTION

When we travel on the Big Long River, we can enjoy scenic views and exciting white water rapids. Trips on this river can be divided into two types, that is, oar-powered and motorized trips. Travel days last from 6 to 18 days. Figure 1 show that a boat travels along the Big Long River. There are scenic views, campsites along the river.

Given the rise in popularity of river rafting, we need to schedule an optimal mix of trips of varying duration and propulsion. In the process of modeling, we should firstly identify the scenic views' and campsites' location and then consider how to choose the scenic views or campsites based on their attractiveness. At the same time, we also consider the river's limited resources.

THE MODELS

Basic analysis:

- **Division points:** Generally, we can divide the river into N segments according to the travel days (Underhill *et al.*, 1986), as shown in Fig. 2.

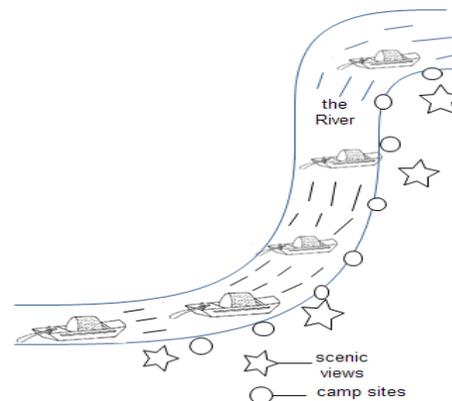


Fig. 1: Overview of the big long river

From the Fig. 2 we can find that different travel days will cause different division of segments. According to reference (Underhill *et al.*, 1986), we know the trips will stop within a certain radius of division points to choose their campsites.

Also, because the camp sites distributes uniformly along the river. Therefore, the distribution density of campsites is:

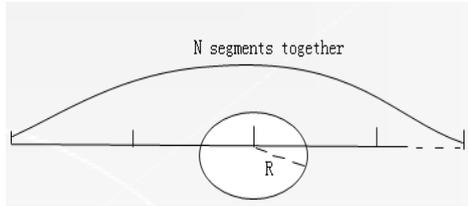


Fig. 2: Divide the river into N segments according to the travel days

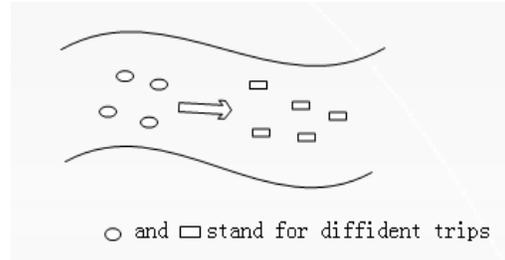


Fig. 4: On-river contacts

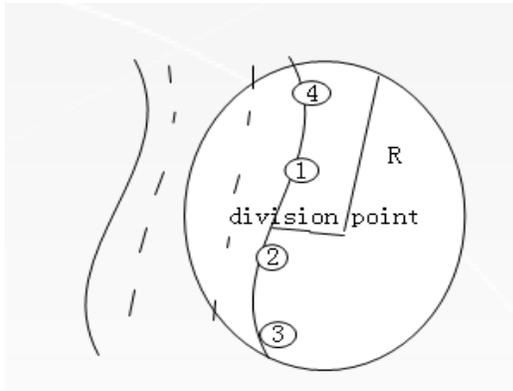


Fig. 3: Different attractiveness among the uniformly distributed campsites

$$\rho = 225/Y \quad (1)$$

where,

Y = The number of campsites along the river

ρ = The distribution density of campsites

- **Principle of contacts in the campsites:** We randomly generated attractiveness for campsites. When the trips arrive at a division point, they will choose the campsites according to the order of attractiveness within a certain radius of the division point. The principle is shown in Fig. 3.

When a number of trips arrive at the same division point in the same day, it involves the problem of how to choose campsites. We consider that each trip have no idea about whether the most popular campsite have been chose by other. So they will go to find another camp site according to the order of attractiveness, until they find one not occupied. The number that a trip meet others during the time finding campsites, we define it as contacts. As shown in Fig. 3. The order ①, ②, ③, and ④ are randomly generated to express the order of attractiveness. The higher is the order, the more attractive is the campsites. A trip will choose more preferred. Analysis is as follows:

The first arrived trip will go to find the most attractive campsites ①. Because there are no other trips within the radius of division point, the contacts is 0.

