

## Plastic Damage Model to Evaluate the Fracture Size of Semi-Rigid Base Pavement

<sup>1</sup>Cao Peng, <sup>1</sup>Feng Decheng, <sup>1</sup>Jing Ruxin and <sup>2</sup>Zhao Yin

<sup>1</sup>School of Transportation Science and Engineering, Harbin Institute of Technology,  
Harbin, 150090, China

<sup>2</sup>CCCC Second Highway Consultants Co. Ltd. Wuhan, 430056, China

**Abstract:** A simple supported beam model has been presented to simulate the response of semi-rigid pavement structure, which are consistent of the upper layer, middle layer, bottom layer, base and sub base course, during the cycle vehicle loading. This mechanics model coupled with plastic-damage mechanics model could simulate the limit broken of the pavement structure in condition that soil base layer losing bearing capacity gradually. In the meanwhile, numerical calculations based on preceding mechanics model, using the FEM software ABAQUS, have been used to define the broken size of beam. The results indicated that: when the size of simple supported beam expanded to 10 m, brittle damage could happen immediately, Just the standard vehicle loading (about 0.7 Mpa) has been implement once. Objective of this study is to provide a physical and rather concrete explanation for the style and concept of the semi-rigid pavement brittle broken.

**Keywords:** Elastic-plastic damage, numerical simulation, semi-rigid base, soil failure, three-point bending beam

### INTRODUCTION

Most highways are made of semi-rigid base structure in our country, in which the semi-rigid pavement structure as a bearing course. Grassroots cracking affect the service life of the road directly and the semi-rigid base crack research has been a hot issue of road.

In overseas, Monismith determined the semi-rigid pavement fatigue life prediction equation through a large number of experiments (Monismith and Coetzee, 1982) and it is worth noting that the semi-rigid pavement fatigue life and experiment the load applied has a very close relationship (Pu, 2007); the Dutch scholar Molenaar used SHRP-NL data obtained the cement stabilized road base material fatigue life equation (Molenaar, 2007) and it also emphasizes the fatigue life of pavement is subjected to a load has a small effect when the stress level less than a certain value. The domestic scholars have also conducted a systematic study of cement stabilized road base material fatigue life, Wang Hong-chang from Southeast University based on finite element and neural network in the two phase of the study method predict the service life of semi-rigid pavement (Wang, 2005).

The above research method for pavement crack propagation fatigue equation of actual establishment, predicted the service life of pavement, or that the base course cracks already exist and have certain distance and then calculate the fatigue cracks in the surface layer

of extended to predict the service life, but little research on the way to produce base crack. One of the important reasons is that cement stabilized base belongs to quasi-brittle material and the stress immediately fall, then cracks when stress exceeds the ultimate strength, so it is difficult to explain the problem by fatigue method. In this study, it described the reason of unrecoverable deformation when soil in the wheel base under load below base (or sub-base) from the overall structure of semi-rigid pavement research and predict the reason why base structure cracks may be sub grade permanent deformation resulting in the middle of the vacant, then in this case use a “simply supported beam” mechanical model to describe it. Based on the above analysis, the base (or sub-base) failure mode can be explained by following method: when sub grade under cyclic loading in the vehicle unrecoverable deformation increase and further expand, making the “simply supported beam” length increase, when the “simply supported beam” reaches a certain size, even if the base under the standard load, it may cracks.

### METHODOLOGY

**The relationship between sub grade failure and semi-rigid base crack:** Chinese road Standard based on the elastic multi-layer theory and according to the following assumptions for system of displacement and stress (Guo and Feng, 2001).

Basic assumptions are as follows:

- Each layer is continuous, complete elastic, homogeneous, isotropic and the small deformation.
- The last layer is infinite in horizontal direction and vertical direction and each layer which on it is infinite in horizontal direction and finite in depth.
- The stress, strain and displacement of each layer in horizontal direction and the last layer in infinite depth are zero.
- The displacement between layers is completely continuous, or the interlayer only vertical stress and displacement continuity without friction.
- Excluding the weight.

