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Synchronization of Coupled Chaotic Neurons with Unknown Time Delays via Adaptive Backstepping Control

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Abstract: In this study, an adaptive Neural Network (NN) based backstepping controller is proposed to realize chaos synchronization of two gap junction coupled FitzHugh-Nagumo (FHN) neurons with uncertain time delays. In the designed backstepping controller, a simple Radial Basis Function (RBF) NN is used to approximate the uncertain nonlinear part of the error dynamical system. The weights of the NN are tuned on-line. A Lyapunov-Krasovskii function is designed to overcome the difficulties from the unknown time delays. Moreover, to relax the requirement for boundness of disturbance, an adaptive law to adapt the disturbance in real time is given. According to the Lyapunov stability theory, the stability of the closed error system is guaranteed. The control scheme is robust to the uncertainties such as approximate error, ionic channel noise and external disturbances. Chaos synchronization is obtained by proper choice of the control parameters. The simulation results demonstrate the effectiveness of the proposed control method.

Keywords: Backstepping control, chaos synchronization, FitzHugh-Nagumo (FHN) model, RBF neural networks, time delay

INTRODUCTION

Chaos synchronization refers to dynamical synchrony of several chaotic systems through special coupling or by means of control. Over the last two decades, chaos synchronization has been widely studied in various fields (Pecora and Carroll, 1990; Lakshmanan and Murali, 1995; Pecora *et al*., 1997; Chen and Dong, 1998; Boccaletti *et al*., 2002; Wu, 2002). Experimental evidence demonstrates that synchronous neuronal oscillations underlie many cortical processes (Gray *et al*., 1989; Steriade *et al*., 1993; Roelfsema *et al*., 1997) and play a key role in the biological information processing (Meister *et al*., 1991; Harris-Warrick *et al*., 1992; Kreiter and Singer, 1996). Moreover, chaos synchronization involving millions of neurons seems essential for rapid communication in the brain (Freeman, 1991). Therefore, chaos synchronization in neural systems has attracted particular attention (Garfinkel *et al*., 1992; Schiff *et al*., 1994; Elson *et al*., 1998; La Rosa *et al*., 2000; Dhamala *et al*., 2004; Wang *et al*., 2004, 2006; Cornejo-Pérez and Femat, 2005; Deng *et al*., 2006; Zhang *et al*., 2007; Aguilar-López and Martínez-Guerra, 2008; Che *et al*., 2011). Without control, identical coupled neurons can eventually synchronize only when the coupling strength is above a certain critical value (Wang *et al*., 2004, 2006; Che *et al*., 2011) which may be beyond the physiological condition. In experiments, two coupled living neurons have been controlled to be synchronous by an external depolarizing DC current (Gray *et al*., 1989; Elson *et al*., 1998). Theoretically, many control methods, such as backstepping control (Deng *et al*., 2006), nonlinear control (Wang *et al*., 2006), adaptive control (Cornejo-Pérez and Femat, 2005), feedback control (Zhang *et al*., 2007) and sliding mode control (Aguilar-López and Martínez-Guerra, 2008) have been developed to achieve chaos synchronization of neuronal systems. In our recent studies, we have proposed adaptive sliding mode control (Che *et al*., 2011; Yu *et al*., 2012) for chaos synchronization of neuronal models, taking account into some uncertain factors in practice, such as unmodelled dynamics, ionic channel noises and external disturbances. However, none of the aforementioned works consider the uncertain communication time delays, which is ubiquitous in real neuronal system.

