Research Journal of Applied Sciences, Engineering and Technology 5(1): 79-87, 2013
ISSN: 2040-7459; e-ISSN: 2040-7467

Submitted: February 18, 2012 Accepted: April 30, 2012 Published: January 01, 2013

Designing a Robust Power System Stabilizer via Probabilistic Analysis of System Eigenvalues

[1]Rouhollah Falahati-Naghibi, [2]Younes Ghasemian and [3]Reza Ahmadi Kordkheili
[1]Lecturer at Payam Noor University, Babol Branch, Babol, Iran
[2]Sama technical and vocational training college, Islamic Azad University, Tonekabon Branch, Tonekabon, Iran
[3]Department of Electrical and Computer Engineering, Babol Noshirvani University of Technology, Babol, Iran

Abstract: Power system stability has always been a serious issue for system operators. Power System Stabilizer (PSS) is a common equipment to improve stability conditions in generators. PSS improve system stability by shifting eigenvalues of system. A common method in traditional PSS is to presume grid parameters, as well as load characteristics as constant values. The fact is, however, in real operating conditions, system and load parameters are not constant. Therefore, different variables and uncertainties such as load value, network design and changes in controller parameters have to be considered. Proper statistic distribution is a common method for modeling uncertainties of parameters. This study focuses on designing PSS via analysis of probabilistic eigenvalues. Then, to prove design authenticity, the method is applied on a test 8-machie system.

Keywords: Eigenvalues, normal dispatch function, probabilistic analysis, robust stabilizer

INTRODUCTION

The common eigenvalue analysis is based on stable operation and constant parameters of system (Kundur, 1994; Kundur *et al*., 1989; Lin *et al*., 1996; Mori *et al*., 1993; Tse and Tso, 1998; Wang and Semlyen, 1990; Yu, 1983). However, operating the power system is based on continuous load dispatch and random variable factors. This implicates that more uncertainty sources and more variables have to be taken into account in system calculation and system planning. This study is based on analysis of probabilistic eigenvalues and it is assumed that variables have a unique distribution.

Probabilistic theory was first introduced in 1978 as a tool to study dynamic stability of power system (Burchett and Heydt, 1978a,b). Except widespread studies within 1979 to 1982, which led to different papers, there is not much work in this field of research.

In early probabilistic theory, which was suggested for analyzing system dynamic behavior, a probabilistic dispatch was suggested for the real part of eigenvalues. The suggested dispatch was based on probabilistic nature of random parameters of system. Random variables may have either a unique distribution or any other kind of dispatch (Burchett and Heydt, 1978a,b). Then, using normal dispatch of sensitive eigenvalues, stability probability of system is calculated. This method is widely used in different studies for calculating random load dispatch (Tse and Tso, 1993). Measurement inaccuracy, inaccuracy due to estimation of controller parameters and inaccuracy of load predictions are uncertainties which are consideredin this study (Burchett and Heydt, 1978a,b; Sauer and Heydt, 1978). In probabilistic dynamic studies, presented in (Burchett and Heydt, 1978a,b), uncertainties in parameters of generator, as well asuncertainties in controller parameters are considered. Considering the complexity in calculation of eigenvalues for parameter dispatch, it seems that rotor angle and damping factor are the only parameters which have been regarded for two-machine system presented in (Burchett and Heydt, 1978a,b). Parameters such as the effect of load uncertainty on the current of generator seem to be ignored for simplicity.

In (Loparo and Blankenship, 1979) second order statistic indices have been used to evaluate dynamic stability of system. In (Brucoli *et al*., 1981) “dynamics stability limits” curves were used to investigate the stability of a grid-connected machine. In 1988, a comprehensive study on different operating conditions was performed using “Gram series” (Maslennikov and Ustinow, 1997). Dynamic model of induction motors was introduced in 1960, in which intelligent methods are used

Corresponding Author: Rouhollah Falahati-Naghibi, Lecturer at Payam Noor University, Babol Branch, Babol, Iran. Tel.: 0098-9112129151

to determine system equivalent matrix, Aeq, via eigenvalues and eigenvectors (Gibbard, 1988).

Distortion sources in a multi-machine system can be listed as:

- Changes in system operating parameters
- Uncertainty of controller parameters
- Changes in structure and parameters of system and output of generator

With respect to statistic inherent of distortion sources, the first source can be modeled as continuous random variables with respect to existing data and prediction inaccuracy. The second distortion can also be modeled via continuous random variables, which these variables express measurement inaccuracy and inaccuracy in estimation of controller parameters. The best way to explain the third distortion sources is to model it with discrete random variables.

This study only focuses on the effects of uncertainties caused by the first source.