The sub grade is regarded as a linear elastic material, but the actual situation of soil mass in general does not have this property and soil mechanics properties is complex, often depends on the soil type, stress history and other factors. According to (Eisenberg and Phillip, 1971) put forward the concept that surface of soil yield and loading under the load, for the soil without stress history in the first load, the initial yield surface is zero, while the yield surface continues to increase in follow loading until the yield surface reaches the failure surface. For sub grade structure, this kind of viewpoint has certain applicability. For semi-rigid pavement, sub base and sub grade contact, both deformations in the initial loading. Sub base isn't fracture when sub base deformation in the elastic stage, after unloading, the part above base can be restored to its initial state, but sub grade in unrecoverable deformation because its initial yield surface is very small. This unrecoverable deformation will increase and unrecoverable sub grade is also to be expanded in the future loading. This incongruity deformation of base and sub grade will lead to the soil which under sub base hangs and expand to a certain range. When failure size reaches a certain level, the part above base will become a "simply supported beam". At this time, if loading on the middle of the "simply supported beam" again, the tension stress of sub base will exceed the material ultimate strength, resulting in base occurrence of quasi brittle failure crack.

**The simply supported beam model of semi-rigid base crack:** In order to explain soil under base in which case, the disposable wheel load can be lead to sub base in ultimate fracture behavior; this study presents a plane simply supported beam theory. When it in ultimate failure, the sub base and base below semi-rigid base has been separated, the semi-rigid base and the parts above it form a simply supported beam, as shown in Fig. 1.

From material mechanics analysis, the tensile stress of simply supported beam middle bottom is largest

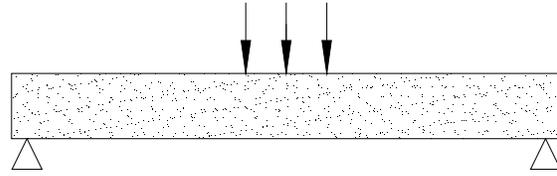


Fig. 1: Three-point bending beam model

when its middle under load. Combined with the actual road conditions, simply supported beam is easy to tensile failure when loading on the middle of it. Based on the above assumptions, this study analysis on mechanical state of sub base in different lengths of simple supported beam (5, 7.5 and 10 m, respectively) by the middle of the wheel load, to judge the range of hanging below simple supported beam that sub base will in quasi-brittle fracture failure.

**Cement stabilized sub base damage simulation:**

Since Lemaitre-Chaboche established plastic damage theory under continuum mechanics based on experiment, plastic damage coupling mechanics model is an important tool to study material damage and fracture in solid mechanics (Jean and Rodrigue, 2005). As the mathematical form is simple in elastic-plastic damage coupling mechanics model of solid mechanics and it can reflect the weakening behavior of material, it is widely used in cement concrete materials for quasi brittle failure (Bazant and Cedolin, 1991; Cao, 2011; Feng *et al.*, 2011).

In this study, it is used elastic-plastic damage model to simulate the quasi-brittle fracture for simply supported beam in the wheel load when soil hanging by the international finite element software ABAQUS. There is a detailed description of cement concrete elastic-plastic damage model in document (Zhuang and Zhang, 2005). The following introduce the constitutive equation of quasi-brittle fracture and the selection of the model parameters which used in the study briefly.

According to the assumption of small deformation, strain can be decomposed into the following formula (Simo and Hughes, 2000):

$$\varepsilon_{ij} = \varepsilon_{ij}^e + \varepsilon_{ij}^p \tag{1}$$

$\varepsilon_{ij}$ ,  $\varepsilon_{ij}^e$ ,  $\varepsilon_{ij}^p$  is respectively strain tensor, elastic strain tensor, plastic strain tensor. According to the assumption of strain equivalence, the stress update formula is as follows:

$$\sigma_{ij} = E_{ijkl}^0 (1 - D) (\varepsilon_{kl} - \varepsilon_{kl}^p) \tag{2}$$

