

A Discrete Velocity Traffic Kinetic Model Including Desired Speed

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Abstract: We introduce the desired speed variable into the table of games and formulate a new table of games and the corresponding discrete traffic kinetic model. We use the hybrid programming technique of VB and MATLAB to develop the program. Lastly, we compared the proposed model result and the detector data. The results show that the proposed model can describe the traffic flow evolution.

Keywords: Desired speed, table of games, traffic kinetic model

INTRODUCTION

In the system of traffic flow models, there are three description scales, which are macroscopic, mesoscopic and microscopic traffic flow models. The basic problem of studying the mesoscopic traffic flow model is to establish the evolution equation of the speed distribution function. The first mesoscopic models of vehicular traffic flow was the Prigogine and Herman (1971), which was established by the kinetic theory of gases. Continuous Prigogine-Herman models are difficult to solve because of the occurrence of integro-differential term. Lu *et al.* (2011) formulate a discrete traffic kinetic model by integrating cell transmission mechanism and continuous Prigogine-Herman model, which can capture not only the number of vehicles, but also the velocity probability distribution. Recently, methods of discrete mathematical kinetic theory had been developed to model vehicular traffic flow. This approach on the one hand converts the Boltzmann's integro-differential equation into a set of partial differential equations, on the other hand relaxes the continuum hypothesis and includes the granular nature of vehicular traffic. Further details on methods of discrete mathematical kinetic theory refer to Bellomo (2008), Bellomo and Dogbe (2011) and Tosin (2008). Kinetic-type models with discrete velocities for traffic flow have been proposed. There are three methods of velocity discretization. The first method is the discretization method with fixed velocity grid (Delitala and Tosin, 2007). The second method is the discretization method with adaptive velocity grid (Coscia *et al.*, 2007). The third method makes the coupling of fixed velocity grid and adaptive velocity grid (Bianca and Coscia, 2011). For the third method, it means that the number of velocity class is constant and when the density is less than the critical density, the velocity is discretized by a fixed grid and when the density is larger than the critical density, the

velocity is discretized by an adaptive grid. Bonzani and Mussone (2009) deals with the identification of the parameters of Delitala-Tosin model using experimental data obtained on the highway Padova-Venezia. Vehicular traffic flow is composed of many driver-vehicle units. The driver-vehicle units can modify their dynamics according to specific strategies due to their ability, which are called active particles, which are different from the classical particles in the Newtonian dynamics. Gramani (2009) and Bellouquid *et al.* (2012) modeled the driver-vehicle unit as an active particle. In particular, these two papers include in the generalized velocity distribution function an activity variable, which describes the driving skills, to model the individual behaviors. The corresponding model structure is called discrete mathematical kinetic theory for active particles.

According to their individual character, technical proficiency of driving, gender and age, car drivers can be divided into different groups. Different drivers have different desired speed which exists in their mind. The individual character of the car driver can be divided into three types: adaptation, conservatism and adventure, which are described by the requirements of the space headway. An adaptive driver approximately keeps the space headway equal to the safety space headway in the car following process; Conservative driver, in most cases, keeps the space headway greater than the safe space headway in the car following process; Aggressive driver keep the space headway less than safety space headway. Actually, under the same road conditions and traffic environment, an aggressive driver has a higher expected speed, a conservative driver has a lower expected speed and an adaptive driver has the desired speed between them. In the real traffic flow, we can find that traffic flow may have a different flow rate and average speed in the same density, which is caused by the difference of the desired speed.

Therefore, the desired speed is an important variable, which should be introduced into the mesoscopic model and describes the effect of the driving behavior on traffic flow performance. It is worth mentioning that, an important improvement of the Prigogine-Herman model by Paveri-Fontana (1975) is also to introduce the desired speed into the model. In the study, we introduce the desired speed variable into the methods of discrete mathematical kinetic theory and propose a new model.

