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Research on Multi-Objective Minimum Spanning Tree Algorithm Based on Ant Algorithm

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Abstract: The ant algorithm is a branch of the artificial intelligence, which is developed from natural rules and of strong adaptability and expandability. Compared to other conventional algorithms, the algorithm has an unparalleled advantage that is demonstrated on combinatorial optimization problems through its application in the multi-objective minimum spanning tree.

Keywords: Ant algorithm, multi-objective optimization, minimum spanning tree, multi-objective minimum spanning tree, vartificial intelligence

INTRODUCTION

The minimum spanning tree problem is to construct a minimum cost spanning tree with the weight graph and is significant to network optimization. There are some typical solving algorithms for the minimum spanning tree in the graph theory, such as the Kruskal algorithm, the Prim algorithm and so on. In fact, A great many models in leaning activities such as road paving, erection of power grid, network building, “firefighting robot” etc. may come down to the minimum spanning tree model, so the Kruskal algorithm and the Prim algorithm (Xie *et al*., 2003) can be used for solution of the problem, in which time complexity is linear. However, problems in the real life are not so simple, an edge of the tree may have more than one weight, for example, environment and induction factors should be considered in the fixed-point path of “firefighting robot”; in erection of power grid, line expense should be taken into account as well as erection difficulty; in network connection, network delay should be taken into consideration besides transmission stability and safety. Therefore, the problem is converted into the Pareto optimal solution of the multi-objective minimum spanning tree; this is a NP problem, which is hardly solved by means of the Kruskal algorithm, the Prim algorithm and other conventional algorithms, with a highly complicated search algorithm.

PROBLEMS ON MULTI-OBJECTIVE MINIMUM SPANNING TREE

Minimum Spanning Tree (MST) Ma and Jiang (1998): In the graph theory, a connected subgraph without any loop (cycle) is called a tree for short. Assume T = (N, Er) is a subgraph of $|N|3$, the following six definitions about the tree are equivalent:

- T is connected without any loop
- T has $|N|$-1 edges without any loop
- T is connected with $|N|$-1 edges
- T is connected and each edge is the cut edge
- Any two points of T is connected with the only path
- T has no loop, but if an edge is added between any pair of nonadjacent points, the only loop will be formed

Assume that there is an undirected graph G = (N, E, W), in which $W = \sum_{e \in E} w_e$ is a weight function; if T = (N, Er, Wr) includes all vertexes of G, then T is a generating tree or spanning tree of G, in which $W = \sum_{e \in E} w_e$ is a weight of the tree T; the spanning trees of the graph G are not unique and the spanning tree with the minimum weight is the minimum one of G.

Multi-objective minimum spanning tree (multi-criteria MST): If each edge in a graph has multiple weights, the corresponding problem is called the multi-objective minimum spanning tree problem. This kind of

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problems is intended to find out all Pareto optimal or effective minimum spanning trees, obviously, these problems belong to NP-complete problems (Ma and Jiang, 1999) even if constraint conditions are added.

Assume a connected undirected graph G = (V, E) is present, where V= $\{v_1, v_2, \ldots v_n\}$ is a limited set which stands for vertexes of edges in the graph G, E = $\{e_{1,2}, e_{1,3}, \ldots, e_{ij}, \ldots e_{n-1n}\}$

$$e_{ij} = \begin{cases} 1, \ if\ there\ is\ a\ edge\ bet\ ween\ v_i\ and\ v_j \\ 0, \mathrm{Ot\,her} \end{cases}$$

(i, 1, 2, …n-1; j = 1, 2, …, n) is the set of edges in the graph G. If the edge e_{ij} is present, then:

The edge has m corresponding properties with positive values, which are expressed with $W_{ij} = \{w_{ij}^1, w_{ij}^2, w_{ij}^m\}$; in practical problems, w_{ij}^k(k = 1, 2, …, m) may be distance cost etc.

$$X = (x_{1,2}, x_{1,3}, \ldots x_{ij} \ldots x_{n-1,n})$$

$$令，\ x_{ij} = \begin{cases} 1, \ e_{ij} = 1 \text{ and is choosen} \\ 0, \mathrm{Ot\,her} \end{cases}$$

min f_1 (x) = $\sum w_{ij}^1 x_{ij}$; (i = 1, 2,…n-1;j = 1,2…n) Indicates a tree of the graph G. X is the set of all x , then the mc-MST problems may be expressed as follows:

$$\begin{aligned} &\min f_1(x) = \sum w_{ij}^1 x_{ij}; \\ &\min f_2(x) = \sum w_{ij}^2 x_{ij}; \\ &\cdots \\ &\min f_m(x) = \sum w_{ij}^m x_{ij}; i = 1, 2, \cdots n-1; j = 1, 2 \cdots n \\ &Constraint\ \ condition \quad x \in X \end{aligned}$$

where, f_i (X) is the i-th minimized objective required for the problem.