PROBABILISTIC EQUATIONS FOR RANDOM VARIABLES

Numerical characteristic of random variables: In most practical issues, random variables can be modeled with mean value and variance of values.

Exponent (Average value): If the discrete random variable, X, consists of x_1, x_2,…. and the probability for each variable is defined as $P(X = x) = p_i$, then exponent of X, shown by E[X] or $\overline{X}$, would be:

$$E[X] = \sum_{i=1}^{\infty} x_i p_i \tag{1}$$

If X is a continuous random variable, with f(x) as its dispatch function, then exponent of X would be:

$$E[X] = \int x f(x) dx \tag{2}$$

Variance: Variance of a discrete random variable can be obtained as:

$$\sigma^2 = E\left[(x - \overline{X})^2\right] = \sum_{i=1}^{\infty} \left(x_i - \overline{X}\right)^2 p_i \tag{3}$$

in which σ is deviation. Variance represents the distribution of probabilistic values around the average value.

Also, for continuous random variable, X:

$$\sigma^2 = E\left[(x - \overline{X})^2\right] = \int_{-\infty}^{\infty} (x - \overline{X})^2 f(x) dx \tag{4}$$

Covariance: Covariance for X and Y, as two discrete random variables, can be expressed as:

$$C_{XY} = E\left[(x - \overline{X})(y - \overline{Y})\right] \tag{5}$$

in which $\overline{Y}$ is the exponent of Y.

Positive covariance is an index stating that variations of X and Y are in the same direction.

Usually, when X is big in value, Y would also be big and vice versa. Therefore, $(x - \overline{X})(y - \overline{Y})$ is usually positive. Negative value for covariance represents dissimilarity in changes of X and Y.

Normal dispatch: If probabilistic dispatch for continuous variable, X, is defined as:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\overline{X})^2/2\sigma^2} \quad -\infty < x < \infty \tag{6}$$

Then X obeys its normal dispatch. Determining normal dispatch is based on exponent and variance. The value of $\overline{X}$ depicts the function position in align with horizontal axis. It also determines the deviation of curve, σ. Dispatch function for a normal dispatch is presented by:

$$P(X \le x) = \frac{1}{2\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-(u-\overline{X})^2/2\sigma^2} du \tag{7}$$

Considering $\overline{X} = 0$ and σ = 1, shown by N (0, 1), standard normal dispatch will be obtained. For $N(\overline{X}, \sigma^2)$, we can write:

$$\tilde{x} = (X - \tilde{X})/\sigma \tag{8}$$

In which $\tilde{X}$ is achieved via N (0, 1), as:

$$f(X) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2} \tag{9}$$

And dispatch function would be:

$$P(X \le x) = \frac{1}{2\sqrt{\pi}} \int_{-\infty}^{x} e^{-u^2/2} du \tag{10}$$

Random vector: The vector X, consisting of n random variables ($X = [X_1, X_2, \ldots, X_m]^T$), is called an n-variable random vector. Assuming (X, Y) as a two-variable random vector and *x* and *y* as real numbers, combined dispatch function can be presented by:

$$P(X \le x, Y \le y) \tag{11}$$

If X and Y are continuous random variables, probabilistic dispatch function can be presented by:

$$f(x,y) = \frac{\partial^2 P(X \le x, Y \le y)}{\partial x \partial y} \quad (12)$$

So:

$$P(X \le x, Y \le y) = \int_{-\infty}^{y}\int_{-\infty}^{x} f(u,v)\,du\,dv \quad (13)$$

Correlation and independency: Correlation and independency affect the behavior of random variables significantly. Correlation factor for X and Y, ρ, can be considered as standard covariance of X and Y:

$$\rho = \frac{C_{XY}}{\sigma_X \sigma_Y} \quad (14)$$

Therefore, it can be mentioned that:

- Covariance is dependent to measurement scale, while correlation factor is independent from measurement.
- Correlation factor is similar in sign with covariance; thus, it changes between -1 and 1.
- The values 1 and -1 for correlation factor represent a linear relation of the two variables.
- If $\rho = 0$, variables X and Y have no correlation.

X and Y will be independent, if and only if:

$$P(X \le x, Y \le y) = P(X \le x)P(Y \le y) \quad (15)$$

$$f(x, y) = f_x(x)f_y(y) \quad (16)$$

In which $f_x(x)$ and $f_y(y)$ are probabilistic density functions for X and Y. It is clear that the data of each variable does not affect the probability of other variable. It can be concluded that, if X and Y are independent random variables, they will be non-correlated.