THE PROPOSED TRAFFIC KINETIC MODEL BASED ON THE IMPROVED TABLE OF GAMES

The main tools of the discrete mathematical kinetic theory are composed of encounter rate and table of games. The evolution equation of the velocity distribution function is hyperbolic partial differential equations in the inhomogeneous condition. The evolution equation for the inhomogeneous case in (Delitala and Tosin, 2007) is:

$$\frac{\partial f_t}{\partial t} + \frac{\partial(V_i f_i)}{\partial x} = J(f) \tag{1}$$

$$J(f) = \sum_{h=1}^{2n-1} \sum_{k=1}^{n-1} \eta_{hk} A_{hk}^i f_h f_k - f_i \sum_{k=1}^{2n-1} \eta_{ik} f_k$$

f_i = The speed distribution of the speed class i .

In general, three types of vehicles are involved in the interactions: Test vehicle which is the representative of the whole system, Field vehicles which interact with test and candidate vehicles and Candidate vehicles which may acquire, in probability, the state of the test vehicle by interaction with the field vehicles. Encounter rate describes the number of interactions of the vehicles with different velocity. For example, η_{hk} denotes the encounter rate of vehicles with velocities V_h and V_k . $H = 1/(1-u)V_h$, u is defined as the ratio between the number of vehicles and maximum number of vehicles. Table of games describes the velocity transition probability after vehicle interaction. For example, A_{hk}^i is the probability density that the candidate vehicle with velocity V_h reaches the velocity V_k , after the interaction with the field vehicle with velocity V_k . The mathematical structure of the evolution equations is that the variation of velocity distribution is equal to the increase amount minus the decrease amount. Because of the property of table of games, the series of model of discrete mathematical kinetic theory are conserved.

There are three models for the table of games. The first one is Delitala and Tosin (2007) model. The speed discretization is fixed and volume and road condition are introduced to the table of games. The interactions between any speed class are possible, but the transition speed after interaction is the adjacent speed class. The second one is the discretization method with adaptive velocity grid (Coscia *et al.*, 2007). The model assumes that the speed transition probability is fixed and only the adjacent speed classes can interact and transform. The third one makes the coupling of

fixed velocity grid and adaptive velocity grid (Bianca and Coscia, 2011). This model does not restrict the speed class of the interaction and transformation.

In this study, we improve the table of game by introducing the desired speed based on the Delitala and Tosin (2007) model:

When $V_h < V_k$

$$A_{hk}^i(u) = \begin{cases} 1 - \frac{|\omega-v|}{\omega} \alpha(1-u), i = h, \text{maintain} \\ \frac{|\omega-v|}{\omega} \alpha(1-u), i = h+1, \text{accelerate} \end{cases} \tag{2}$$

When $V_h > V_k$

$$A_{hk}^i(u) = \begin{cases} 1 - \alpha(1-u), i = k, \text{decelerate (a)} \\ \frac{|\omega-v|}{\omega} \alpha(1-u), i = h+1, \text{accelerate (b)} \\ \frac{v}{\omega} \alpha(1-u), i = h, \text{maintain (c)} \end{cases} \tag{3}$$

where, the sum of (b) and (c) is the probability of overtaking.

When $V_h = V_k$

$$A_{hk}^i(u) = \begin{cases} \frac{|\omega-v|}{\omega} \alpha u, i = h-1, \text{decelerate} \\ 1 - \frac{|\omega-v|}{\omega} \alpha, i = h+1, \text{maintain} \\ \frac{|\omega-v|}{\omega} \alpha(1-u), i = h+1, \text{accelerate} \end{cases} \tag{4}$$

$$A_{11}^i = \begin{cases} 1 - \frac{|\omega-v|}{\omega} \alpha(1-u), i = 1, \text{maintain} \\ \frac{|\omega-v|}{\omega} \alpha(1-u), i = 2, \text{accelerate} \end{cases} \tag{5}$$

$$A_{nn}^i(u) = \begin{cases} \frac{|\omega-v|}{\omega} \alpha u, i = n-1, \text{decelerate} \\ 1 - \frac{|\omega-v|}{\omega} \alpha u, i = n, \text{maintain} \end{cases} \tag{6}$$

where,

- ω = The desired speed
- v = The current speed of the vehicle
- α = The condition of the road, $\alpha \in (0,1)$ the higher of value, the better of condition of road
- u = The saturation degree.