Then the second arrived trip will also go to find the most popular campsites ①, but only to find that it has been occupied, so the second arrived trip have to find the most popular campsites ② remained. So the contacts are 1. The third arrived trip will firstly go to find the ① and ② that have been occupied and finally will choose ③ as their campsites. So the contacts are 2. As the same reason, the contacts of the fourth arrived trip is 3, the contacts of the K_{th} arrived trip is $K-1$. Therefore, for those with the same travel days, we assume the number of trips arrives at the same division point is K . Then the total contacts within the radius of division point is C :

$$C = 0 + 1 + \dots + (K - 1) = \frac{K(K - 1)}{2} \quad (2)$$

So with this equation, we can count the contacts between trips arrive at the same division point at the same day.

- **Principle of contacts on the river:** Contacts not only occur in the campsites but also on the river. When a higher speed trip surpasses another trip, one contact is counted. As shown in Fig. 4.

If there are l_1 trips that have higher speed, surpass l_2 trips, then the total contacts D on the river is:

$$D = l_1 \times l_2 \quad (3)$$

Model: Develop the best schedule:

The principle of simulation model: We have done some of the basic analysis during the traveling, but we haven't known the relationship between schedule and contacts. So we decide to use the step-to-step method, that is, we firstly simulate when there are only one type of trip to explore the relationship. Then we add another type of trip to the first simulation, in which we find that the second simulation have already enough to express all necessary principles of the relationship. So when developing schedule, we use these principles directly to get the optimal schedule.

In the following we will mainly explain the principles by one and two kinds of travel days.

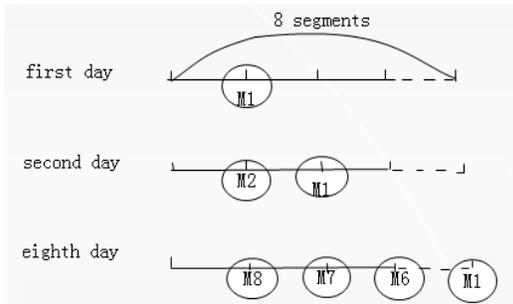


Fig. 5: The contacts between motorized and oar-powered boats

- One kind of travel days:** According to the reference (Underhill *et al.*, 1986) and consider the actual living habits, we decide to tack one week as a period of the schedule. We let M_i as the number of trips launch every day during a period, a : bas the proportion between motorized and oar-powered boats, in each day, trips must arrive the corresponding division point to find their camp sites. So, from 6-18 nights, there are 13 kinds of travel days, we randomly select one kind to explain the situation of the trip. And now we will take 8 nights for example to calculate the total contacts, as shown in Fig. 5.

By analyzing Fig. 5, according to principle of contacts in the camp sites, we can calculate the contacts:

The first day:

$$0 + 1 + \dots + M_1 = \frac{M_1(M_1 + 1)}{2}$$

The second day:

$$(0 + 1 + \dots + M_1) + (0 + 1 + \dots + M_2) = \frac{M_1(M_1 + 1)}{2} + \frac{M_2(M_2 + 1)}{2}$$

Analogy to infer the eighth day:

$$\frac{M_1(M_1 + 1)}{2} + \frac{M_2(M_2 + 1)}{2} + \dots + \frac{M_8(M_8 + 1)}{2}$$

So the total contacts C is:

$$C = \frac{M_1(M_1 + 1)}{2} \times 8 + \frac{M_2(M_2 + 1)}{2} \times 7 + \dots + \frac{M_8(M_8 + 1)}{2} \times 1 \quad (4)$$

Through computer simulation, we can the contacts of all kinds of different situations.