Compared to common MST problems, the objective functions of the mc-MST problem are not unique; however, it is multiple objectives and frequent conflict between them that make it impossible to generate the minimum tree gradually through finding out the edge with the smallest weight by means of the typical MST algorithm. If the typical algorithm is applied after multiple objectives are converted into a single objective, only one solution is obtained instead of a group of Pareto optimal solutions of the problem, furthermore, conversion from multiple objectives to a single objective is also a challenge for the decision maker.

APPLICATION OF THE ANT ALGORITHM IN THE MULTI-OBJECTIVE MINIMUM SPANNING TREE

Background of the ant algorithm: The ant algorithm is a new bionic algorithm from the living world in the nature and comes into the world for only several years. As a general stochastic optimization method, the algorithm involves behavioral characteristic of the ant in the insect kingdom and has been crowned with success in a series of difficult combinatorial optimization problems through the inherent search mechanism. Because the artificial ant concept is used in the simulation, so sometimes the system is known as the ant system.

According to observation and research of entomologists, it is discovered that ants in the biological world can find out the shortest path from their ant nest to the food source without any visible hint, which is varied with change of the environment; they adaptively search for a new path to generate a new selection. As an insect, the ant can release a peculiar ectocrine (pheromone) along the passed path in finding food source, which makes other ants within a certain range be able to perceive it and produces effects on their subsequent actions. The more ants there are passing some paths, the more pheromone is left so as to increase intensity of pheromone and this selection process is known as autocatalytic behavior of ants. Because the theory is a positive feedback mechanism, so the ant colony may be considered to be the so-called reinforcement learning system.

Since the ant algorithm becomes effective in the famous Travelling Salesman Problem (TSP) and the sequencing problem of work pieces, it has spread to other problem fields such as the graph coloring problem, large scale integrated circuit design, the load balancing problem in the communication network, vehicle scheduling problem and so on, with quite excellent performance in many respects.

Theory of the ant algorithm: The fundamental principle of the artificial ant algorithm for the optimization field absorbs some distinguishing features of ant colony behavior in the biological kingdom:

- Probable of detecting conditions in a small area and estimating whether there is food or pheromone track of other ants
- Probable of releasing their own pheromone
- The quantity of the remaining pheromone will become less and less as time goes. Since ants in the natural world are almost blind, they do not know where to go find food and gain it, nor do they know how to return their own nest after finding the food, therefore, they depend on only track of the special material (pheromone) emitted by their same species to decide where to go. It is interesting that ants are able to find the best path from their nest to the food source without any prior knowledge, even if obstacles are placed on the path, they can still find the new best path quickly again. Here, a visual

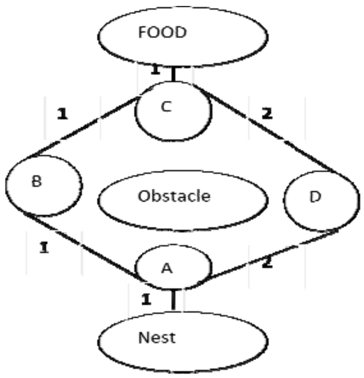

Fig. 1: Ants move to the food source from the nest

diagram is utilized to explain the path search theory and the mechanism of the ant colony.

Assume that there are two paths around the obstacle, through which the food source can be reached from the ant nest (Fig. 1): Nest-ABD-Food and Nest-ACD-Food, Ji and Xin (1999) their lengths are 4 and 6 respectively. An ant can move for a unit length within unit time and at the beginning there is no pheromone left on all paths.

At t = 0 $t = 0$, 20 ants begin to move for A from the nest. They select the left road or the right one with the same probability, therefore, on the average, there are 10 ants go from the left side, other 10 ants from the right.

At t = 4, the first group of ants arriving at the food source will return.

At t = 5, two groups of ants will meet each other at the point D; at the moment quantity of the pheromone on the BD is same as that on the CD, because there are 10 ants on each road selecting the corresponding road, of which 5 ants select BD consequently, while other 5 ants select CD.

At t = 8, the first five ants will return to the nest, but there are 5 ants on the AC, CD and BD each.

At t = 9， the first five ants move to A, they will encounter selection of the left road or the right again.

At this time, the track number on AB is 20, while 15 on AC, therefore, majority of ants will select the left path to enhance the pheromone on the path; as the process goes on, difference of pheromone quantity between the two paths will become increasingly large until most ants select the shortest path, just because one path is shorter than the other path; for this reason, in the same time interval, there will be more chances to select the shorter path.

