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An Imprecise Probability Model for Structural Reliability Based on Evidence and Gray Theory

[1]Bin Suo, [2]Ying Yan, [1]Chao Zeng and [1]Jun Li
[1]Institute of Electronic Engineering, China Academy of Engineering Physics, Mianyang 621900, China
[2]School of Economics and Management, Southwest University of Science and Technology, Mianyang 621010, China

Abstract: To avoid the shortages and limitations of probabilistic and non-probabilistic reliability model for structural reliability analysis in the case of limited samples for basic variables, a new imprecise probability model is proposed. Confidence interval with a given confidence is calculated on the basis of small samples by gray theory, which is not depending on the distribution pattern of variable. Then basic probability assignments and focal elements are constructed and approximation methods of structural reliability based on belief and plausibility functions are proposed in the situation that structure limit state function is monotonic and non-monotonic, respectively. The numerical examples show that the new reliability model utilizes all the information included in small samples and considers both aleatory and epistemic uncertainties in them, thus it can rationally measure the safety of the structure and the measurement can be more and more accurate with the increasing of sample size.

Keywords: Epistemic uncertainty, evidence theory, gray theory, imprecise probability model, structural reliability

INTRODUCTION

In reliability assessment of a structure, some factors, such as the complexity of internal mechanism, the limitation of people's knowledge and errors in manufacturing, will produce a variety of uncertainties. How to handling these uncertainties is a serious problem associating with the confidence of reliability analysis results. There are three types of reliability model at present, which are probabilistic, fuzzy and non-probabilistic reliability model, respectively. Probabilistic reliability model is constructed on the basis of probability theory and has been widely used in reliability assessment for various structures (Park *et al*., 2004; Christopher and Masoud, 2009). But in this model, the inherent stochastic characteristics of structure are considered as the only source of uncertainties. However, recent researches show that despite inherent stochastic characteristics, limitation of people's knowledge is another source to produce uncertainties and this type of uncertainty is so called "epistemic uncertainty" (Oberkampf *et al*., 1998, 2001). Probability theory is effective on handling aleatory uncertainty, but is not applicable for epistemic uncertainty (Helton, 1997; George and Richard, 2001). On the other hand, the precondition of using probabilistic reliability model is there having sufficient information to represent the probability distribution of uncertain quantity. But it is hard to meet this condition in many occasions. Above two shortages limit the application of probabilistic reliability model. Fuzzy reliability model is constructed on the basis of probabilistic reliability model and avoids the insufficiency of hypothesis of discrete finite state (Utkin and Gurov, 1996; Giasi *et al*., 2003; Biondini *et al*., 2004). But fuzzy reliability model is also having the aid of statistical method to obtain membership function, which is similar with probabilistic reliability model. The only difference between them is that the former uses subjective probability and the later uses objective probability (Lu and Feng, 2000). However, in general cases, obtaining the bounds of an uncertain quantity are easier than the probability distribution. In this view, Ben-Harm, Elishakoff and other researchers proposed the non-probabilistic reliability model based on interval analysis and convex model (Ben-Haim, 1994; Ben-Haim *et al*., 1996; Elishakoff, 1994; Luo *et al*., 2009). Although the model is simple in calculation, the results are always too conservative. Furthermore, the model only uses the lower and upper bounds of uncertain quantities and wastes the other valuable information, so it was difficult to be applied in engineering practice.

In engineering practices, as the limitation of experimental expenses and time, experimental samples and data are always inadequate. This makes the obtained information both involving aleatory and epistemic uncertainties. The existing data is insufficient

Corresponding Author: Bin Suo, Institute of Electronic Engineering, China Academy of Engineering Physics, Mianyang 621900, China

to construct the probability distribution of uncertain quantity, but can get additional information more than merely an interval range. Evidence theory is the generalization of probability theory (Dempster, 1967; Shafer, 1976). Researches show that evidence theory has strong ability on handling both aleatory and epistemic uncertainties simultaneously (Helton *et al*., 2004). So the motivation of this study is constructing a new imprecise probabilistic reliability model to evaluate structure reliability with insufficient experimental data using evidence theory and gray theory, in which gray theory is used to estimate the confidence interval of experimental data. The new model will consider both aleatory and epistemic uncertainties, so it is expected to get more reasonable results than the other reliability models.

