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Analysis Method and its Application of Weighted Grey Relevance Based on Super Efficient DEA

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Abstract: For the objectivity issue of grey relevance analysis method determining weight, a new weighted grey relevance analysis method based on super efficient DEA has been proposed. The new method combines the advantage of super efficient DEA analysis and grey relevance analysis and takes grey relevance analysis as the central model and super efficient model as the sub-model, which determines the weight vector of all the relevance coefficient from every observation point of each sub factor, then calculates the comparatively best relation grade and obtains objective superior sequence of all factors. Super DEA can improve the resolving power of the model to grey relation grade, breakthrough the limit of weight sum being one and make the average weight more flexible. The validity of the combined algorithm has been proved by the case study of the influencing factors of vacant commercial housing.

Keywords: Grey relevance method, relation grade, super efficient DEA, weight

INTRODUCTION

In many attribute decision issues, deciders evaluate and sequence similar decision units by analyzing and comparing the attribute of different natures (Yuny, 2004; Mavrotas and Trifillis, 2006). Grey relevance analysis method is a typical tool of attribute distinction and study, which judges the evaluating object's distribution law according to similarity degree of the value frequency of the analyzed object and common distribution value frequency. It has the characteristics of little required test data, easy process and simple calculating process etc., (Liu and Yu, 2007; Guan *et al*., 2006). However, traditional grey relevance analysis method always can't determine the attribute weight objectively when solving multi-attribute decision and once determined, weight coefficient will be applied to the grey relevance calculation of all decision units. This kind of average weight often cannot achieve the best results.

DEA is a comparatively valid nonparametric method to evaluate Decision Making Unit (DMU) and a common and significant analysis tool and study means as well (Liu *et al*., 2006; Liang *et al*., 2006). It can be determine the comparatively best weight coefficient and relation grade of every grey relevance attribute by solving the comparative distance of each DMU and data's efficient frontier. However, obtained comparative best relation grade by traditional CCR (Charnes, Cooper and Rhodes) model may exist the phenomenon of simultaneous validity, namely, efficiency value is one. Under this circumstance, the comparison of DMU cannot sequence it adequately. This study applies an improved DEA model, super efficient DEA model to solve the problem. Different from general DEA model, super efficient DEA model eliminates the evaluating unit itself from the evaluating unit collection and compares the evaluating unit with the linear combination of all the other evaluating units, whose super efficient value can sequence the comparatively best grey relation grade adequately and has better resolving power. The study has proved the combined the validity of the algorithm by the case of the influencing factors of Beijing vacant commercial housing.

MULTI-ATTRIBUTE DECISION MODEL BASED ON GREY RELEVANCE ANALYSIS

Multi-attribute decision issue: For multi-attribute decision issue, evaluation and study are done to decision unit by analyzing and comparing different attributes, with the mathematical model as follows:

$$D = \begin{array}{c} \\ C_1 \\ C_2 \\ \vdots \\ C_m \end{array} \begin{array}{c} \begin{array}{cccc} A_1 & A_2 & \cdots & A_n \end{array} \\ \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \vdots & \vdots & & \vdots \\ x_{m1} & x_{m2} & \cdots & x_{mn} \end{bmatrix} \end{array}$$

$$Y = [y_1\ y_2\ \ldots y_n]$$

$$\omega = [\omega_1\ \omega_2 \ldots \omega_n] \tag{1}$$

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in which,
$A_1, A_2, \ldots, A_n$: n observation points
$C_1, C_2, \ldots, C_m$: m attributes
x_{ij} : The value of attribute number i from observation point number j
ω : Weight vector
ω_k : The weight of observation number k

Establishment of multi-attribute decision model based on grey relevance analysis: According to the above definition, the data will be dealt as follows:

$$x_1^{(0)}(k) = x_{1k} / \overline{x_1}\ ;\ k = 1,2,\ldots,n$$

$$x_2^{0}(k) = x_{2k} / \overline{x_2}\ ;\ k = 1,2,\ldots,n$$

$$\vdots$$

$$x_m^{(0)}(k) = x_{mk} / \overline{x_m}\ ;\ k = 1,2,\ldots,n \quad (2)$$

In which: $\overline{x_i}$ refers to the average of observation point number n of attribute number i, with the following *m* new sequences obtained:

$$x_1 = \{x_1^{(0)}(1), x_1^{(0)}(2), \ldots, x_1^{(0)}(n)\} \quad (1)$$

$$x_2 = \{x_2^{(0)}(1), x_2^{(0)}(2), \ldots, x_2^{(0)}(n)\} \quad (2)$$

$$\vdots$$

$$x_m = \{x_m^{(0)}(1), x_m^{(0)}(2), \ldots, x_m^{(0)}(n)\} \quad (3)$$

The above process of equalization can eliminate the dimension influence of different attributes. Similarly, the equalization of the essence attribute can obtain the following:

$$x_0 = \{x_0^{(0)}(1), x_0^{(0)}(2), \ldots, x_0^{(0)}(n)\}$$

In which: $x_0^{(0)}(k) = y_k / \overline{Y}(k = 1,2,\ldots,n)$, $\overline{Y}$ refers to the average of observation point number n of essence attribute.