Behavior of random variables: Considering X, Y and Z as random variables and a and b as real numbers, the following equations can be written based on exponent and covariance. These can also be used for random vectors.
If Z = Ax + b, then:

$$\overline{Z} = a\overline{X} + b \quad (17)$$

$$C_Z = a^2 C_X \quad (18)$$

Considering a = 1 and b = $-\overline{X}$ in Eq. (18) reveals that $\Delta X (= X - \overline{X})$ has the same variance as X does, which means:

$$C_{\Delta X} = C_X \quad (19)$$

If Z=X+Y, then:

$$\overline{Z} = \overline{X} + \overline{Y} \quad (20)$$

$$C_Z = C_X + C_Y + 2C_{XY} \quad (21)$$

For Z=XY:

$$\overline{Z} = \overline{XY} + C_{XY} \quad (22)$$

If X and Z are n-variable vectors, which $X = [X_1, X_2, \dots, X_n]^T$ and $Z= [Z_1, Z_2, \dots, Z_n]^T$ and if Z can be explained as a nonlinear function of X, as:

$$Zk= F(X) \text{ Or } Z_k = F_k(X) \text{ for } k = 1, 2, \dots, n \quad (23)$$

Then, exponent and covariance for Z can be achieved via Tailor series, as:

$$\begin{cases} \overline{Z}_k = F_k(\overline{X}) + \frac{1}{2}\sum_{i,j}\left(\frac{\partial^2 Z_k}{\partial X_i \partial X_j} C_{X_{ij}}\right) \\ C_Z = D C_X D^T \end{cases} \quad (24)$$

D represents first-order derivative for matrix C_x and $C_{X_{ij}}$ stands for element (i, j) in matrix C_x. Equation (24) states that exponent of Z can be corrected by covariance.

PROBABILISTIC LOAD DISPATCH

Changes in power of generators, changes of load power and distortions in voltages of PV buses are distortion factors in probabilistic eigenvalues analysis. Primary operating condition of generators and primary voltage of buses are determined by probabilistic load dispatch. In a normal dispatch, the necessary parameters are exponent and covariance.

In the N-bus system, the injected power vector, S, for voltage vector of V, which $V = [V_1, V_2, \dots, V_{2N}]^T$, is the second-order function of voltage and can be written as (Stankovic and Lesieutre, 1989):

$$S = G(V) = G'\left[V_1V_2, \dots, V_iV_j, \dots, V_{2N}V_{2N}\right] \quad (25)$$

Proper distribution of Eq. (25) around the exponent of $\overline{V}$ would make the second-order term similar to G (V):

$$\begin{aligned} S &= G'\left[\overline{V}_1\overline{V}_1, \dots, \overline{V}_i\overline{V}_j, \dots, \overline{V}_{2N}\overline{V}_{2N}\right] + J_V \Delta V \\ &+ G'\left[\Delta V_1 \Delta V_1, \dots, \Delta V_i \Delta V_j, \dots, \Delta V_{2N} \Delta V_{2N}\right] \end{aligned} \quad (26)$$

Or:

$$S = G(\overline{V}) + J_V \Delta V + G(\Delta V) \tag{27}$$

In which JV is Jacobean matrix. Assuming $\overline{\Delta V_i \Delta V_j} = C_{V_{i,j}}$ and $\overline{\Delta V} = \overline{V - \overline{V}} = \overline{V} - \overline{V} = 0$, exponent of S can be expressed as:

$$\begin{aligned} \overline{S} &= \overline{G(V)} + J_V \overline{\Delta V} + \overline{G(\Delta V)} = \overline{G(V)} + \overline{G(\Delta V)} \\ &= G'(\overline{V}\overline{V}_1 + C_{V_{1,1}}, \ldots, \overline{V}_{2N}\overline{V}_{2N} + C_{V_{2N,2N}}) \end{aligned} \tag{28}$$

$C_{Vi,j}$ represents covariance of V_i and V_j. The effect of voltage covariance can be considered from Eq. (28) by adding covariance term. Therefore, mismatch power can be explained as:

$$\Delta \overline{S} = \overline{S}_0 - \overline{S} \tag{29}$$

$\overline{S}_0$ is the exponent vector of bus injected power. Implementing linearization will lead to:

$$\Delta S = J_V \Delta V \tag{30}$$

Covariance matrix, C_V, will then be achieved:

$$\begin{aligned} C_V &= \overline{\Delta V \Delta V^T} = J_V^{-1} \overline{\Delta S \Delta S^T} (J_V^{-1})^T \\ &= J_V^{-1} C_S (J_V^{-1})^T \end{aligned} \tag{31}$$

Consequently, the correction equations for probabilistic load dispatch are:

$$\Delta \overline{S} = J_V \Delta \overline{V} \tag{32-a}$$

$$C_V = J^{-1}{}_V C_S (J^{-1}{}_V)^T \tag{32-b}$$

In probabilistic load dispatch calculations, due to different effects of exponent of voltage and covariance of injected power, Eq. (32-a) and (32-b) have different roles in calculation of C_V from C_S. While Eq. (32-a) is the inner loop in the calculation, Eq. (32-b) acts as the outer loop in the calculation.