PROBABILISTIC RELIABILITY MODEL

In reliability analyzing for an engineering structure, limit state equation can be constructed according to the designed functions and requirements. In probabilistic reliability model, if $X = \{x_1, x_2,\ldots, x_n\}$ is the vector of n random variables for stochastic factors, then the structure limit state equation is:

$$Z = g(\mathbf{X}) = g(x_1, x_2, \cdots, x_n) = 0 \tag{1}$$

According to the two states hypothesis in probabilistic reliability theory, failure surface $\{X|g(X) = 0\}$ divides the basic variable space into two areas. If $g(X) > 0$, then the structure is in reliable state and safe domain is:

$$\Omega_s = \{\mathbf{X} \mid g(\mathbf{X}) > 0\} \tag{2}$$

Otherwise the structure is in failure state and its failure domain is:

$$\Omega_f = \{\mathbf{X} \mid g(\mathbf{X}) < 0\} \tag{3}$$

If $f(X)$ is the joint probability density function of X, then the structure failure probability is:

$$P_f = P(F) = \int_{\Omega_f} f(\mathbf{X}) d\mathbf{X} \tag{4}$$

where, F is the proposition $F = \{X \in \Omega_f\}$. Correspondingly, the reliability is:

$$R_s = 1 - P_f \tag{5}$$

Generally, probability distribution of variable x_i can be obtained by statistic method with experimental data. However, if the size of experimental data is small, it is hard to determine the distribution pattern and distributed parameters for x_i. And more, for a same set of data, sometimes different distribution patterns may all be fitted (Ben-Haim, 1994). But these different distribution patterns have different header and tail information, which are very important for the accuracy of structure reliability calculation. Choosing an ill-suited distribution pattern will result in great error in reliability calculation (Ben-Haim, 1993; Elishakoff, 1995). So in the situation of small sample data, conventional probabilistic reliability model faces enormous challenges.

IMPRECIS PROBABILITY MODEL FOR STRUCTURAL RELIABILITY UNDER SMALL SAMPLES

The essence of small sample data is aleatory and epistemic uncertainties are coexisting. To a variable x_i, aleatory uncertainty is its inherent characteristics and this type of uncertainty cannot be eliminated. However, epistemic uncertainty is caused by inadequate data and knowledge and can be removed when there are enough data or knowledge to construct the precise probability distribution of x_i. The root cause of the defect of conventional probability reliability model is that it ignores the epistemic uncertainties in variables when there are not enough samples.

In this section, gray distance measure is used to estimate the gray confidence interval of small sample data. Then the Basic Probability Assignments (BPAs) are constructed and structural reliability is calculated by evidence reasoning.

Gray confidence interval estimation: If the sample set of x_i is $A = \{a_1, a_2, \ldots, a_m\}$, then the interval of x_i should be estimated with A first. Traditional methods are working on the basis of large sample size and the prior distribution of the variable x_i (Chien, 1997; David, 1997; Law, 2006). But to small sample data, the prior distribution is generally unknown, so the traditional interval estimation methods are not available. Confidence interval of variable x_i can be estimated using gray theory in the following procedures:

- Calculating the gray distance measure of each sample a_i to each other sample a_j with (Ke *et al*., 2007):

$$dr(a_i, a_j) = \frac{\xi \left\| d(A, a_j) \right\|_\infty}{|a_i - a_j| + \xi \left\| d(A, a_j) \right\|_\infty} \tag{6}$$

where, ξ is the resolution ratio. Generally $\xi = 0.5$ (Deng, 1990) and:

$$\left\| d(A, a_j) \right\|_\infty = \max_k \left\{ |A(k) - x_j|, k \in [1, m] \right\} \tag{7}$$

Take the mean value of these gray distance measures as the distance measure of a_i ($i \in [1, m]$) to the whole sample space:

$$J_i = \left(\sum_{j=1}^{m} dr(a_i, a_j) \right) \Big/ m \tag{8}$$

Finally, normalize J_i ($i \in [1, m]$) as:

$$w_i = J_i \Big/ \sum_{j=1}^{m} J_i \tag{9}$$

where, w_i is the weight of sample a_i to gray estimate $\hat{X}_\iota$.