Formulas from (1) to (6) constitute the proposed model.

AN EXAMPLE

Data collection: The data is collected by the laser detector, which uses the principle of laser ranging. Laser ranging uses electromagnetic waves to measure the distance between the laser gun and the vehicle by

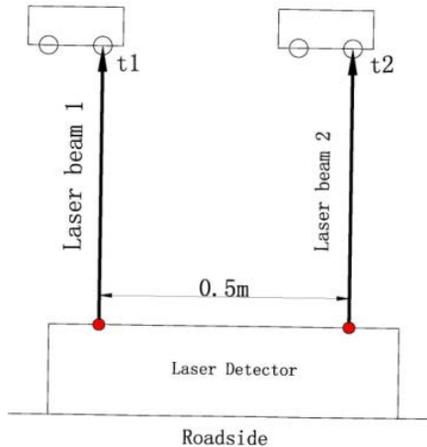


Fig. 1: The principle of the laser detector

Table 1: Data form of the laser detector

| Time | Driving direction | Lane number | Speed | Vehicle type |
|--------------|-------------------|--------------------|-------------------|--------------|
| Space headwa | Time headwa | The number of axes | The axis distance | |
| y | | | | |

Table 2: The type of the vehicles

| Type | Number | Proportion |
|--------------------|--------|------------|
| Large size trucks | 44 | 0.03 |
| Medium size trucks | 254 | 0.15 |
| Cars | 1317 | 0.79 |
| Non-motor vehicles | 57 | 0.03 |
| Sum | 1672 | 1 |

Table 3: Statistics of the speed (km/hr)

| Type | V _{max} | V _{min} | V _{mean} | V _{85%} |
|--------------------|------------------|------------------|-------------------|------------------|
| Large size trucks | 59.70 | 20.54 | 41.53 | 49.94 |
| Medium size trucks | 72.48 | 21.39 | 46.00 | 56.18 |
| Cars | 89.28 | 21.05 | 53.91 | 65.66 |

Table 4: Statistics of the headway

| | Lane 1 | Lane 2 | Lane 3 | Sum |
|----------------------------|--------|--------|--------|------|
| Average time headway (s) | 5.15 | 4.39 | 4.89 | |
| Space headway (m) | 77 | 66 | 73 | |
| Density (Veh/Km) | 13 | 15 | 14 | 42 |
| Number of vehicles (Veh) | 358 | 710 | 604 | 1672 |
| Percentage of the lane use | 0.21 | 0.42 | 0.36 | 1 |

emitting a laser beam to the vehicle, receiving the reflected wave and reporting the time difference. The speed of electromagnetic wave is 300,000 Km/h. The principle is illustrated in the Fig. 1. The distance between laser beam 1 and laser beam 2 is set as 0.5 m. So the speed of the vehicle is calculated as:

$$v = 0.5 / (t_2 - t_1) \tag{7}$$

The unit is m/s.

The measured data form of the laser detector is illustrated in Table 1.

Data analysis: We did the data collection on the highway with unidirectional three lanes. The number of vehicles on the unidirectional three lanes is 1672 Veh/h from PM16:30 to 17:30. Based on the average time