DAMPING RATIO

For a specific eigenvalue $\lambda_i = \alpha_i + j\beta$, damping ratio, ξ_i, can be calculated as:

$$\xi_i = -\alpha_i / \sqrt{\alpha_i^2 + \beta_i^2} \tag{33}$$

Exponent and variance for ξ_i will be calculated using $\overline{\lambda}_i$ and C_λ. If ξ_C remains in defined range (ξ_i for this report is 0.1[50]), probabilistic dispatch of damping ratio would be:

$$P_i\{\xi_i \geq \xi_c\} = \int_{\xi_c}^{\infty} f(\xi_i) d\xi_i \tag{34}$$

Determination of probability of stability: Equation (35) changes normal dispatch of x to a standard normal dispatch, which $x \in N(\overline{X}, \sigma^2)$:

$$z = (x - \overline{X}) / \sigma \tag{35}$$

It should be mentioned that $z \in N(0,1)$.
Dispatch function for z will be:

$$P(z < z_c) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z_c} e^{-u^2/2} du \tag{36}$$

In which z_c is calculated by:

$$\begin{aligned} z_c &= -\overline{\alpha}_i / \sigma_{\alpha_i} \quad for\ \alpha_i \\ z_c &= (\xi_c - \overline{\xi}_c) / \sigma_{\xi_i} \quad for\ \xi_i \end{aligned} \tag{37}$$

Probability in Eq. (36) can be achieved via probabilistic dispatch table.

NUMERICAL RESULTS

The 8-machine system presented in Fig. 1 is a part of the 36-bus system of PSASP software, in which DC links are eliminated. System parameters and power of buses are presented in Appendix. Buses 1 to 16 represent load buses, while buses 17 to 24 are generator buses. The remaining bus is considered as reference bus. Modeling

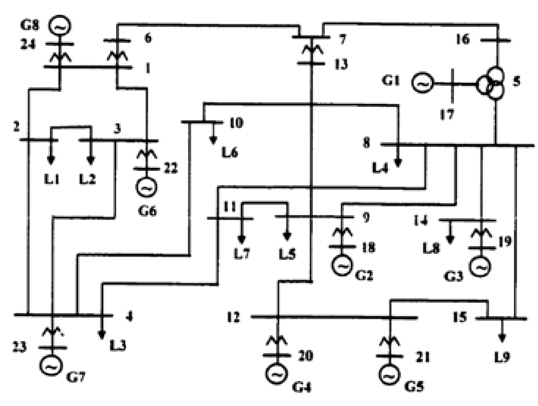

Fig. 1: 8-machine system

Table 1: Eigenvalue, $\lambda = \alpha \pm j\beta$, with the smallest damping ratio, $\xi = -\alpha/|\lambda|$

No	α(rad /s)	β(rad/s)β	ξ
54.55	-0.6941610	-12.1345066	0.0571122
63.64	-4392042	-6.9477981	0.063089
72.7	-0.3620442	-7.5619372	0.04782
80.8	-0.1784845	-6.0561261	0.029458

Table 2: Sensitivity for eigenvalues of transformer Tap (p.u); $\partial\lambda/\partial T_{i,j}$,$\partial^2\lambda\partial T^2_{i,j}$