- Two kinds of travel days:**
- Intersection principle:** When the duration between two kinds of travel days has a big diffidence, the division points of the two kinds of travel days could

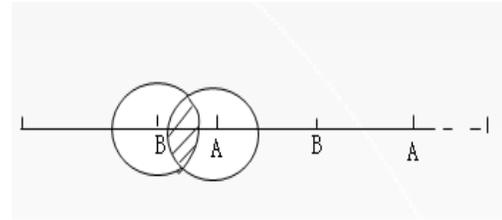


Fig. 6: The intersection principle between two kinds of travel day

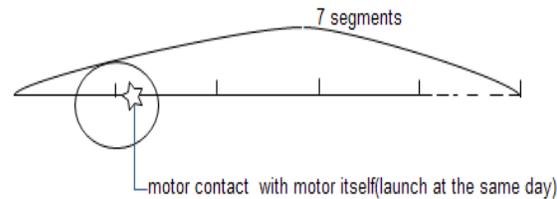


Fig. 7: Motorized boats contacted at the same division point and at the same day

be very close, so they will have intersection principle within the radius of their division points. As shown in Fig. 6.

In Fig. 6 A and B stand for the division points of trips of two kinds of travel days, each kind of trip find its camp sites within the radius of their division points, shaded part stand for the same part of two kinds of trips. When one kind of trip arrive at its division point, the trip have the priority to choose the most popularity camp sites within the radius of the division point, when another kind of trip arrive, the trip choose the camp sites still according to the principle of contacts, but cannot choose the camp sites have been occupied by the kind of trip anymore. According to this principle, we can calculate the contacts even with intersections.

Calculate the contacts: In order to understand the camping principle, we randomly selected two types to calculate the contacts. We take motorized boats of travel 7 days and oar-powered rubber rafts of travel 14 days into account.

When choosing the campsite, there will be two types of contact. The first one is contacted at the same division point and at the same day when choosing the campsite. We illustrate by motorized boats contacts, as shown in Fig. 7:

The second is motorized boats (launch at i day) contact with oar-powered rubber rafts (launch at z day) when choosing a campsite. Because motorized boats is faster than oar-powered rubber rafts, so motorized boats will catch up with oar-powered rubber rafts. When they contact at the division point, as shown in Fig. 8:

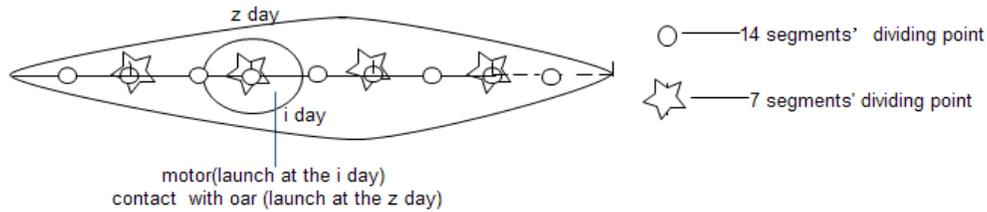


Fig. 8: Motorized boats (launch at *i* day) contact with oar-powered rubber rafts (launch at *z* day) at the division point when choosing a campsite

From Principle of contacts, motorized boats launched at *i* day will contact at the division point when choosing a campsite, their contacts is:

$$A_i = \left[0+1+\dots \left(\frac{a_i}{a_i+b_i} \times M_i - 1 \right) \right] \tag{5}$$

The same, oar-powered rubber rafts launched at *i* day will contact at the division point when choosing a campsite, their contacts is:

$$B_i = \left[0+1+\dots \left(\frac{b_i}{a_i+b_i} \times M_i - 1 \right) \right] \tag{6}$$

From Principle of contacts, motorized boats launched at *i* day will contact with oar-powered rubber rafts launched at *z* day when choosing a campsite, their contacts is:

$$C_{i,z} = \left[0+1+\dots \left(\frac{a_i}{a_i+b_i} \times M_i + \frac{b_z}{a_z+b_z} \times M_z - 1 \right) \right] \tag{7}$$

Combined with real life situations, we know the launched cycle is a week. We calculated the contacts in 14 days. Let is *T* the total contacts.

At the first day, the trips' contacts *T*₁ is:

$$T_1 = A_1 + B_1$$

At the second day, the trips' contacts *T*₂ is:

$$T_2 = A_1 + B_2 + C_{2,1}$$

The same as above, at the 14 day, the trips' contacts *T*₁₄ is:

$$T_{14} = B_2 + B_4 + \dots + B_{14} + C_{14,13} + C_{13,11} + \dots + C_{9,3}$$

So the total contacts *T* is:

$$T = T_1 + T_2 + T_3 \dots + T_{14} + D \tag{8}$$

where, *D* are the contacts on the river from principle of contacts in the campsites.