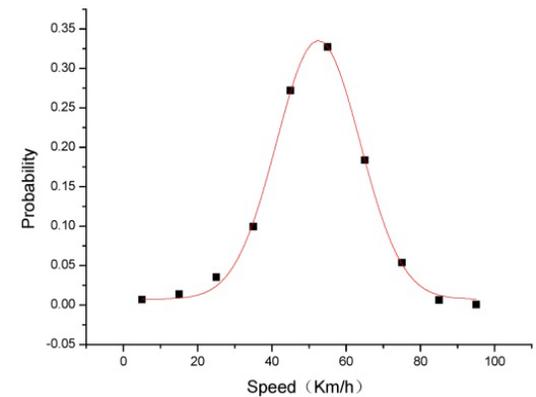
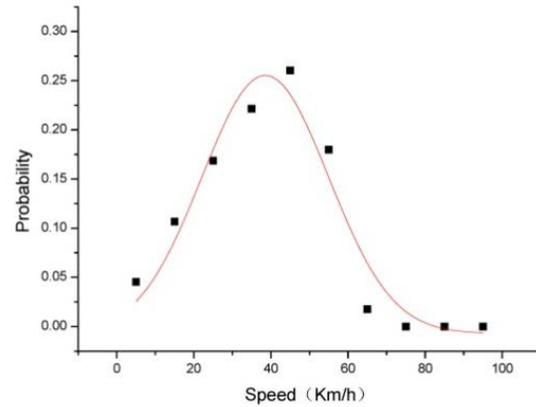


Fig. 2: The normal distribution fitting of the model result and detector data

headway, we derive the average space headway and the density. The density is 42 vehs/km. After sorting out the data, we divide all vehicles into four types: large size trucks, medium size trucks, cars and non-motor vehicles. The proportion of vehicle types is illustrated in the Table 2.

For the proposed model in the above section, the desired speeds are the input parameters. We calculate 85% speed of the detector data as the desired speed. The results are illustrated in Table 3.

The statistics of the headway is illustrated in Table 4. Table 4 shows some characteristics of Chinese driver behavior. More drivers choose the middle lane. They may overtake from right lane or left lane, which may be dangerous for overtaking by the right lane.

We calculate the jam density by the minimum time headway from the laser detector, which is 92Veh/Km. So $u = 42/92 = 0.45$.

Model performance analysis The study uses the hybrid programming technique of VB and MATLAB to develop the program. The speed is divided into 10 classes. The desired speed is set as No.6 and No.7. $u = 0.45$ and the condition of road is set as $\alpha = 0.9$. The results are illustrated in the Fig. 2. The normal distribution fitting equation of speed distribution based on the model result is:

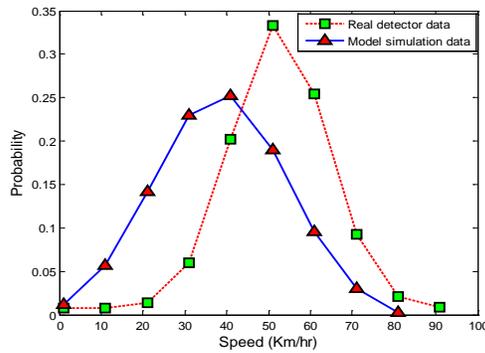


Fig. 3: The comparison between the model result and the detector data

$$y = -0.00704 + \left(\frac{10.79691}{32.81307 \cdot \sqrt{\pi}/2} \right) * e^{-2 * \left(\frac{x-38.53182}{32.81307} \right)^2} \quad (8)$$

The correlation parameter is $R^2 = 0.93908$

The normal distribution fitting equation of speed distribution based on the detector data is

$$y = 0.00751 + \left(\frac{9.24948}{22.50196 \cdot \sqrt{\pi}/2} \right) * e^{-2 * \left(\frac{x-52.50155}{22.50196} \right)^2} \quad (9)$$

The correlation parameter is $R^2 = 0.99778$

According to the fitting equation, the comparison between the model and the detector is illustrated in Fig. 3. For the real detector data, $\mu_1 = 0.1$, $\sigma_1 = 01116$. For the model simulation data, $\mu_1 = 0.1$, $\sigma_1 = 01198$. The model result has the better agreement with the real detector data.

CONCLUSION

From the detector data, the statistics tell us there are different desired speeds for the different vehicle type and different drivers. The difference between the desired speeds causes the lane-changing and overtaking. So introducing the desired speed into the traffic kinetic model is important. One characteristic of Chinese driver behaviour is that more drivers choose the middle lane. They may overtake from right lane or left lane, which may be dangerous for overtaking by the right lane.

In this study, we propose a discrete traffic kinetic model considering desired speed. An example shows that the proposed model can describe the speed distribution evolution.

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