- Calculate the gray estimate $\hat{X}_\iota$ with weighted accumulation method as:

$$\hat{x}_i = \sum_{i=1}^{m} w_i a_i \tag{10}$$

- Calculate the gray confidence interval of x_i. From Eq. (6), dr ($a_0, \hat{X}_\iota$) is the gray confidence. For a given confidence α (α∈ [1, 1/3]) , by the Inequality:

$$dr(a_0, \hat{x}_i) \geq \alpha \tag{11}$$

the confidence interval of $\hat{X}_\iota$ at confidence α can be obtained as [inf (α_0), sup (α_0)].

Calculation of focal elements and BPAs: After get the interval estimation of variable x_i, i.e., [inf (α_0), sup (α_0)], the data set A = {a_1, a_2, …, a_m}, inf (α_0) and sup (α_0) are arranged from small to large order as:

$$B = \left\{ a_{(1)}, a_{(2)}, \cdots, a_{(k)} \right\} \tag{12}$$

where, k = m+2. The mean of above set is:

$$\bar{a} = (a_{(1)} + a_{(2)} + \cdots + a_{(k)}) / k \tag{13}$$

Constructing N minimum intervals A_{ij} = [$\bar{\alpha} - \Delta_{j,\bar{\alpha}} + \Delta_j$] ($1 \leq j \leq N$) with their mid-values at $\bar{\alpha}$, where $\Delta_1 < \Delta_2 < \ldots < \Delta_N$. These intervals are the focal elements of variable x_i. The probability of samples being included in each interval is P_j ($1 \leq j \leq N$). If there are M samples in the interval A_{ij} ($1 \leq j \leq N$), then:

$$p_j = M / N \tag{14}$$

And

$$Bel(A_{ij}) = p_j (1 \leq j \leq N) \tag{15}$$

Then the BPA of focal element A_{ij} is:

$$m(A_{ij}) = \begin{cases} Bel(A_{i1}) & (j = 1) \\ Bel(A_{ij}) - Bel(A_{i(j-1)}) & (2 \leq j \leq N) \end{cases} \tag{16}$$

Denotes the frame of discernment of X = {x_1, x_2, ···, x_n} to be Θ_X and its focal elements to be C_P ($1 \leq P \leq N^n$), then c_p is a Cartesian product:

$$c_p = A_{1j_1} \times A_{2j_2} \cdots \times A_{nj_n} \tag{17}$$

where, $j_1, j_2, \ldots, j_n \in [1, N]$. By Dempster's rule, BPA of C_p is:

$$m(c_p) = m(A_{1j_1}) \times m(A_{2j_2}) \cdots \times m(A_{nj_n}) \tag{18}$$

Structural reliability calculation with evidence theory: Let Θ_z be the frame of discernment in failure domain and Bel_z (F) and Pl_z (F) are belief and plausibility function of structure failure respectively, then:

$$Bel_Z(F) \leq P_Z(F) \leq Pl_Z(F) \tag{19}$$

Bel_z (F) is the lower bound of P_f, Pl_z (F) is the upper bound and [Bel_z (F), Pl_z (F)] is the interval estimation of P_f. If information content about F is great enough, then Bel_z (F) will be close to Pl_z (F) indefinitely and | Bel_z (F)-Pl_z (F)| will tend to be an infinitesimal. So the mean of the two functions is the approximation of P_f:

$$\tilde{P}_f = (Bel_Z(F) + Pl_Z(F)) / 2 \tag{20}$$

Denote Z_0 = {z: $z < 0$, z∈Θ_z}, then according to evidence theory, Bel_z (F) and Pl_z (F) can be expressed as:

$$Bel_Z(F) = Bel_X[f^{-1}(Z_0)] = \sum_{c_i \subset f^{-1}(Z_0)} m(c_i) \tag{21}$$

$$Pl_Z(F) = Pl_X[f^{-1}(Z_0)] = \sum_{c_i \cap f^{-1}(Z_0) \neq \varnothing} m(c_i) \tag{22}$$

Then structure failure probability is obtained by Eq. (20). If it is nessisary, structure reliability can also be calculated with Eq. (5).