$\sigma_{ij}$  is cauchy stress,  $E_{ijkl}^0$  is elastic tensor of material with no damage,  $D$  is damage variable. The above two formulas at a rate as follows:

$$\dot{\epsilon}_{ij} = \dot{\epsilon}_{ij}^e + \dot{\epsilon}_{ij}^p \quad (3)$$

$$\dot{\sigma}_{ij} = E_{ijkl}^0 (1 - D) (\dot{\epsilon}_{kl} - \dot{\epsilon}_{kl}^p) \quad (4)$$

The strain rate in Eq. (4) can be expressed by Jaumann objective rate as follows:

$$\sigma^{\nabla J} = \dot{\sigma}_{ij} - W \sigma_{ij} - \sigma_{ij} W^T \quad (4)$$

In Eq. (5)  $W$  is rotation tensor,  $\sigma^{\nabla J}$  is Cauchy stress of Jaumann objective rate, the above formula can guarantee the presence of large deformation and the objectivity of constitutive integration (Belytschko *et al.*, 2001). According to flow rule the plastic strain increment as follows:

$$\dot{\epsilon}_{ij}^p = \dot{\lambda} \frac{\partial G}{\partial \sigma_{ij}} \quad (5)$$

$\dot{\epsilon}_{ij}^p$  is plastic strain rate tensor,  $\dot{\lambda}$  is plastic damage multiplier.

Concrete damage plasticity model use a non-associated plastic flow rule in ABAQUS program, the form of plastic potential function  $G$  is Drucker-Prage hyperbolic function:

$$G = \sqrt{(\tau \sigma_{i0} \tan \varphi)^2 + \bar{q}^2} - \bar{p} \tan \varphi \quad (6)$$

In this formula:  $\varphi$  is dilatancy angle of  $\bar{p} - \bar{q}$  plane which is 36 in this study;  $\sigma_{i0}$  is uniaxial tensile fracture stress;  $\tau$  is parameter of offset characterization which is 0.1 in this study.  $\bar{p}$  is mean stress of effective stress space,  $\bar{q}$  is the second stress invariant of effective stress space.

The Lubliner yield function (Lee and Fenves, 1998; JTG D50-2006, 2006)  $F$  of effective stress space in ABAQUS program is:

$$F = \frac{1}{1 - \alpha} (\bar{q} - 3\alpha\bar{p} + \beta(\bar{\epsilon}^{pl}) < \hat{\sigma}_{\max} > - \gamma < -\hat{\sigma}_{\max} >) - \bar{\sigma}_c(\bar{\epsilon}_c^{pl}) = 0 \quad (7)$$

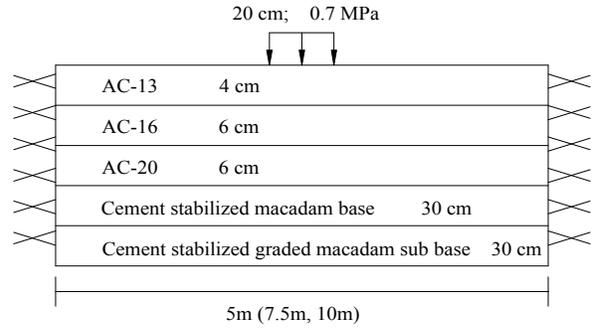


Fig. 2: Pavement model

In this formula:

$$\alpha = \frac{(\sigma_{b0}/\sigma_{c0}) - 1}{2(\sigma_{b0}/\sigma_{c0}) - 1}, 0 \leq \alpha \leq 0.5$$

$$\beta = \frac{\bar{\sigma}_c(\bar{\epsilon}_c^{pl})}{\bar{\sigma}_t(\bar{\epsilon}_t^{pl})} (1 - \alpha) - (1 + \alpha), \gamma = \frac{3(1 - k_c)}{2k_c - 1}$$

In this formula,  $\hat{\sigma}_{\max}$  is the maximum principal stress component of effective stress;  $\sigma_{b0}/\sigma_{c0}$  is the ratio of the initial limit of two-way compression and initial limit of one-way compression which is 1.16 in this study;  $k_c$  is the ratio of  $q_{(TM)}$  (the second invariant on yield surface of concrete spatial radial tensile stress) and  $q_{(CM)}$  (the second stress invariant on compression meridian) under hydrostatic pressure  $P$  when principal stress component which is 0.667 in this study.