Through the examples, the simulation principles have been stated clearly. Therefore, we directly simulate the calculations combine with the optimization models using these principles.

Multi-objective optimization model: Because the schedule should meet the travelers' requirements and the company's interests, so it is a multi-objective optimization model. Therefore, we consider the satisfaction degree of the travelers and the satisfaction degree of the company, calculated the maximum value of the composite index of the satisfaction degree and achieved the best schedule.

- **The satisfaction degree of the traveler:** According to the reference (Troy and Bo, 2000), the satisfaction degree of the traveler is base on the average contacts every day. When $T_{mean} \leq 2$, the satisfaction degree of the traveler is 1. When $2 < T_{mean} \leq 10$, we set the satisfaction degree of the traveler is between 1 to 0.02. When $T_{mean} > 10$, we set the satisfaction degree of the traveler is 0.02.

$$R_1 = \begin{cases} 1 & T_{mean} \leq 2 \\ -0.1225 \times T_{mean} + 1.2450 & 2 < T_{mean} \leq 10 \\ 0.02 & T_{mean} > 5 \end{cases} \tag{9}$$

where, *T*_{mean} are the average contacts every day?

The satisfaction degree of the company: The company's main aim is profit, so we can use the company's revenue to portray the satisfaction degree of the company.

According to website (Canyon Explorations Expeditions, 2012), we can calculate the average price of trips is \$ 2861.

The number of trips launch per day is 18 (GCRRA, 2012), the company's average revenue per day is $18 \times 2861 = 51506$. When the number of trips launch per day by the company is more than 22, the company have more revenue and we set the satisfaction degree of the company is 1. When the number of trips launch per day by the company is between 10 to 22, we set the satisfaction degree of the company is between 1 to 0.02.

When the number of trips launch per day by the company is less than 10, we set the satisfaction degree of the company is 0.02. We use a linear equation to represent the satisfaction degree of the company:

$$R_2 = \begin{cases} 1 & A_{mean} \geq 22a \\ 0.0817 \times A_{mean} - 0.7967 & 10a < A_{mean} < 22a \\ 0.02 & A_{mean} < 10a \end{cases} \quad (10)$$

where, A_{mean} is the company's average revenue per day, a is the average price of trips.

- **Composite index of the satisfaction degree:** In order to combine the satisfaction degree of the travelers with the satisfaction degree of the company reasonably, we define a composite index of the satisfaction degree. We believe that the travelers' requirements are more important than the company's interests, therefore the composite index of the satisfaction degree can be defined as:

$$R = 0.7R_1 + 0.3R_2 \quad (11)$$

where, R is the composite index of the satisfaction degree? By maximizing the composite index of the satisfaction degree, we can achieve the best schedule.

Model II: The carrying capacity of the river: According to the reference (Changing Lakes, 2010), the definition of carrying capacity is: "Carrying capacity can be thought of as a threshold value, if exceeded, would lead to an undesirable set of conditions or problems." But for our topic, carrying capacity is a threshold value affected by the travelers' requirements and the River resources in fact. Therefore, we can determine the carrying capacity, combining it with the travelers' requirements and the river resource.

To the travelers' requirements, we will relate to the satisfaction degree of the travelers and the satisfaction degree of the company.

To the river resource, we can relate to the boating density. Boating density is the ratio of boat area to the river's useable area. When boating density exceeds the threshold, it would destroy the ecological balance of rivers. Therefore, by using the fuzzy logic method, we can definite the satisfaction degree of the boating density.

We set d as boating density. According to the reference (Douglas, 2004), we use the fuzzy logic method. When $d \leq 0.02/acre$, the satisfaction degree of boating density is 1. When $0.02/acre < d \leq 0.049/acre$, the satisfaction degree of boating density is 0.7. When $0.049/acre < d \leq 0.1/acre$, the satisfaction degree of boating density is 0.3. When $d > 0.1/acre$, the satisfaction degree of boating density is 0.01.