Suppose $M = \{1,2,\ldots,m\}$, $N = \{1,2,\ldots,n\}$, then:

$$\Delta_1 = \min_{i \in M} \{\min_{i \in N} |x_0^{(0)}(k) - x_i^{(0)}(k)|\}$$

$$\Delta_2 = \max_{i \in M} \{\max_{i \in M} |x_0^{(0)}(k) - x_i^{(0)}(k)|\}$$

$$\Delta_3 = |x_0^{(0)}(k) - x_i^{(0)}(k)|$$

$$d_{0i}(k) = \frac{\Delta_1 + \lambda\Delta_2}{\Delta_3 + \lambda\Delta_2}\ ;\ k = 1,2,\ldots,n \quad (4)$$

In which: $d_{0i}(k)$ refers to the comparative distance of x_i and x_0 at observation point number k, which is called the relation grade of x_i and x_0 at observation point number k; λ refers to the resolving coefficient, usually being $0 \le \lambda \le 1$ and $\lambda = 0.5$. Suppose weight ω_k 0, k ε N, given $r_{0i} = \sum_{k=1}^{n} \omega_k d_{0i}(k)$, i ε M, r_{0i} is the relation grade of x_i and x_0, which reflects the relation degree between attribute number *i* and the essence attribute. By comparing r_{0i}, sequence can be done to the relation degree between every sub-attribute and essence attribute and needed information can be offered to deciders.

To calculate r_{0i}, the grey relevance weight vector ω_k needs to be determined. At the time of determining weight vector, there mainly exist the following problems:

- The common practice is applying the method of expert marking, experience value and AHP. However, these methods contain many human factors and their results are always too subjective.
- Once the weight coefficient is determined, it will be applied to the relation grade calculation of every attribute. This kind of average method makes the weight coefficient good for some attributes but bad for other attributes. Therefore, the obtained relation grade lacks fairness.
- Traditional methods require the weight sum to be one, which is based on traditional weight concept but the relation grade of every attribute may not be the best.

The application of DEA method and its combination with grey relevance analysis can solve the above problems.

GREY RELEVANCE ANALYSIS MODEL BASED ON SUPER EFFICIENT DEA

Super efficient DEA: In the traditional CCR model, it will occur that the comparison of DMU is impossible when more than one DMU is valid. The reference collection of super efficient DEA model excludes the evaluated DMU_0 itself and compares the evaluated unit with the linear combination of all the other evaluated units. The maximum ratio value of some DMU input increased but comparative validity kept is called super efficient value, which may obviously be over one. For the DMU that is valid in the result of CCR model, they can be properly solved by comparing super efficient value obtained by super efficient DEA.

Suppose there is n DMU; the input index is m; the output index is s_0 and $X_j = (x_{1j}, x_{2j}, \ldots, x_{mj})^T$ is

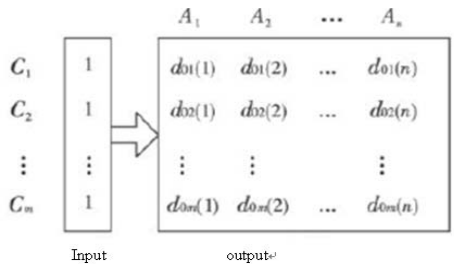

Fig. 1: Input/output structure based on super efficient DEA and grey relevance analysis

the input data collection; $Y_j = (y_{1i}, y_{2j}, ..., y_{sj})^T$ is the output data collection, in which $j = 1, 2, \ldots, n$. The following is the DEA model (opposed to DEA (CCR) model) of evaluation unit DMU_0 number j_0 (the inferior character is 0 and it's the same in the following):

$$\text{Max } h_0 = \sum_{r=1}^{s} \mu_r y_{ro}$$

$$s.t \sum_{i=1}^{m} \omega_i x_{i0} = 1$$

$$\sum_{i=1}^{m} \omega_i x_{ij} - \sum_{r=1}^{s} \mu y_{rj} \geq 0$$

$$j = 1, 2, \ldots, n \ (j \neq 0)$$

$$\omega_i, \ \mu_r \geq 0, \ \forall \ i \ \ r \tag{5}$$

Super efficient DEA improves grey relevance analysis: To calculate r_{0i}, grey relevance analysis is modified by the application of the mentioned DEA model. To establish *m* dimension input vector with input value being one and take every attribute in multi-attribute decision as a DMU, the obtained output value is the relevance grade of the unit. The input/output structure is as Fig. 1:

Based on the grey relevance analysis improved by super efficient DEA, the following mixed model can be obtained:

$$\max \ r_{0i} = \sum_{k=1}^{n} \omega_k d_{oi_0}(k)$$

$$s.t. \omega_0 \times 1 = 1$$

$$\omega_0 \times 1 - k - \sum_{k=1}^{n} \omega_k d_{0i}(k) \geq o$$

$$i = 1,2,..., m(i \neq i_0)$$

$$\omega_k, \omega_0 \geq 0, \ \forall \ k \tag{6}$$

In which: ω_0 refers to input value weight; $\omega_k (k = 1,2,..., n)$ refers to output value weight, namely, the weight of all attributes in grey relevance, every attribute corresponds to n weight values. Format (6) can be simplified as the following:

$$\max \ r_{0i} = \sum_{k=1}^{n} \omega_k d_{oi_0}(k)$$

$$s \ t \sum_{k=1}^{n} \omega_k d_{0i_0}(k) \leq 1$$

$$i = 1,2,..., m(i \neq i_0)$$

$$\omega_k \geq 0, \ \forall \ k \tag{7}$$

The optimum solutions obtained by solving format (7) are ω_k^* and r_{0i}^*, being the weight of number i_0 attribute and the comparatively best grey relation grade. In the traditional grey relevance analysis, the sum of weight is required to be one. And once the weight is determined, it will be applied to calculate very attribute's relation grade, which will lead to the impossibility of obtaining the comparatively best relation grade of every attribute. Format (7) overcomes these deficiencies and for every attribute, the model gives related weight vector and the comparatively best grey relation grade, making the result more objective and reasonable. Format (7) that calculates m attributes m times and sequences the obtained r_{0i}^* value can obtain the sequence of relation grade between m attributes and the essence attribute, namely, the larger the r_{0i}^* value is, the more related is the sub-attribute to the essence attribute.

Through the mathematical description of multi-attribute decision issues and model establishment by the use of grey relevance analysis method, then grey relevance model is modified and improved by applying super efficient DEA method. The mixed model obtained by the improved two-step multi-attribute decision model establishment not only has the advantages like a little data and simple calculation of grey system but also overcomes the deficiency of weight by the application of super efficient DEA. Meanwhile, the mixed model has its advantages over best relation grade, resolving and sequencing, making it an excellent method for the solving of multi-attribute decision issues.

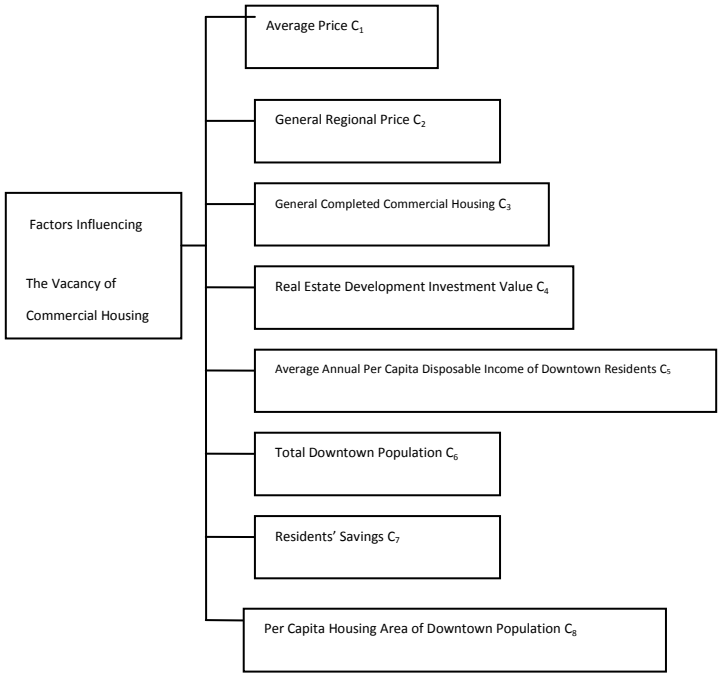

Fig. 2: Factors influencing the vacancy of commercial housing

Table 1: Factors that influenced the vacancy of commercial housing of a certain city in 2002 to 2007