In this study, we propose an adaptive Neural Network (NN) based backsteppting controller for chaos synchronization of two coupled FitzHugh-Nagumo (FHN) neurons with uncertain time delays. The FHN model is a two-dimensional simplification of the widely

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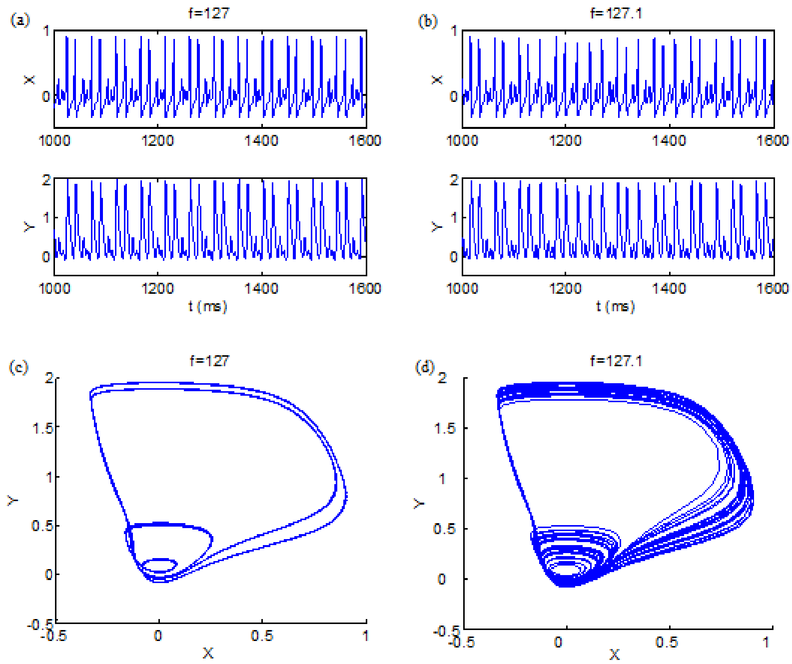

Fig. 1: Examples of periodic and chaotic responses to different stimulate frequencies. Time series of the system states *X*, *Y* (*a*, *b*) and corresponding phase portraits on *X*-*Y* plane (*c*, *d*) showing periodicity and chaos at frequencies $f = 127$ (*a*, *c*) and $f = 127.1$ (*b*, *d*), respectively

known Hodgkin-Huxley model (Hodgkin and Huxley, 1952) describing the signal transmission across axons in neurobiology. Under external sinusoidal electrical stimulation, the individual FHN model may exhibit various behaviors including chaos (Wang *et al*., 2004). In the framework of backstepping controller design, we first use a Radial Basis Function (RBF) NN to approximate the uncertain nonlinear function of the dynamical system, since the RBFNN has the ability to uniformly approximate continuous functions to arbitrary accuracy (Sanner and Slotine, 1992). Then a Lyapunov-Krasovskii function is designed to overcome the difficulties caused by unknown time delays. Further, we propose an adaptive algorithm to approximate the uncertainties and disturbances of the dynamical system, which relaxes the requirement for boundness of disturbances. According to the Lyapunov stability theorem, the proposed controller guarantees closed-loop stability for the synchronization error system. Thus the chaos synchronization of the coupled neural system is obtained. Compared with the single control schemes (Cornejo-Pérez and Femat, 2005; Deng *et al*., 2006; Wang *et al*., 2006; Zhang *et al*., 2007; Aguilar-López and Martínez-Guerra, 2008), the present hybrid control approach has the advantages of adaptive technique and robust control, which makes this approach attractive for a wide class of nonlinear systems with both uncertain nonlinearities and disturbances.

DYNAMICS OF FHN NEURONS

The model of a single FHN neuron is described as:

$$\begin{aligned} \dot{X} &= X(X-1)(1-rX) - Y + I \\ \dot{Y} &= bX \end{aligned} \qquad (1)$$

where X and Y are rescaled membrane voltage and recovery variable, respectively. I is the external electrical stimulation in form of $I = A/\omega \cos(\omega t)$ with A, $\omega = 2\pi f$ the amplitude and frequency, respectively. Throughout this study, we fix parameters b, r, A at values 1,10 and 0.1, respectively. This system exhibits a chaotic behavior for $124.5 < f < 131$ (Che *et al*., 2011). Several periodic windows embed in chaos. Figure 1 gives examples of time series of *X*, *Y* (*a*, *b*) and corresponding phase portraits on X-Y plane (c, d) showing periodicity and chaos at f = 127 (a, c) and f = 127.1 (b, d), respectively.