**Engineering analysis:** In order to make sure the range of soil hanging below base (sub base) under standard load that occur quasi-brittle fracture, this study use elastic-plastic damage coupling mechanics model to analysis the simply supported beam model with different sizes. Combination of a highway structure in Heilongjiang Province, the simply supported beam model is made up by 5 structures; they are AC16 Upper Layer (4 cm), AC16 Mid-surface (6 cm), AC 20 Binder Course (6 cm), cement stabilized macadam base (30 cm), cement stabilized graded macadam sub base (30 cm), shown in Fig. 2. The sub base material is defined by elastic-plastic damage model and other materials use linear elastic model to simulate. The materials' parameters of each layer are shown in Table 1, 2 and 3.

In this study, it use simply supported beam model to simulate the state of parts above sub base when soil failure, the finite element model is a plane strain model

Table 1: Parameters of material

Pavement structure	Modulus of elasticity /(MPa)	Poisson's ratio
Upper layer	870	0.25
Mid-surface	910	0.25
Binder course	1031	0.25
Cement stabilized macadam base	1200	0.20
Cement stabilized graded macadam sub base	300	0.30

Table 2: Tension stiffening of cement stabilized crush stone

Yield stress/ (Mpa)	0.5	0.0001
Fracture strain/ ( $\mu\epsilon$ )	0.0	0.0160

Table 3: Tension damage of cement stabilized crush stone

Damage value	0.0	0.900
Fracture strain/ ( $\mu\epsilon$ )	0.0	0.016

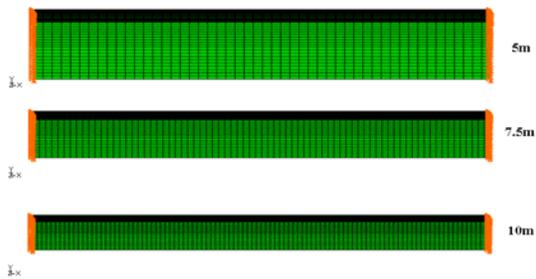


Fig. 3: FEM models of three conditions

and limit the lateral and vertical displacement at both ends. It applies vertical stress 0.7 MPa in the middle of simply supported beam and the range of 20 cm. There are three groups of simply supported beam finite element model; they are 5, 7.5, 10 m, respectively. They use 2500, 3750, 5000 four-node quadratic plane strain reduced integration elements (Fig. 3), calculated on the server with an 8 CPU (Intel (R), Xeon (R), 2.13 GHz) and 16 GB memory.

### ANALYSIS OF FINITE ELEMENT RESULTS

The simply supported beams with three different lengths are simulated; Fig. 4 is Von-mises equivalent stress contours of three models after analysis. In order to facilitate comparison, the deformation of three models magnified 10 times. From the figure it can be found that the Von-mises equivalent maximum stress is about 1 MPa when the length of simply supported beams is 5 m and locate in the boundary which means the simply supported beam mid-span stress is not concentrated. But the Von-mises equivalent maximum stress is 1.4 MPa when the length of simply supported beams is 7.5 m and locates in the middle. The Von-mises equivalent maximum stress has reached 3 MPa when the length of simply supported beams is 7.5 m and appears in the top