T_{ap}	T_{apo}	$\dot{\lambda}_{55}$	$\ddot{\lambda}_{55}$	$\dot{\lambda}_{64}$	$\ddot{\lambda}_{64}$	$\dot{\lambda}_{73}$	$\ddot{\lambda}_{73}$	$\dot{\lambda}_{81}$	$\ddot{\lambda}_{81}$
T6,1	1.025	0.163+j0.582	0.929-j0.886	-0.214-j1.161	1.041+j1.669	0.179+j2.142	-1.829+j12.78	-0.834+j3.388	0.793-j25.66
T1,24	1.075	0.364+j7.093	-10.09+j9.068	-0.230-j0.939	5.291-j9.946	-0.056+j2.456	-41.70-j141.0	-1.114+j4.835	49.10-j309.7
T3,22	1.075	-0.004-j8.425	1.719-j407.7	-0.013+j0.046	0.520+j1.474	0.066+j0.599	16.63-j14.75	-0.167+j0.284	-4.905+j52.61
T4,23	1.075	-0.144+j3.450	-0.435-j11.34	-0.081-j1.483	1.104+j5.732	0.514+j3.216	20.28-j215.1	0.896-j6.613	-15.48-j26.56
T8,5	1.027	0.327+j0.182	26.78+j30.44	-0.206-j2.057	2.102+j8.967	-0.470-j1.142	3.889-j40.67	-0.289-j3.280	-19.76-j156.3
T16,5	1.027	-0.203+j0.065	0.865+j1.302	0.056-j0.317	0.597+j1.683	-0.342-j2.964	-0.601+j9.056	0.625-j3.397	0.911-j17.83
T7,13	1.025	-0.065-j0.186	0.455+j0.298	0.122+j0.756	0.133+j0.744	-0.095-j0.777	0.398+j6.890	0.435-j1.840	-1.413-j1.961
T9,18	1.025	0.136-j0.197	0.034-j0.319	0.006+j0.034	0.832-j0.493	0.006-j0.142	6.996-j24.79	-0.003+j0.080	3.672+j6.659
T12,20	1.025	-0.028+j0.026	0.677+j1.258	-0.764+j0.034	59.51+j23.36	0.028-j1.440	-9.995+j44.23	0.739+j0.728	-49.34-j78.71
T12,21	1.025	-0.038+j0.028	0.150+j0.465	1.410+j2.839	72.47-j27.04	-1.386-j0.497	-27.88+j88.34	0.000-j2.608	-37.46-j46.22
T14,19	1.075	0.087-j0.021	-0.163+j0.503	0.011+j0.113	0.833-j0.808	0.113+j0.122	4.414-j20.27	-0.019+j0.449	2.573-j4.342

all generators with the fifth-order model of synchronous machine will lead to 84 eigenvalues, from which are 40 real eigenvalues and 44 complex eigenvalues. The real part is negative for all eigenvalues. Furthermore, four pairs of complex eigenvalues, shown in Table 1, have the minimum damping ratio. Therefore, only the results of these 4 modes are presented.

In analysis of probabilistic eigenvalues, changes in injected power of buses affect all operating parameters, such as current, voltage and rotor angle. On the other hand, these parameters are involved in forming matrix A. Therefore, they will affect eigenvalues. Using system admittance matrix, known by Y, for producing matrix A and for calculating sensitivity of eigenvalues, enable us to calculate eigenvalues via state matrix of A. With respect to the fact that eigenvalues are random variables, complex eigenvalues, e.g., $\lambda_i = \alpha_i + j\beta_i$, distribute within a specific area of s-plane. Discussing the distribution obedience can be done via probabilistic density function (p.d.f). With respect to its random values, the stability or instability of mode i^{th} cannot be determined by sample values.

To demonstrate the point, p.d.f of α53 and α55, mentioned in Table 2, are shown in Fig. 2, which is based on normal dispatch.

Despite the lower exponent of $\overline{\alpha}_{53}$ in comparison with the exponent of $\overline{\alpha}_{65}$, mode 55 is a rigid stable mode, which is due to the fact that all samples of α55 have negative values. At the same time, mode 53 is an instable mode.

Stability order for i can be evaluated by:

$$P\{\alpha_i < 0\} = \int_{-\infty}^{0} f(\alpha_i) d\,\alpha_i \tag{38}$$

In which f(0) represents p.d.f for αi, where αi is a random variable. The system stability can be evaluated via probabilistic stability of critical eigenvalues.

For 8-machine system of Fig. 1, it is possible to assume different daily load curves for buses (SL1-SL9,PG1-PG7 and QG3-QG6). The voltage of reference bus changes with respect to A2 curve. All generators are modeled by third-order model of synchronous machine.

Table 3: Selected real eigenvalues for 8-machine system

No	$\overline{\alpha}$	σ_α	α*	P_α
14	- 25.005	0.1628	153.62	1
21	- 11.250	0.0181	620.09	1
27	- 5.221	0.0106	490.21	1
28	- 2.093	0.1126	18.58	1
49	- 0.382	0.0503	7.58	1

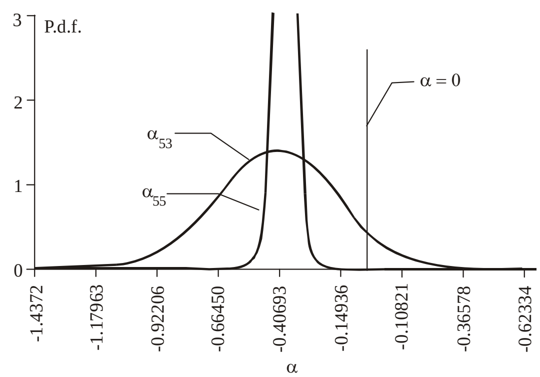

Fig. 2: Probabilistic dispatch function for α53 and α55

The results reveal that, within 74 eigenvalues are 30 real eigenvalues and 22 complex eigenvalues. Some eigenvalues are presented in Table 3.
The 74^{th} eigenvalue has a positive real part.