In neural systems, a gap junction is an electrical synapse that is a mechanical and electrically conductive link between two adjacent neurons. Through gap junctions, neurons can communicate with each other and the synaptic current is proportional to the difference of membrane potentials between a neuron and its neighbors.

Now we consider two FHN neurons coupled via gap junction with uncertain communication time delay as follows:

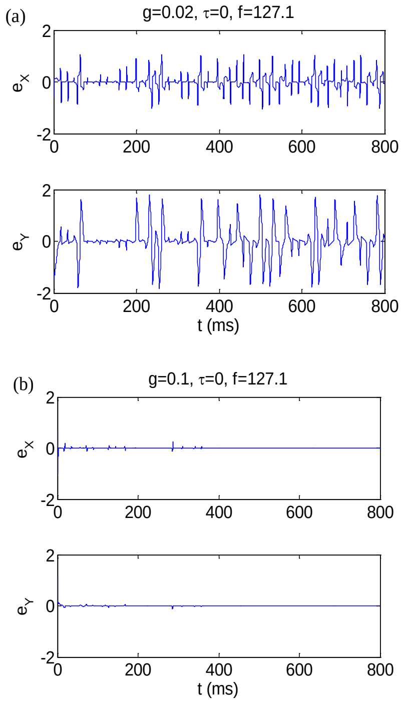

Fig. 2: Time series of states errors ($e_x = X_2 - X_1$, $e_Y = Y_2 - Y_1$) for different coupling strengths without time delay: (a) g = 0.02, the two neurons are not synchronous (b) g = 0.1, the two neurons are synchronous

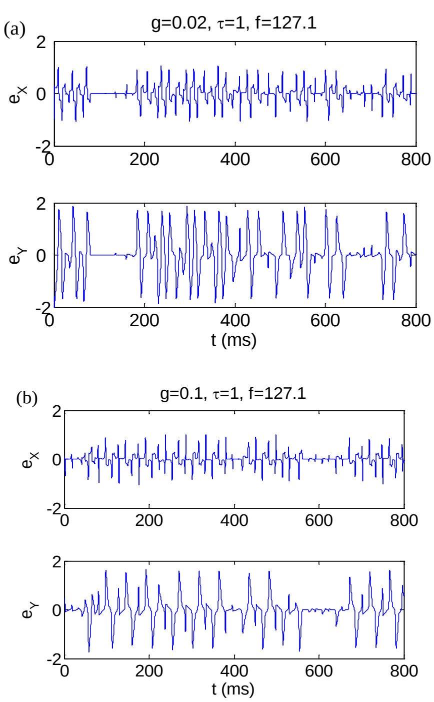

Fig. 3: Time series of states errors ($e_X = X_2 - X_1$, $e_Y = Y_2 - Y_1$) for different coupling strengths with time delay τ = 1 (a) g = 0.02 (b) g = 0.1. The two neurons are not synchronous for both cases

$$\begin{aligned} \dot{X}_i &= X_i(X_i-1)(1-rX_i)-Y_i+g(X_j(t-\tau)-X_i)+I \\ \dot{Y}_i &= bX_i \end{aligned} \qquad (2)$$

where i, j = 1, 2, ($i \# j$), $g \geq 0$ is the coupling strength of gap junction, $\tau \geq 0$ is the time delay in the interaction.