The updating equation of track intensity is expressed as follows:

$$\tau_{ij}^{new} = \rho \bullet \tau_{ij}^{old} + \sum_k \triangle\tau_{ij}^k$$

Meaning of each parameter is provided as follows:

- m = Number of ants
- n_{ij} = Visibility of side arc (i, j), i.e., $1/d_{ij}$
- τ_{ij} = Track intensity of side arc (i, j)
- $\triangle\tau_{ij}^k$ = Track pheromone quantity per unit length left by the ants on the side arc (i, j)
- P_{ij}^k = Diversion probability of the ant k is proportional to $\tau_{ij}^{\alpha} \bullet \eta_{ij}^{\beta}$, where j is the node which is not accessed:
- α = Relative importance of track (α ≥ 0)
- β = Relative importance of visibility (β ≥ 0)
- ρ = Durability of track (0 ≤ ρ < 1).1- ρis interpreted as track evaporation.
- Q = A constant indicating track quantity left by ants

For example, in the typical TSP problem, main steps for solution through the artificial ant method may be described as follows:

Step 1: nc ← 0 ; (nc is the number of iterations or search times)

Initialing of each τ_{ij} and $\triangle\tau_{ij}$; m ants are placed on n vertexes.

Step 2: The starting point of each ant is put in the current solution set

Each ant k(k = 1, …, m) is moved to the next vertex J according to the probability P_{ij}^k.
The vertex j = put in the current solution set:

Step 3: The objective function value Z_k(k = 1, …, m) of each ant is computed, with a record of the best solution

Step 4: Track intensity is modified as per the updating equation

Step 5: For each side arc (i,j), $\triangle\tau_{ij} \leftarrow 0; nc \leftarrow nc+1$.

Step 6: if nc are less than the predetermined iterations without any degradation (that is to say, the obtained solutions are same), go to Step 2

Time complexity of the algorithm is $O(nc \bullet m \bullet n^2)$. For TSP, the empirical result may be expressed as follows: when m is approximately equal to n, the effect is best, the time complexity at the moment is $O(nc \bullet n^3)$; because of graph symmetry of the algorithm and no special requirements in the objective function, the algorithm may be applied to various asymmetry problems and nonlinear problems. Now, there is no theoretical basis for parameter setting in the algorithm yet, all published experimental results are applicable for specific problems only. The earliest data is acquired by solving some examples in the problem library (TSPLIB) of TSP: $0 \leq \alpha \leq 5, 1 \leq \beta \leq 5$, 0.1≤ 0.99 (0.7 or so is preferred), $10 \leq Q \leq 10000$.

APPLICATION

Application to path problems:

- **Multi-objective TSP:** In practic al problems, many factors are often considered at the same time, such as the shortest distance and time, the minimum cost, the smallest risk and so on; it means that there are multiple weight properties between cities; therefore, it is of great actual significance to study the multi-objective TSP. There is no doubt that the multi-objective combinatorial optimization problem is difficult to solve, which is more complicated than the single objective problem and is hardly studied at home and abroad, especially practical algorithms is scarce; because the solution at the moment is "the compromise solution" or "the non-inferior solution", so the solution of the multi-objective TSP may be defined as follows:

Assume that there is a cycle solution H no other cycle solution Q, then $Z_\tau \leq Z_\tau$ (H), r = 1, 2, …,L, in which one equation at least is strictly tenable (Zis the corresponding objective function value), so H is a non-inferior solution or Pareto solution.

It is not difficult to generalize the solving thought of the ant algorithm to the multi-objective condition, which can be achieved only when each weight is reflected as per the corresponding probability as the ant moves. If there is weight priority in the actual problem, the algorithm can meet the requirement better. The multi-objective bottleneck TSP can be solved with similar methods and the preliminary computation experiment is performed on the PC, which is compared with the simulated annealing algorithm and other methods; the result shows that more solutions can be obtained by means of the ant algorithm.

- **Multi-objective shortest path:** The ant algorithm itself is the mechanism, through which an ant colony searches the shortest path. So it is natural that the algorithm be applied to solving the shortest path, such as the conventional shortest path, K shortest paths and so forth. However, these problems have been well solved for long, thus the ant algorithm's advantage is not shown. If the algorithm thought is introduced in solving the multi-objective shortest path problem, it will play a similar part compared with solution of the multi-objective TSP. If the objective function is modified as the bottleneck form, the multi-objective (bottleneck) shortest problem will be solved. Computer experiments show that these attempts are feasible and effective.
- **Quadratic Assignment Problem (QAP):** Original formulation of the problem may be described as follows: there are n known locations and n known factories, the distance matrix between any two locations is assumed to be D = $[d_{ij}]_{n\bullet n}$, the freight volume matrix between any two factories is F = $[f_{ij}]_{n\bullet n}$, these n factories are required to be built on the n known locations respectively with the minimum total expense; in the process, the cost to construct the factory i at the location k and the factory j at the location l is $f_{ij}d_{ij}$ (construction cost of each factory will not produce an essential effect on difficulty in problem solving, so it is often ignored). The problem becomes extremely difficult due to nonlinearity of the objective function.