CONCLUSION

Probabilistic dispatch of an eigenvalue, as well as damping ratio can be determined via estimation of exponent and covariance. Figure 2 presents p.d.f for 53^{rd} and 55^{th} damping ratios which are shown in Table 3.

Damping factor probability and damping ratio are shown in the last two columns of Table 3, 4 and 5. α*stands for standard exponent, $-\alpha/\sigma_\alpha$.

For matrix A of 8-machine system, a comparison of eigenvalues has been implemented in two different conditions: with covariance correction and without covariance correction. The results are shown in Table 3 and 5. In our test system, without covariance correction exponent has a negative real value for all eigenvalues. The biggest value in this case is -0.085.

Implementing covariance correction will affect the results. For instance, the biggest value for exponent of

Table 4: Selected complex eigenvalues for 8-machine system

No	$\bar{\alpha}$	$\bar{\beta}$	σ_α	σ_β	α*	P_α
53	0.364	0.695	0.2683	0.8240	1.36	0.9115
55	- 0.338	0.201	0.0386	0.0362	8.78	1
57	- 0.332	0.164	0.1063	0.0362	3.13	0.9991
61	- 0.262	15.019	0.1279	1.1880	2.05	0.9798
67	- 0.111	9.353	0.0335	0.2474	3.13	0.9995
74	0.180	4.226	0.3060	2.9099	-0.59	0.2810

Table 5: Selected damping ratio for 8-machine system

No	$\bar{\xi}$	σ_ξ	ξ*	P_ξ
53	0.4644	0.1713	2.13	0.9830
55	0.8595	0.0137	55.26	1
57	0.8967	0.0211	37.68	1
61	0.0175	0.0072	-11.54	0
67	0.0119	0.0038	-22.96	0
74	-0.0426	0.0587	-2.43	0.0078

eigenvalues will be 0.18 with covariance correction, as shown in Table 4.

To reduce the calculation task and time, in systems with numerous machines and, consequently, numerous eigenvalues, only a modicum number of eigenvalues will be investigated.

Appendix:

Equations of a third-order machine: Machine equations in Park dq frame can be presented by:

$$pE'_q = [E_{fd} - (X_d - X_{d'})I_d - E'_q]/T'_{do}$$
$$p\delta/\Omega_0 = \Delta\Omega - \Delta\Omega_{ref}$$
$$p\Omega = [P_m - P_e]/M$$
$$V_t^2 = V_d^2 + V_q^2$$
$$V_d = -R_a I_d = X_q I_q$$
$$V_q = E'_q - X'_d I_d - R_\alpha I_q$$
$$P_e = V_d I_d + V_q I_q + \left(I_d^2 + I_q^2\right)R_a$$

Load model: In order to use reduced-order system admittance, Y_{GG}, system equations are written as follows:

$$\begin{bmatrix} I_G \\ I_M \end{bmatrix} = \begin{bmatrix} Y_{0GG} & Y_{GM} \\ Y_{MG} & Y_{MM} \end{bmatrix}$$

In which G represents generator-buses of system and M stands for other buses:

$$V_M = Y_{MM}^{-1}(I_M Y_{MG} V_G)$$
$$I_G = (Y_{0GG} - Y_{GM}Y_{MM}^{-1}Y_{GM})V_G + Y_{GM}Y_{MM}^{-1}I_M$$
$$= Y'_{GG}V_G + I_{gl}$$
$$= Y'_{GG}V_G + Y_{gl}V_G$$

For which $Y_{G'G}(Y_{0GG} - Y_{GM}Y_{MM}^{-1}Y_{GM})$ is reduced-order admittance matrix. I_{gl} and Y_{gl} represent loadcurrent and load equivalent admittance in a generator-bus, respectively. Y_{gl} is a diagonal matrix:

$$Y_{gl,ii} = I_{gl,i}/V_{G,i}$$
$$I_{gl} = \left[Y_{0GG}Y_{GM}\right]V - Y'_{GG}V_G$$

Therefore,

$$I_G = Y_{GG}V_G$$

Test system data: The base power is 100Mwatt for test system. For voltage-dependent loads, constants a and b would be 2.