In our previous study (Che *et al*., 2011), we have investigated the case of the time delay τ = 0. If the individual neurons are chaotic (for example when f = 127.1), then the chaos synchronization occurs only when the coupling strength of gap junction satisfies certain condition. Figure 2 gives examples of time series of states errors under different coupling strength g = 0.02 and g = 0.1, respectively. The synchronization cannot occur when g = 0.02, but it occurs when g = 0.1. However, when the time delay τ>0, whether chaos synchronization occurs or not depends on the values of τ. As shown in Fig. 3, with the same time delay τ = 1, the coupled neurons cannot synchronize in both cases of g = 0.02 and g = 0.1, respectively.

CHAOS SYNCHRONIZATION VIA ADAPTIVE BACKSTEPPING CONTROL

For the sake of clarity and without lost of generality, the gap junction coupled FHN system with time delays under control can be expressed as:

$$\begin{aligned} \dot{X}_1 &= X_1(X_1-1)(1-rX_1)-Y_1+g(X_2(t-\tau)-X_1)+I+d_1 \\ \dot{Y}_1 &= bX_1 \\ \dot{X}_2 &= X_2(X_2-1)(1-rX_2)-Y_2+g(X_1(t-\tau)-X_2)+I+d_2+u \\ \dot{Y}_2 &= bX_2 \end{aligned} \qquad (3)$$

where, d_1 and d_2 are added to simulate disturbances or noises in ionic channels. u is the added control force (synchronization command) such that the dynamical behaviors of the two coupled FHN neurons are synchronous.

Let $e_x = X_2$-X_1, $e_Y = Y_2$-$Y_1$and $d = d_2$-d_1, the error dynamical system of the coupled neurons can be expressed as:

$$\begin{aligned} \dot{e}_X &= f(\mathbf{x})-e_Y-ge_X-ge_X(t-\tau)+d+u \\ \dot{e}_Y &= be_X \end{aligned} \qquad (4)$$

where,

$$f(\mathbf{x}) = X_2(X_2-1)(1-rX_2)-X_1(X_1-1)(1-rX_1)$$

and

$$x = [X_1, X_2]^T$$

The problem to realize the synchronization between two neurons is now transformed to a problem of how to choose a control law u (t) to make e_x and e_y generally converge to zero when time tends to infinity. Here back stepping design is used to achieve the goal.

To perform the backstepping design, the following change of coordinates is made: $z_1 = e_Y$ and $z_2 = e_x - \alpha$, where α is an intermediate control. The control law $u(t)$ is designed in the last step to stabilize the whole closed-loop system. Then Eq. (4) becomes to be:

$$\begin{aligned} \dot{z}_1 &= b(z_2 + \alpha) \\ \dot{z}_2 &= f(\mathbf{x}) - e_Y - ge_X - ge_X(t-\tau) - \dot{\alpha} + d + u \end{aligned} \tag{5}$$

Step1: We first consider the z_1-subsystem as:

$$\dot{z}_1 = b(z_2 + \alpha) \tag{6}$$

Choose the Lyapunov function candidate $V_1(t)$ as follows:

$$V_1 = \frac{1}{2} z_1^2 \tag{7}$$

The time derivative of $V_1(t)$ along (6) is:

$$\dot{V}_1 = bz_1(z_2 + \alpha) \tag{8}$$

The intermediate control $\alpha(t)$ is designed as:

$$\alpha = -kz_1 / b \tag{9}$$

then

$$\dot{V}_1 = -kz_1^2 + bz_1 z_2 \tag{10}$$

where k>0 is the gain constant and the coupling term bz_1z_2 will be handled in the next step, then we will have $\dot{V}_1 = -kz_1^2 \leq 0$.