When the ant algorithm is used for solving, η_{ij} in transition probability is equal to $1/s_{ij}$, where:

$$s_{ij} = (\sum_{p=1}^{n} f_{ip})(\sum_{q=1}^{n} d_{jq})$$

QAP is the second NP problem following TSP, which is solved through the ant algorithm, relevant experimental results and detail can be seen in the earlier documents on the ant algorithm, they will not be covered here.

Application to the optimal tree problem:

- **Degree-constrained minimum tree problem:** It is well known that the minimum spanning tree problem is a common basic problem in network optimization and proven methods have been available for effective solving. If degree of each vertex in the tree is limited, namely not exceeding the predetermined value, then nature of the problem will become entirely different, this is the so-called Degree-constrained Minimum Spanning Tree problem (DCMST), the combined meaning is to find out the spanning tree from all spanning trees (up tonn n^{n-2}), of which the vertex degree satisfies the constraint and the total weight is minimum. Solving difficulty of the problem varies with different vertex degree constraints. There are numerous examples in our real world, such as pipeline laying, circuit design, communication system waiting and computer network system waiting.

Quite good results can be achieved in solving these difficult optimization problems with the ant algorithm, especially when the vertex degree constraint is relatively rigorous; the core of the algorithm is that each ant moves as per formation rules of the spanning tree, with the same transition probability as the typical TSP. A large number of numerical tests show that it is obviously effective.

- **Multi-objective optimal tree problems:** The so-called Pareto effective tree can be derived from the typical minimum spanning tree concept, namely to seek the optimal tree of multiple objectives' properties. In addition, there is the so-called MIN-MAX vertex degree spanning tree problem in engineering design such as circuit layout design, which is intended to find out a spanning tree to minimize the maximum vertex degree in the tree. If the total weight of the spanning tree is also required to be minimal, then the problem will become the optimization problem with double objectives; the ant algorithm for these multi-objective optimal tree problems can be designed based on the ant algorithm thought for the multi-objective shortest path problem and the degree-constrained minimum tree problem. Because implementation detail is similar to that of path problems, so it will be not covered again; certain results are obtained through the preliminary testing.

SOFTWARE IMPLEMENTATION AND COMPUTER EXPERIMENTS

A complete set of ant algorithm is implemented by means of DELPHI and is embedded in the integrated software package (operations research and management science) OPMS For Windows, which is developed by ourselves; the software runs on Windows 98 and includes more than 80 algorithm modules, available for solving of practical problems. In addition, a large number of computer experiments are carried out on Pentium computers, in which the ant algorithm in the software is used to solve some combinatorial optimization problems mentioned in this study and is compared with other algorithms in the software package; it can be seen by comparison that the new thought of the ant algorithm shows the optimizing ability in solving NP challenges.

CONCLUSION

At present, besides those accepted hot evolutionary methods such as the genetic algorithm, the simulated annealing algorithm, the tabu search algorithm, the artificial neural network and so on, the emerging ant algorithm begins to play a important role and provides a new and effective solving algorithm for the complicated system optimization problems; although some thoughts are still in the infancy, the people have been indistinctly aware that, mankind comes into being from the nature, so it seems that inspiration of problem solving should come from the nature as well. This novel systematic thought which is presented and improved by European scholars is attracting more and more people's attention and is studied by them and its application scope begins to spread throughout many fields. According to the available retrieved reference documents, those scholars studying and using the ant algorithm are mainly from Belgium, Italy, UK, France, Germany and other European countries; in Japan and USA research of the ant algorithm has been begun in recent years. In some university graduate academic dissertations have covered projects on the algorithm. Among all accomplished PhD Dissertations from all universities of USA in 1999, one dissertation is related to the ant algorithm. In our country, there are a small number of reports and research achievements just from late 1998 to early 1999, which are confined to the TSP problem only.

Nowadays, the ant algorithm thought has gradually become an independent branch in the scope of the multi-objective minimum spanning tree algorithm, which has been discussed as a special topic for times. In 1998, the First International Workshop on the Ant Algorithm is held in University of Brussels, Belgium and the Third International Workshop on the algorithm still took place there in 2000. Although the ant algorithm research is still at the experimental stage, the novel system optimizing thought from imitating of natural living beings has a promising future undoubtedly in view of current application effect and more intensive research study needs to be further developed.

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