If $Z = g(X)$ is monotonic, then vertex method (Dong and Shah, 1987) can be used. Denote the mapping of c_i

in Z as d_i. From the monotonicity of $g(X)$, the upper and lower bounds of d_i are only possible to obtain when X is the hypercube vertexes of c_i. Denote:

$$d_i = g(c_i) = [l_i, u_i] \tag{23}$$

Then:

$$d_i = g(c_i) = [\min_j\{g(v_j): j = 1,\cdots,n\}, \max_j\{g(v_j): j = 1,\cdots,n\}] \tag{24}$$

where, v_j (j = 1, 2⋯, n) are the vertexes of hypercube c_i. By Eq. (24), all d_i ($i \in m^n$) in Θ_Z can be calculated. Then

$$Bel_Z(F) = \sum_{d_i \subseteq Z_0} m(d_i) = \sum_{\sup(d_i)<0} m(d_i) \tag{25}$$

$$Pl_Z(F) = \sum_{d_i \cap Z_0 \neq \varnothing} m(d_i) = \sum_{\inf(d_i)<0} m(B_i) \tag{26}$$

If $Z = g(X)$ is non-monotonic, then vertex method is unusabel because of large computational errors. In this case, sampling method can be used to calculate Bel_Z (F) and Pl_Z (F). Denote:

$$N\_Bel_Z = \{i : c_i \subset g^{-1}(Z_0)\} \tag{27}$$

$$N\_Pl_Z = \{i : c_i \cap g^{-1}(Z_0) \neq \varnothing\} \tag{28}$$

Then Eq. (21) and (22) can be expressed as:

$$Bel_Z(F) = \sum_{i \subset N\_Bel_Z} m(c_i) \tag{29}$$

$$Pl_Z(F) = \sum_{i \subset N\_Pl_Z} m(c_i) \tag{30}$$

The procedure to estimate N_ Bel_z (F) and N_ Pl_z (F) using sampling method is as follows:

Step 1: Randomly generating N_s ($N_s \to \infty$) $\boldsymbol{X}$ in c_k ($c_k \subset \Theta_X$ and calculate Z with Eq. (1). If $Z<0$ hold for any Z, then $i \in N\_Bel_Z$ and $i \in N\_Pl_Z$. If $Z<0$ hold for some Z and $Z \geq 0$ hold for the others, then $i \in N\_Pl_Z$.

Step 2: For $1 \leq i \leq N^n$, repeate the step 1.

Then Bel_z (F) and Pl_z (F) can be calculated with Eq. (29) and (30).

After calculating Bel_z (F) and Pl_z (F), approximation of P_f can be obtained with Eq. (20).

NUMERICAL EXAMPLES

Example 1: The limit state equation of a structure is:

$$Z = g(\boldsymbol{X}) = x_1^3 + x_2^3 - 18 = 0 \tag{31}$$

where, x_1 and x_2 obey normal distribution N (1, 0.2) and N (3, 0.09), respectively. Randomly generate 10 samples with the probability distribution function of each variable, as shown in Table 1.

Set gray confidence $\alpha = 0.8$ and gray confidence intervals of x_1 and x_2 are obtained as (0.89, 23.47) and (-4.25, 22.24). Construct 11 subintervals with their mid-values at $\overline{x_1}$ and $\overline{x_2}$, respectively and P_j = j/11 ($1 \leq j \leq 11$).

Then focal elements and BPAs of x_1 and x_2 are calculated, the results are shown in Table 2.