Therefore, we use an equation to represent the satisfaction degree of the boating density:



Fig. 9: Division of daily time

$$R_3 = \begin{cases} 1 & d \leq 0.02 / acre \\ 0.7 & 0.02 / acre < d \leq 0.049 / acre \\ 0.3 & 0.049 / acre < d \leq 0.1 / acre \\ 0.01 & d > 0.1 / acre \end{cases} \quad (12)$$

At last, we combine the travelers' requirements with the River resources, that is, we consider three indicators to determine the carrying capacity of the river. Three indicators is the satisfaction degree of the travelers, the satisfaction degree of the company and river's boating density. We give them three weights $\lambda_1, \lambda_2, \lambda_3$ and the composite index of the three indicators can be defined as:

$$R' = \lambda_1 R_1 + \lambda_2 R_2 + \lambda_3 R_3 \quad (13)$$

where, $\lambda_1 + \lambda_2 + \lambda_3 = 1$.

Because boating density play a leading role on the river's carrying capacity, for facilitating the calculation, we set λ_1 is 0.6, λ_2 is 0.2, λ_3 is 0.2. When the composite index of the three indicators reached the maximum, we can find the most suitable boating density R^3 . According to the definition of boating density, we know that:

$$n = d \times A_{river} \quad (14)$$

where,

n = River's carrying capacity

d = Boating density

A_{river} = The river's area. So we can calculate the total number of the trips on the river.

SOLUTIONS TO THE REQUIREMENTS

- **Determine values for some parameters: Time segment of every day:** According to reference (Underhill *et al.*, 1986), we divide one day into 10 segments from 10:00 AM to 3:00 PM. Each segment lasts 30 min, as shown in Fig. 9:

Ratio of Num. of trips launched during each time segment per day: According to reference (Underhill *et al.*, 1986), we define the ratio of Num. of trips launched during each time segment per day as 0.0533, 0.1333, 0.3033, 0.1567, 0.12, 0.1100, 0.0833, 0.0753, 0.0683 and 0.0633. For example, if one

Table 1: Every day's launched trips

Weekday	Mon.	Tue.	Wed.	Thu.	Fri.	Sat.	Sun.
No of launched trips	19	18	19	18	17	18	17

Table 2: Num. of launched trips during each time segment per day

Time seg.	1	2	3	4	5	6	7	8	9	10
Mon.	0	3	6	3	2	1	2	1	0	1
Tue.	1	2	5	3	2	2	1	1	1	0
Wed.	0	3	6	3	2	2	1	0	1	1
Thu.	1	2	5	2	2	2	1	1	1	1
Fri.	0	2	5	3	2	2	1	1	1	0
Sat.	1	2	4	3	2	2	1	1	1	1
Sun.	0	2	5	3	1	2	1	1	1	1

Table 3: The type of trips (motorized or oar-powered)

Time segment	1	2	3	4	5	6	7	8	9	10
Motorized	0	2	4	2	1	1	1	1	0	1
Oar-powered	0	1	2	1	1	0	1	0	0	0
Total	0	3	6	3	2	1	2	1	0	1

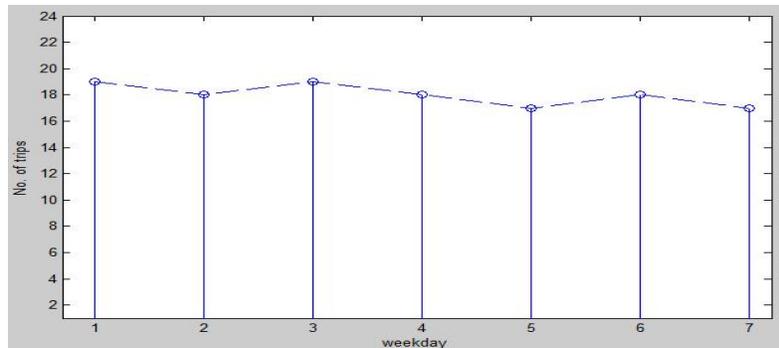


Fig. 10: Num. of trips in a week. The horizontal axis is weekday while the vertical axis is the No. of trips

day's total launched trips is 30, then the Num. of trips launched during the first time segment is $30 \times 0.0333 \approx 1$.