Influencing factors	2003	2004	2005	2006	2007	2008	2009	2010
C_1/Yuan/m^2	4919.00	5062.00	4764.00	4737.00	5052.90	6274.00	8280.00	12436.00
C_2/0.1 billion Yuan	3161.00	3710.50	4330.40	5023.80	6060.30	6886.30	7870.30	9006.20
C_3/10,000 m^2	1365.60	1707.40	2384.40	2593.70	3067.00	3770.90	3193.90	2891.70
C_4/0.1 billion Yuan	522.10	783.80	989.40	1202.50	1473.30	1525.00	1719.90	1995.80
C_5/Yuan	10349.70	11577.80	12463.90	13882.60	15637.80	17653.00	19978.00	21989.00
C_6/10,000 capita	1058.00	1080.00	1118.00	1151.00	1187.00	1286.00	1333.30	1379.90
C_7/0.1 billion Yuan	2923.20	3536.30	4389.70	5293.50	6122.30	8315.80	9515.00	9155.30
C_8/m^2/capita	16.75	17.62	18.20	18.67	19.09	19.45	20.06	20.30
Vacant area of commercial housing/ 10,000 m^2	627.40	774.00	919.00	1123.40	1044.10	1374.20	1039.70	1136.20

CASE STUDY

The factors influencing the vacancy of commercial housing are complicated, affected by economy, society, population, environment, policy etc. Its main influencing factors are shown in Fig. 2.

According to the statistical bulletin of the national economy and social development and the statistical yearbook of a certain city in the year 2007, the factors that influenced the vacancy of commercial housing in the 2003 to 2010 are displayed in Table 1. Attribute $C_1 \sim C_8$ is the sub-attribute of influencing factors; the last line is the data of vacant area of commercial housing, acting as the essence attribute of grey relevance analysis.

Through analyzing the data in chart 1 by grey relevance, the relation grade of every attribute can be obtained. The result and comparison of the weight optimization and relation grade calculation by the respective application of traditional grey relevance analysis method, CCR model and the mentioned mixed model are demonstrated in Table 2 As for grey relevance analysis, based on traditional weight determining method, the sequence of eight influencing factors is $C_2 \succ C_5 \succ C_3 \succ C_7 \succ C_1 \succ C_6 \succ C_8 \succ C_4$, where "$\succ$" means superiority, namely, the sub-attribute is more related to the essence attribute. It can be told that the average price of commercial housing C_1 is too inferior, which is not in compliance with the market practice. As for CCR model, the relation grade of C_1、C_2、C_3、C_5 and C_7 are all one, which makes it impossible to sequence adequately. By the application of super efficient DEA model, the factors can be optimized and districted, with the

Table 2: Results and comparison obtained by grey relevance analysis improved by super efficient DEA

Influencing factors	2003	2004	2005	2006	2007	2008	2009	2010	Grey relation grade	Grey relation grade based on CCR	Grey relation grade based on super efficient DEA
C_1	1	0. 831	0. 670	0. 549	0. 612	0. 524	0. 975	0. 584	0. 718	1.000	1. 241
C_2	1	0. 944	0. 914	0. 833	0. 799	0. 988	0. 547	0. 492	0. 815	1.000	1. 464
C_3	1	0. 984	0. 781	0. 902	0. 634	0. 638	0. 596	0. 766	0. 788	1.000	1. 158
C_4	1	0. 790	0. 700	0. 662	0. 465	0. 579	0. 381	0. 333	0. 614	0. 812	0. 812
C_5	1	0. 897	0. 794	0. 691	0. 868	0. 675	0. 786	0. 762	0. 809	1.000	1. 236
C_6	1	0. 825	0. 711	0. 589	0. 650	0. 508	0. 717	0. 665	0. 708	0. 916	0. 916
C_7	1	0. 977	0. 965	0. 980	0. 700	0. 606	0. 386	0. 432	0. 756	1.000	1. 133
C_8	1	0. 847	0. 727	0. 598	0. 667	0. 494	0. 686	0. 627	0. 705	0. 462	0. 462

commercial housing and the average annual per capita disposable income of downtown residents are the major factors influencing the vacancy of commercial housing, total downtown population, real estate development investment value and per capita housing area of downtown population having little influence on it.

CONCLUSION

The study researches the issue of multi-attribute, solves the relation grade of every attribute by the application of grey relevance analysis, improves grey relevance analysis by super efficient model and optimizes weight vector and grey relation grade. The obtained mixed model has the following advantages:

- It overcomes the problem of objective determination of the weight vector of multi-attribute decision. Improved model by the application of nonparametric evaluating method DEA makes the weight more objective.
- In traditional calculation, all the attributes from different observation points correspond to the same weight and the sum of the weight of a certain attribute form all observation points is one. The mixed model in the study breakthroughs the limitation of the sum being one, which is more flexible and able to obtain the comparatively best relation grade of all attributes.
- Super efficient DEA can improve the resolving power of traditional DEA model and sequence the effective relation grade adequately. Further study can expand grey relevance to the range of fuzzy number and take the optimization of the grey relevance analysis of observation points by super efficient DEA into consideration.

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