As Eq. (31) is monotonic, vertex method is used in this example to calculate the belief and plausibility functions of the structure failure probability. The results are Bel_z (F) = 0 and Pl_z (F) = 0.0165. So from Eq. (20) structure failure probability is $\widetilde{P_f}$ = 0.0083 and from Eq. (5) structure reliability is $\widetilde{R_s}$ = 0.9917.

As the sample size is small, the results are imprecise. Increases the sample size, the results will be more accurate. Let m be 20, 30, 40 and 50, respectively and calculate the structure reliability with proposed method. The results are compared with that of Monte Carlo method with 10^6 samples and shown in Table 3. If the results of Monte Carlo method, R_s, can be taken as the exact value, then it can be seen from Table 3 that the results of proposed method is very close to the exact value when m increases to 50.

Table 1: Experimental data of x_1 and x_2

x_1					$\overline{x_1}$
1.36	3.380	6.760	8.850	9.370	9.86
9.78	10.24	12.57	14.67	21.75	
x_2					$\overline{x_2}$
-2.85	2.980	6.720	8.390	9.570	9.50
10.93	11.25	12.29	15.64	20.08	

Table 2: Focal elements and BPAs of x_1 and x_2

BPA	Focal elements of x_1	Focal elements of x_2
1/11	9.78, 10.24	8.39, 9.570
1/11	9.37, 10.24	8.39, 10.93
1/11	8.85, 10.24	8.39, 11.25
1/11	8.85, 12.57	6.72, 11.25
1/11	6.67, 12.57	6.72, 12.29
1/11	6.67, 14.67	6.72, 15.64
1/11	3.38, 14.67	2.98, 15.64
1/11	1.36, 14.67	2.98, 20.08
1/11	0.89, 14.67	-2.85, 20.08
1/11	0.89, 21.75	-2.85, 22.24
1/11	0.89, 23.47	-4.25, 22.24

Table 3: Results of reliability with different sample size in example 1

m	Inf (R_s)	Sup (R_s)	$\widetilde{R_s}$	R_s
10	0.9835	1.0000	0.9918	0.9943
20	0.9865	0.9977	0.9921	0.9943
30	0.9900	0.9979	0.9939	0.9943
40	0.9911	0.9964	0.9937	0.9943
50	0.9931	0.9954	0.9942	0.9943

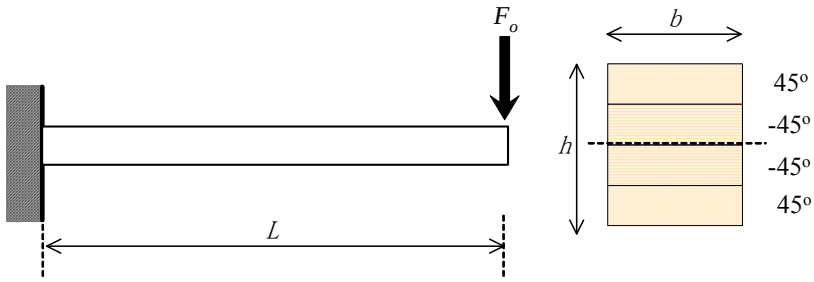

Fig. 1: Composite cantilever beam structure model

Example 2: A composite cantilever beam with a point load is shown in Fig. 1 (Bae *et al*., 2004). To simplify the calculation of tip displacement of the composite beam, a symmetric laminated beam is used with one composite material and (±45) s angle plies. The tip displacement is obtained by the classical laminated plate theory (Reddy, 1997) as:

$$\delta_{Tip} = \frac{F_0 L^3}{2h^3} \cdot \left(\frac{E_L^2 - 4G_{LT}E_T v_{LT}^2 + E_L(E_T + 4G_{LT} + 2E_T v_{LT})}{E_L G_{LT}(E_L + E_T + 2E_T v_{LT})} \right) \tag{32}$$

where,

h : The height (3.83 cm) of the beam
L : The length (50.7 cm) of the beam
F_0 : The applied load per width (360 kN)
G_{LT} : The shear modulus (9.38 GPa)
v_{LT} : The Poisson's ratio (0.03)
The Young's moduli E_L : Considered as uncertain variables and follow normal distribution N (143, 11.8)
The Young's moduli E_T : Considered as uncertain variables and follow normal distribution N (33, 6.2)