The total Num. of X trips for 6 months: It satisfy the equation $X = 6 \times 4 \times 7 \times N_{\text{mean}}$ (we suppose each month has four weeks and one week has 7 days), where N_{mean} is the average Num. of trips launched per day.

Firstly we let the area of the Big Long River is 3.53×10^4 acres (BaiduBaike, 2012). According to the Num. of camp sites $Y = 141$, the ratio between the Num. of motorized boats and oar-powered raft is 3:1 (Underhill *et al.*, 1986).

- **Developing the best schedule:** We apply Genetic Algorithm and obtain greatest composite index $R = 0.69$ with one week as calculation unit. The corresponding every day's launched trips are shown in Table 1.

According to ratio of num. of trips launched, we can calculate the Num. of launched trips during each time segment per day, as shown in Table 2.

We combine the composite index of the satisfaction degree. Take Monday for example, we can calculate the type of trips (motorized or oar-powered) that launched, as shown in Table 3.

Because we suppose ratio of Num. of trips launched during each time segment per day is constant and the ratio between the Num. of motorized boats and oar-powered raft is constant, we can know trips launched during each time segment per day if we know the Num. of trips launched. We can also know the type of trips. Therefore, we will analyze the Num. of trips launched per day as follows.

When the composite index is greatest, we draw the Num. of trips launched per day in a week, as shown in Fig. 10.

From Fig. 10, we know the Num. of trips launched per day varies a bit. Moreover, the Num. of trips is less than 22, so the company's satisfaction degree is less than 1. We can conclude the weight of traveler's satisfaction degree is greater.

Determining the carrying capacity of the river: With the Genetic algorithm, we obtain the optimal solution boating density $d = 0.087$. Therefore, we can calculate the river's carrying capacity is:

$$n = d \times A_{\text{river}} = 3071$$

that is, the river can carry the maximum Num. 3071 boats.

Table 4: Sensitivity of composite index to $\pm 10\%$ change in mean value

Change of mean value	Mean value after change	Change of composite index
Decrease by 10%	16.2	-2.8%
Increase by 10%	19.8	-4.7%

Table 5: Sensitivity of composite index to $\pm 10\%$ change in Num. of campsites

Num. of campsites	Num. after change	Change of composite index
Decrease by 10%	103	+0.17%
Increase by 10%	125	-0.32%

Table 6: Sensitivity of composite index to $\pm 10\%$ change in radius r

Radius r	Value of r after change	Change of composite index
Decrease by 10%	4.5	+0.37%
Increase by 10%	5.5	-0.45%

SENSITIVITY ANALYSIS

The sensitivity of num. of trips launched: We analyze this aspect's sensitivity by adopting the mean value of trips launched per day in the week, as shown in Table 4.

From Table 4, we can know that composite index is sensitive to mean value. Moreover, whether the mean value increase or decrease, the composite index decreases. We can conclude the mean value of 18 is a Equilibrium value, when the mean value exceed it, the composite index will decrease only.

The sensitivity of num. of camp sites: From Table 5, we know composite index is a little sensitive to Num. of camp sites. And when the composite index increased, the Num. of campsites had decreased. We can conclude that the composite index changes in the opposite with Num. of campsites.

The sensitivity of radius r: From Table 6, we know composite index is also a little sensitive to radius r. And when the composite index increased, the Num. of campsites had decreased. We can conclude that the composite index also changes in the opposite with radius r.

CONCLUSION

In this study, we constructed two models. One is based on the optimal schedule and the other is to calculate the river's carrying capacity.

For the first model, we define two satisfaction degree functions to describe the travelers' needs and

company's interest respectively. Moreover, we define a composite index to combine the two functions. Solution for this model is the best schedule stated in the 'Developing the best schedule' part and the corresponding greatest composite index 0.69.

For the second model, we take three factors which are travelers' needs, company's interest and resources into consideration. For measuring the resources, we define a satisfaction degree function. Finally, we define a composite index to combine the three factors by giving them different weights.

In addition to the solution, we also analyze the composite index's sensitivity and find that composite index is sensitive to mean Num. of launched trips and less sensitive to Num. of campsites and radius r.

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