Step 2: The dynamics of z_2-subsystem is given by:

$$\dot{z}_2 = f(\mathbf{x}) - e_Y - ge_X - ge_X(t-\tau) - \dot{\alpha} + d + u \tag{11}$$

Choose the Lyapunov function candidate $V_2(t)$ as follows:

$$V_2 = \frac{1}{2} z_2^2 \tag{12}$$

The time derivative of $V_2(t)$ along (11) is:

$$\begin{aligned} \dot{V}_2 &= z_2\left[f(\mathbf{x}) - e_Y - ge_X - ge_X(t-\tau) - \dot{\alpha} + d + u \right] \\ &\leq z_2\left[f(\mathbf{x}) - e_Y - ge_X - \dot{\alpha} + d + u \right] + g\left|z_2\right|\left|e_X(t-\tau)\right| \\ &\leq z_2\left[f(\mathbf{x}) - e_Y - ge_X - \dot{\alpha} + d + u \right] + \frac{1}{2}g^2 z_2^2 + \frac{1}{2}e_X^2(t-\tau) \end{aligned} \tag{13}$$

To overcome the design difficulties caused by the unknown time delay τ, we consider the Lyapunov-Krasovskii function $V_U(t)$ as:

$$V_U = \int_{t-\tau}^{t} \frac{1}{2} e_X^2(\tau) d\tau \tag{14}$$

The time derivative of $V_U(t)$ is:

$$\dot{V}_U = \frac{1}{2} e_X^2 - \frac{1}{2} e_X^2(t-\tau) \tag{15}$$

which can be used to cancel the time-delay term on the right-hand side of (13) and thus eliminate the design difficulty from the unknown time delay τ without introducing any uncertainties to the system. Accordingly, we obtain:

$$\dot{V}_2 + \dot{V}_U \leq z_2\left[f(\mathbf{x}) - e_Y - ge_X - \dot{\alpha} + d + u + \frac{1}{2}g^2 z_2 + \frac{1}{2z_2} e_X^2 \right] \tag{16}$$

Therefore, the design of control $u(t)$ is free from unknown time delay τ at present stage.
An ideal desired control can be designed as:

$$u^* = -kz_2 - Q(\mathbf{Z}) - d - \frac{1}{2z_2} e_X^2 - bz_1 \tag{17}$$

where the term $-bz_1$ is used to cancel the coupling term bz_1z_2 in step 1 and

$$Q(\mathbf{Z}) = f(\mathbf{x}) - e_Y - ge_X - \dot{\alpha} + \frac{1}{2} g^2 z_2 \tag{18}$$

with $Z = [X_1, X_2, z_1, z_2]^T$

However, since $f(x)$ is an unknown smooth function, it cannot be used to design the controller. As mentioned in Section 1, RBFNNs have good capability in nonlinear function approximation. According to approximation theory [31], we use a RBFNN $Q_{nn}(Z) = \theta^T \phi(Z)$ to approximate the unknown function $Q(Z)$ in form of:

$$Q(\mathbf{Z}) = \theta^{*T} \phi(\mathbf{Z}) + \varepsilon \tag{19}$$

where the weight vector $\theta \in R^m$ with m being the NN node number and the basis function $\phi(Z)$ chosen as the commonly used Gaussian functions with fixed centers and widths, θ^* is the ideal constant weight vector and ε is the approximation error.
Thus, the ideal control becomes:

$$u^* = -kz_2 - \theta^{*T} \phi(\mathbf{Z}) - \xi - \frac{1}{2z_2} e_X^2 - bz_1 \tag{20}$$

where, $\xi = \varepsilon + d$ is the lumped uncertainty.

Since the ideal weight θ^* is generally unknown, we use its estimate $\hat{\theta}$ instead and denote $\tilde{\theta} = \hat{\theta} - \theta^*$ as the weight estimate error vector. We choose the following adaptive law for online tuning the NN weights:

$$\dot{\hat{\theta}} = \gamma\phi(\mathbf{Z})z_2 \tag{21}$$

Moreover, since the lumped uncertainty ξ and its upper boundness are difficult to determine, we use $\hat{\xi}$ to estimate ξ and choose its adaptive law as:

$$\dot{\hat{\xi}} = \beta z_2 \tag{22}$$

where β is a positive constant.
To this end, we design the actual control as:

$$u = -kz_2 - \hat{\theta}^T\phi(\mathbf{Z}) - \xi - \frac{1}{2z_2}e_X^2 - bz_1 \tag{23}$$