If the tip displacement should not exceed the limit state value of 5.59 cm, otherwise, the beam will fail and then the limit state equation of the structure is:

$$Z = \delta_{Tip} - Limit = 0 \tag{33}$$

and safe domain is:

$$\Omega_s = \{\delta_{Tip} : \delta_{Tip} - Limit < 0\} \tag{34}$$

Randomly generate 20 samples by the distribution function of E_L and E_T, respectively as shown in Table 4.

Set gray confidence $\alpha = 0.8$ and gray confidence intervals of E_L and E_T are obtained as (115.26, 177.65) and (16.14, 48.26), respectively. Construct 22 subintervals with their mid-values at $\bar{E}_L$ and $\bar{E}_T$, respectively and $P_j = j/22$ ($1 \le j \le 22$). Then focal elements and BPAs of E_L and E_T are obtained. As Eq. (33) is non-monotonic, sampling method is used to calculate the structure reliability and the results are shown in Table 5. Let m be 30, 40 and 50, respectively and calculate the structure reliability with proposed method. The results are compared with that of Monte Carlo method with 10^6 samples, which is also shown in Table 5. It can be seen from Table 5 that with the increasing of m, $\widetilde{R_s}$ becomes more and more accurate too.

Table 4: Experimental data of E_L and E_T

E_L					$\overline{E_L}$
118.08	129.27	131.23	132.56	135.72	145.32
138.71	139.74	140.45	140.98	143.06	
143.99	145.25	146.61	149.92	153.09	
156.12	159.29	162.54	167.37	172.47	
E_T					$\overline{E_T}$
17.40	18.87	21.18	23.24	26.72	31.12
28.25	28.64	29.55	30.63	31.63	
31.87	32.34	34.07	34.86	35.04	
36.41	37.11	38.85	39.69	46.14	

Table 5: Results of reliability with different sample size in example 2

m	Inf (R_s)	Sup (R_s)	$\widetilde{R_s}$	R_s
20	0.9876	1.0000	0.9928	0.9977
30	0.9883	0.9990	0.9937	0.9977
40	0.9932	0.9983	0.9958	0.9977
50	0.9963	0.9985	0.9974	0.9977

CONCLUSION

When sample size is too small to construct the probability distribution of the variable, conventional probabilistic reliability model is not applicable. Although non-probabilistic reliability model based on interval analysis and convex model can calculate structure reliability under above situation, but in this model only bounds of data can be used. As many valuable data is discarded, the results of non-probabilistic reliability model are always too conservative to be applied in reality engineering structures. In this study, a new imprecise reliability model is proposed based on evidence theory and gray theory, which not only avoids the problem of constructing the distribution functions of variables, but also utilizes all of the experimental data. As the model

considers aleatory and epistemic uncertainties together, the results of reliability calculation can reflect the actual situation of structure. The numerical examples show the model has high precision and its precision will improve when sample size increases. On the other hand, the new model does not depend on the distribution information about variables. So the new model is very effective under small samples and is a beneficial supplement to conventional probabilistic and non-probabilistic reliability model.

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REFERENCES

Bae, H.R., R.V. Grandhi and R.A. Canfield, 2004. An approximation approach for uncertainty quantification using evidence theory. Reliabil. Eng. Syst. Saf., 85(1-3): 215-225.

Ben-Haim, Y., 1993. Convex models of uncertainty in radial pulse buckling of shells. J. Appl. Mech., 60(3): 683-688.

Ben-Haim, Y., 1994. A non-probabilistic concept of reliability. Struct. Saf., 14(3): 227-245.

Ben-Haim, Y., G. Chen and T.T. Song, 1996. Maximum structural response using convex models. J. Eng. Mech., 122(4): 325-333.