Table A1: Line parameters for 8-machine system

Branch no.	From I	To J	R (p.u.)	X (p.u.)	B/2 or T(p.u.)
1	24	-1	0.0000	0.0150	1.0750
2	1	2	0.1060	0.0740	0.0000
3	1	3	0.0147	0.1040	0.0000
4	2	4	0.0540	0.1900	0.1650
5	-4	23	0.0000	0.0124	1.0750
6	-3	22	0.0000	0.0217	1.0750
7	2	3	0.0034	0.0131	0.0000
8	3	4	0.0559	0.2180	0.1954
9	4	10	0.0214	0.0859	0.3008
10	4	11	0.0150	0.0607	0.2198
11	1	-6	0.0000	0.0180	1.0250
12	6	7	0.0033	0.0343	1.8797
13	7	7	0.0000	0.7320	0.0000
14	7	7	0.0000	0.7320	0.0000
15	-7	13	0.0000	0.0180	1.0250
16	8	11	0.0037	0.1780	0.1640
17	13	9	0.0034	0.0200	0.0000
18	17	5	0.0000	0.0347	0.0000
19	5	-8	0.0000	0.0010	1.0270
20	5	-16	0.0000	0.0010	1.0270
21	7	16	0.0020	0.0250	1.3900
22	10	8	0.0165	0.0662	0.2353
23	11	9	0.0114	0.0370	0.0000
24	9	8	0.0578	0.2180	0.1887
25	14	8	0.0033	0.0333	0.0000
26	-14	19	0.0000	0.0375	1.0750
27	9	12	0.0196	0.0854	0.0810
28	15	8	0.0030	0.0130	0.0000
29	8	8	0.0000	-1.0000	0.0000
30	15	12	0.0030	0.0340	0.1650
31	-12	20	0.0000	0.0438	1.0250
32	-9	18	0.0000	0.0640	1.0250
33	-12	21	0.0000	0.0328	1.0250

Table A2: Data of buses for 8-machine system

Load bus data				Generator bus data			
bus no.	P (p.u.)	Q (p.u.)	curve no.	bus no.	P (p.u.)	Q (p.u.)	curve no.
2	2.870	1.440	1	19	4.300	3.340	2
3	3.760	2.250	4	18	1.600	0.700	5
4	2.270	2.690	6	22	6.000	3.600	30

Table A2: Continue

10	0.719	0.474	1	21	3.060	0.000	2
11	0.700	0.500	1	23	3.100	0.000	1
9	0.864	0.662	5	17	2.000	0.000	3
14	4.300	2.200	3	20	2.250	0.000	2
15	5.200	2.500	2				
8	5.000	2.300	2				
PV bus data							
Bus no.		17	20	21	23	24	(slak bus)
Voltage (p.u.)		1.000	1.00	1.000	1.000	1.000	
Curve no.		1	1	1	1	31	

Table A3: Data of generators for 8-machine system (5^{th} order)

	G1	G2	G3	G4	G5	G6	G7	G8
R_a	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
X_d	1.6330	0.7702	0.3060	0.3163	0.1931	0.3210	0.1380	0.1710
X_q	1.6330	0.7702	0.3060	0.2238	0.1573	0.3210	0.0680	0.1710
X'_d	0.1970	0.1209	0.0480	0.1252	0.0788	0.0382	0.0396	0.1710
X'_q	1.6330	0.7702	0.3060	0.2238	0.1573	0.3210	0.0680	0.1710
X''_d	0.1480	0.0779	0.0311	0.0881	0.0505	0.0238	0.0283	0.1710
X''_q	0.1480	0.0779	0.0311	0.0881	0.0505	0.0238	0.0283	0.1710
T'_{do}	4.0000	6.2000	6.2000	5.5300	5.9500	8.3750	7.2400	10.000
T'_{qo}	9999.0	9999.0	9999.0	9999.0	9999.0	9999.0	9999.0	9999.0
T''_{do}	0.0600	0.1920	0.0500	0.0500	0.0500	0.2240	0.0500	0.1000
T''_{qo}	0.2400	1.8900	0.5000	0.0500	0.0500	1.6600	0.2000	0.2000
M	5.2400	15.6792	39.1999	21.9991	32.5984	29.9979	79.5035	140.81
D	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
S_b	1.0000	2.3500	6.3750	2.8600	3.8840	7.0600	8.8200	18.8000

Table A4: Data of generators for 8-machine system (3^{rd} order)

	G1	G2	G3	G4	G5	G6	G7	G8
R_a	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
X_d	1.6330	1.8100	1.9510	0.9045	0.7500	2.2660	1.2170	0.2820
X_q	1.6330	1.8100	1.9510	0.6400	0.6110	2.2660	0.6000	0.2820
X'_d	0.1970	0.2840	0.3060	0.3580	0.3060	0.2700	0.3490	0.2820
X'_d	1.6330	0.7702	0.3060	0.2238	0.1573	0.3210	0.0680	0.1710
T'_{do}	6.9200	6.2000	6.2000	5.5300	5.9500	8.3750	7.2400	10.000
M	5.2400	12.344	12.298	15.384	16.786	8.4980	18.028	140.980
D	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Table A5: Data of transfer blocks for excitation system and governer in 8-machine system