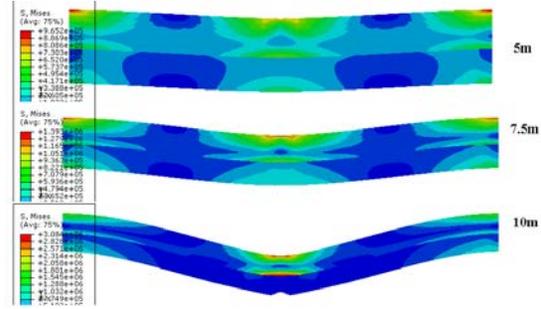


Fig. 4: Distribution of von-mises stress of three conditions

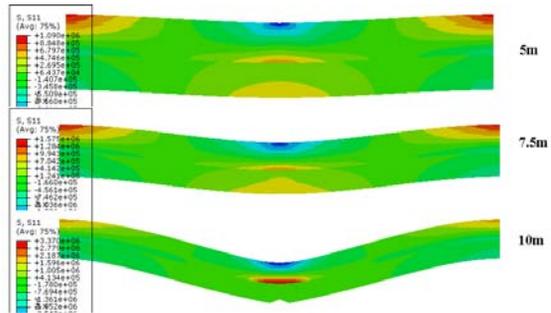


Fig. 5: Distribution of stress  $\sigma_{11}$  in horizontal direction of three conditions

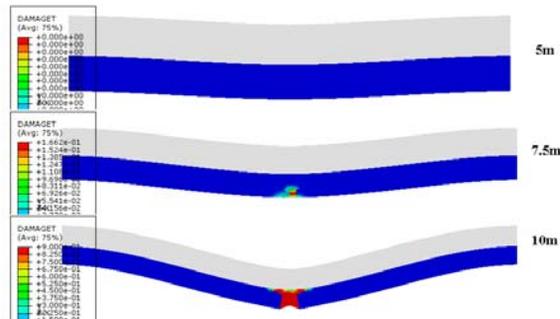


Fig. 6: Distribution of damage of three conditions

of base and the bottom of the sub base, this value has exceeded the sub base material ultimate strength.

As the design specifications using bottom tensile stress as pavement design parameters, there are horizontal normal stress contours of three models in this study (Fig. 5) and the maximum tensile stress appears in the bottom of the base, which is same as described in semi-rigid asphalt pavement design specification (JTG D50-2006, 2006).

Figure 6 is final damage distribution nephogram of three models under load. The sub base didn't appear to damage when the size is 5 m and when the fatigue loads continue to function, soil failure parts gradually increased. When the size of the model is extended to 7.5

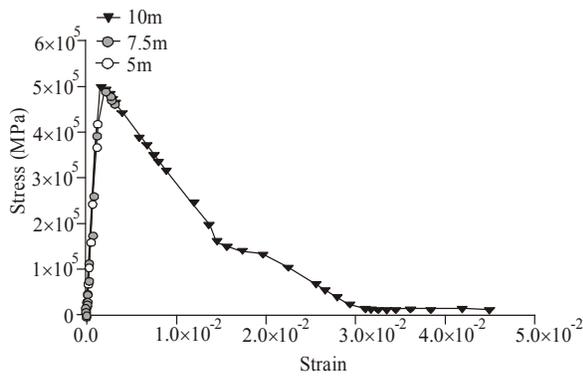


Fig. 7: Stress-strain curve in the mid-section of sub base in three conditions

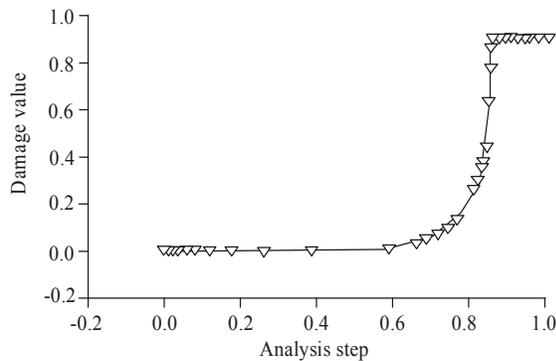


Fig. 8: Damage evolution in the mid-section of sub base in the third condition

m, the bottom of sub base appeared damage region, but it's very small about 0.16 and not through the whole base which means no macroscopic crack is appeared. When the range of soil failure under sun base reaches 10 m, there appears a damaged zone through sub base under standard loads (the damage value is 0.9), which means sub base fracture and appear macroscopic crack. Figure 7 is the stress-strain comparison chart of the midpoint of three models bottom. From the Figure it can be see the sub base material in elastic stage when the size is 5 m. When the size is 7.5 m, the sub base material stress over the peak stress and there is a certain decline, but did not produce stress drop. When the size is 5 m, the sub base material stress produce stress drop and reduce to a small value, which is an important symbol of fracture. Figure 8 is the Damage evolution in the mid-section of sub base when the size is 10 m, the damage value reach 0.9 after analysis.