Biondini, F., F. Bontempi and M.P. Giorgio, 2004. Fuzzy reliability analysis of concrete structures. Comput. Struct., 82(13-14): 1033-1052.

Chien, C.G., 1997. Large-sample results for batch means. Manage. Sci., 43(3): 1288-1295.

Christopher, D.E. and R.R. Masoud, 2009. Integrated reliability and sizing optimization of a large composite structure. Mar. Struct., 22(2): 315-334.

David, W.C., 1997. System reliability confidence intervals for complex systems with estimated component reliability. IEEE T. Reliabil., 46(4): 487-493.

Dempster, A.P., 1967. Upper and lower probabilities induced by a multi valued mapping. Ann. Math. Stat., 38(2): 325-339.

Deng, J.L., 1990. Application of grey system theory in China. Proceeding of Uncertainty Modeling and Analysis Symp. December, Wuhan, China, pp: 285-291.

Dong, W.M. and H.C. Shah, 1987. Vertex method for computing functions of fuzzy variable. Fuzzy Sets Syst., 24(1): 65-78.

Elishakoff, I., 1994. Discussion on: A non-probabilistic concept of reliability. Struct. Saf., 14(4): 227-245.

Elishakoff, I., 1995. Essay on uncertainties in elastic and viscoelastic structures: from AM freudenthal's criticisms to modern convex modeling. Comput. Struct., 56(6): 871-895.

George, J.K. and M.S. Richard, 2001. On measuring uncertainty and uncertainty-based information: Recent development. Ann. Math. Artif. Intelli., 32(1-4): 5-33.

Giasi, C.I., P. Masi and C. Cherubini, 2003. Probabilistic and fuzzy reliability analysis of a sample slope near Aliano. Eng. Geol., 67(2): 391-402.

Helton, J.C., 1997. Uncertainty and sensitivity analysis in the presence of stochastic and subjective uncertainty. J. Stat. Comput. Simul., 57 (1): 3-76.

Helton, J.C., J.D. Johnson and W.L. Oberkampf, 2004. An exploration of alternative approaches to the representation of uncertainty in model predictions. Reliabil. Eng. Syst. Saf., 85(1-3): 39-71.

Ke, H.F., Y.G. Chen and Y. Liu, 2007. Data processing of small samples based on grey distance information approach. J. Syst. Eng. Electron., 18(2): 281-289.

Law, A.M., 2006. How to build valid and credible simulation models. Proceeding of Winter Simulation Conference, Jun, Shanghai, China, pp: 58-66.

Lu, Z.Z. and Y.W. Feng, 2000. Advances in structural reliability studies. Adv. Mech., 30(1): 21-28.

Luo, Y.J., Z. Kang, Z. Luo and A. Li, 2009. Continuum topology optimization with non-probabilistic reliability constraints based on multi-ellipsoid convex model. Struct. Multidiscip. Optimi., 39(3): 297-310.

Oberkampf, W.L., K.V. Diegert and K.F. Alvin, 1998. Variability, uncertainty and error in computational simulation. ASME Proceedings of the 7th AIAA/ASME Joint Thermo physics and Heat Transfer Conference, June, Albuquerque, New Mexico, pp: 59-72.

Oberkampf, W.L., J.C. Helton and K. Sentz, 2001. Mathematical Representation of Uncertainty. Non-Deterministic Approaches Forum, AIAA, April, Seattle, WA, pp: 165-178.

Park, S., S. Choi, C. Sikorsky and N. Stubbs, 2004. Efficient method for calculation of system reliability of a complex structure. Int. J. Solids Struct., 41(18-19): 5035-5050.

Reddy, J.N., 1997. Mechanics of Laminated Composite Plates: Theory and Analysis. CRC Press, Boca Raton, pp: 243-256.

Shafer, G., 1976. Mathematical Theory of Evidence. Princeton University Press, Princeton, pp: 297, ISBN: 069110042X.

Utkin, L.V. and S.V. Gurov, 1996. A general formal approach for fuzzy reliability analysis in the possibility context. Fuzzy Sets Syst., 83(2): 203-213.