Accordingly, we have:

$$\dot{V}_2 + \dot{V}_U \leq -kz_2^2 - \tilde{\theta}^T\phi(\mathbf{Z})z_2 - \tilde{\xi}z_2 - bz_1z_2 \tag{24}$$

Now, we consider the overall Lyapunov function candidate:

$$V = V_1 + V_2 + V_U + \frac{1}{2\gamma}\tilde{\theta}^T\tilde{\theta} + \frac{1}{2\beta}\tilde{\xi}^2 \tag{25}$$

where $\tilde{\xi} = \hat{\xi} - \xi$ is the estimate error of ζ.

Differentiating V with respect to time and noting (10), (21), (22) and (24), we have:

$$\begin{aligned} \dot{V} &= \dot{V}_1 + \dot{V}_2 + \dot{V}_U + \frac{1}{\gamma}\tilde{\theta}^T\dot{\hat{\theta}} + \frac{1}{\beta}\tilde{\xi}\dot{\hat{\xi}} \\ &\leq -k(z_1^2 + z_2^2) \\ &\leq 0 \end{aligned} \tag{26}$$

According to Lyapunov stability theorem, the overall system is stable at the origin, i.e., $z_1, z_2 \to 0$, as $t \to \infty$. Thus we have e_X, $e_Y \to 0$, as $t \to \infty$ and chaos synchronization of the two coupled neurons is obtained under control.

SIMULATION RESULTS

In this section, numerical simulations are carried out for chaos synchronization of the coupled FHN neuron systems via the proposed NN based backstepping control.

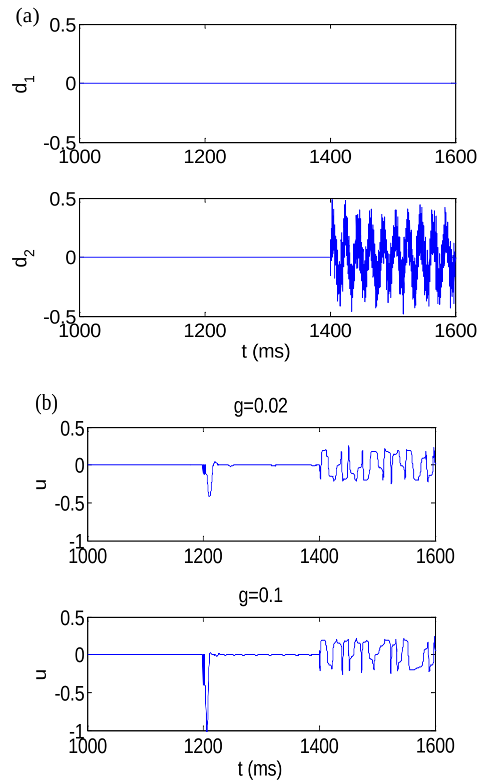

Fig. 4: Time courses of disturbances (d_1 and d_2) and the control input (u). The control input is on after t = 1200 ms, the disturbance is added after t = 1400 ms

We choose the design parameters in the simulations as: k = 1, $\gamma = \beta = 2$. According to Sanner and Slotine (1992), the centers and widths are chosen on a regular lattice in the respective compact sets in our simulations. Specifically, the RBFNN $\hat{\theta}^T\phi(Z)$ contains 7 nodes (i.e., m = 7) with centers c_i (i = 1, 2,…, m) evenly spaced in [-0.5, 1.5]×[-1, 2.5]×[-1,1]×[-1, 1] and widths b_i = 0.5 (i = 1, 2,…, m).