### CONCLUSION

In this study, it based on plastic damage mechanics theory and to simulate semi-rigid quasi-brittle fracture through the simply supported beam model and obtained the following conclusions:

- This study presents a limit fracture model to explain the failure behavior of semi-rigid base and sub base, which provides a new reference to study semi-rigid pavement failure mechanism.
- This study simplifies soil failure process under fatigue load to gradual growth in the vacant position of the parts above sub base. This Simplified way can transform the fatigue load into soil failure process.
- It simplifies parts above soil failure to a simply supported beam and uses the elastic-plastic damage model to simulate the limit damage of parts above sub base under standard load. And the sub base will appear quasi-brittle fracture when the length of simply supported beam is about 10 m.
- Compared with the previous method to calculate the crack tip stress intensity factor, using elastic-plastic damage model to analyze the cement stabilized sub base material limit vandalism is easy and solute efficiency.

### ACKNOWLEDGMENT

The work described in this study was supported by research project funded by the China Ministry of Communications (Project No. 19672016).

### REFERENCES

Bazant, Z.P. and L. Cedolin, 1991. Stability of structures: Elastic, Inelastic, Fracture and Damage Theories, Oxford University Press, Oxford, New York, pp: 124-165.

Belytschko, T., W.K. Liu and B. Moran, 2001. Nonlinear Finite Element for Continua and Structure. John Wiley and Sons Ltd., New York, pp: 78-112.

Cao, P., 2011. Plastic damage model of concrete and development of Abaqus UMAT. MA Thesis, of Shenyang University of Technology, Shenyang, pp: 45-63.

Eisenberg, M.A. and S.A, Phillip, 1971. A theory of plasticity with non-coincident yield and loading surfaces. Acta Mech., 11: 247-260.

Feng, D.C., L. Tian and P. Cao, 2011. Study of longitudinal cracking during settlement of soil based on extended finite element method. Eng. Mech., 28(5): 149-154.

Guo, D.Z. and D.C. Feng, 2001. Mechanics of Elastic Layer System. Harbin Institute of Technology Press, China, pp: 70-93.

Jean, L. and D. Rodrigue, 2005. Engineering Damage Mechanics. Springer, Berlin Heidelberg, New York, pp: 115-143.

- JTG D50-2006, 2006. Specifications for Design of Highway Asphalt Pavement. China Communications Press, China, pp: 43-58.
- Lee, J. and G.L. Fenves, 1998. Plastic-damage model for cyclic loading of concrete structures. *ASCE J. Eng. Mech.*, 124(3): 892-900.
- Molenaar, A.A.A., 2007. Prediction of Fatigue Cracking in Asphalt Pavements: Do We Follow the Right Approach. Paper Presented at the 2007 Annual TRB Meeting, Washington DC, pp: 1206-1267.
- Monismith, C.L. and N.F. Coetzee, 1982. Reflecting cracking: Analyses, laboratory studies and design considerations. *Proceedings of the Association of Asphalt Paving Technologist*, 49: 268-313.
- Pu, B., 2007. Analysis of the Performance of Pavements with a Cement Treated Base. Delft University of Technology, Delft, pp: 25-41.
- Simo, J.C. and T.J.R. Hughes, 2000. *Computational Inelasticity*. Springer, Berlin Heidelberg, pp: 63-87.
- Wang, H.C., 2005. Study on Two-Stage Design of Semi-Rigid Base Asphalt Pavement. Southeast University, Nanjing, 14: 74-85.
- Zhuang, Z. and F. Zhang, 2005. Analysis and Examples of Non-Linear Finite Element Software ABAQUS. Science Press, Duluth, pp: 95-142.