We illustrate chaos synchronization of two coupled FHN neurons when f = 127.1 and g = 0.02 or g = 0.1 with the same time delay τ = 1. We switch on the controller at time t = 1200ms. To evaluate the robustness of the control scheme, we add a disturbance d_2 to the controlled system at time t = 1400ms, i.e., $d_1 = d_2 = 0$ when t< 1200ms, and $d_1 = 0$, d_2 = dnoise +0.2 sin (100πt) when t≥1200ms, where dnoise is Gaussian random noise with mean zero and variance 0.04. Figure 4 shows the time series of the disturbances d_1 and d_2 and the control input u. The corresponding responses of the system are given in Fig. 5. As shown in Fig. 5, before the control is implemented, the two neurons exhibit their own chaotic dynamical behaviors and are not synchronous. After the controller is applied, the errors converge to zero rapidly and the nearly complete synchronization is obtained.

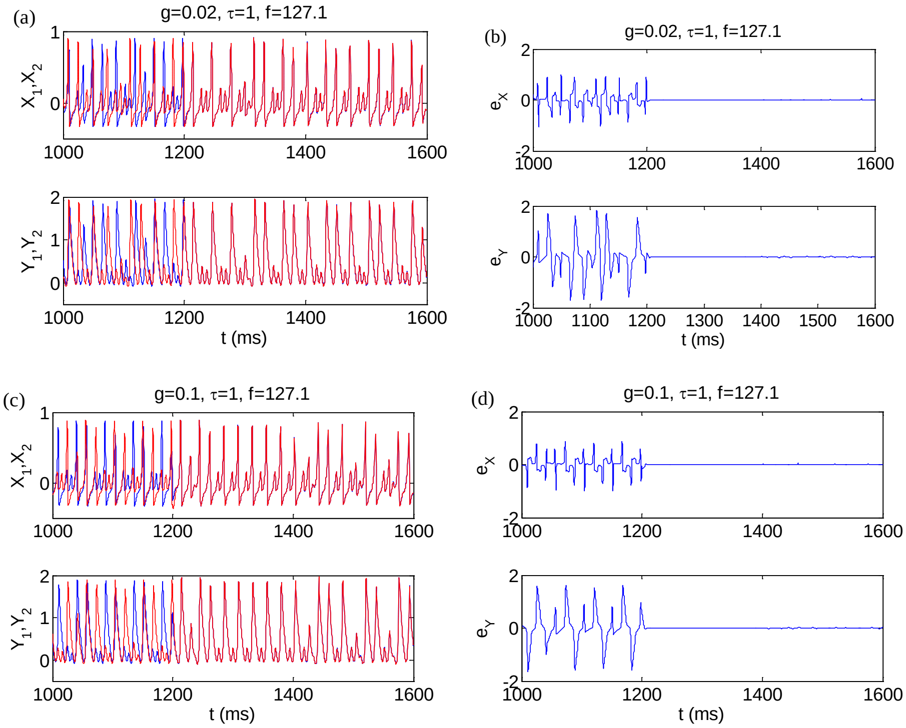

Fig. 5: System responses before and after control with time delay $\tau = 1$ and different coupling strengths (a, b) $g = 0.02$ and (c, d) $g = 0.1$. Time series of system states (a, c) and the corresponding synchronization errors (b, d). Before control, the two neurons are not synchronous. After control, the two neurons become synchronous

The adaptively changed control command suppresses the random disturbance efficiently and the added disturbance has almost no effect on the stable synchronization of the coupled system.

CONCLUSION

In this study, chaos synchronization of two gap junction coupled FHN neurons with uncertain time delays via the adaptive NN based backstepping control has been investigated. The designed controller consists of a simple RBFNN which is tuned on-line. According to the Lyapunov stability theory, the stability analysis has shown that the controller guarantees the stability of both synchronization errors and the NN weights. The synchronization errors can be kept as small as possible by proper choice of control parameters, which means chaos synchronization of the coupled neurons. The control scheme is robust to the approximate errors, ionic channel noise and disturbance. The simulation results have demonstrated the validity of the proposed control method.

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