	From	To	a	T_a	b	T_b		From	To	a	T_a	b	T_b
EXC1	8	9	-1.000	0.000	1.000	0.030	GOV5	5	41	-20.00	0.000	0.000	0.000
	9	21	50.00	0.000	1.000	0.050		41	42	1.000	0.000	0.000	5.000
	21	22	1.000	0.000	1.000	0.500		42	41	-0.800	0.000	0.000	0.000
	22	7	1.000	0.000	0.000	0.000		42	41	0.000	-2.500	1.000	5.000
	22	9	0.000	-0.023	1.000	0.800		42	43	1.000	-1.000	1.000	0.500
EXC2	8	9	-1.000	0.000	1.000	0.030		43	6	1.000	0.000	0.000	0.000
	9	21	50.00	0.000	1.000	0.030	PSS5*	8	9	-1.000	0.000	1.000	0.030
	21	22	1.000	0.000	1.000	0.500		9	21	1.000	1.000	1.000	2.000
	22	7	1.000	0.000	0.000	0.000		21	22	1.000	1.000	1.000	2.000
	22	9	0.000	-0.040	1.000	0.715		22	7	20.00	0.000	1.000	0.020
GOV2	5	41	-20.00	0.000	0.000	0.000	EXC6*	8	9	-1.000	0.000	1.000	0.030
	41	42	1.000	0.000	0.000	0.500		9	21	50.00	0.000	1.000	0.040
	42	41	-1.000	0.000	0.000	0.000		21	22	1.000	0.000	1.000	0.500
	42	43	1.000	0.000	1.000	0.200		22	7	1.000	0.000	0.000	0.000
	43	6	1.000	0.000	0.000	0.000		22	9	0.000	-0.040	1.000	0.715
EXC3	8	9	-1.000	0.000	1.000	0.030	GOV6	5	41	-20.00	0.000	0.000	0.000
	9	21	50.00	0.000	1.000	0.010		41	42	1.000	0.000	0.000	0.400
	21	22	1.000	0.000	0.000	0.500		42	41	-1.000	0.000	0.000	0.000
	22	7	1.000	0.000	0.000	0.000		42	43	1.000	0.000	1.000	0.200
	22	9	0.000	-0.050	1.000	0.715		43	6	1.000	0.000	0.000	0.000
GOV3	5	41	-20.00	0.000	0.000	0.000	EXC7	8	9	-1.000	0.000	1.000	0.030
	41	42	1.000	0.000	0.000	0.500		9	21	50.00	0.000	1.000	0.030
	42	41	-1.000	0.000	0.000	0.000		21	22	1.000	0.000	1.000	0.500

Table A5: Continue

	42	43	1.000	0.000	1.000	0.200		22	7	1.000	0.000	0.000	0.000
	43	6	1.000	0.000	0.000	0.000		22	9	0.000	-0.100	1.000	0.715
EXC4	8	9	-1.000	0.000	1.000	0.030	GOV7	5	41	-20.00	0.000	0.000	0.000
	9	21	1.000	1.000	1.000	2.000		41	42	1.000	0.000	0.000	5.000
	21	22	1.000	1.000	1.000	2.000		42	41	-0.800	0.000	0.000	0.000
	22	7	20.00	0.000	1.000	0.020		42	41	0.000	-2.500	1.000	5.000
GOV4	5	41	-20.00	0.000	0.000	0.000		42	43	1.000	-1.000	1.000	0.500
	41	42	1.000	0.000	0.000	5.000		43	6	1.000	0.000	0.000	0.000
	42	41	-0.800	0.000	0.000	0.000	EXC8	8	9	-1.000	0.000	1.000	0.030
	42	41	0.000	-2.500	1.000	5.000		9	21	50.00	0.000	1.000	0.030
	42	43	1.000	-1.000	1.000	0.500		21	22	1.000	0.000	1.000	0.500
	43	6	1.000	0.000	0.000	0.000		22	7	1.000	0.000	0.000	0.000
PSS4*	5	31	-100.	0.000	0.000	0.000		22	9	0.000	-0.200	1.000	0.715
	31	32	0.000	1.000	1.000	10.00	GOV8	5	41	-20.00	0.000	0.000	0.000
	32	33	1.000	0.300	1.000	0.050		41	42	1.000	0.000	0.000	0.300
	33	8	1.000	0.300	1.000	0.050		42	41	-1.000	0.000	0.000	0.000
EXC5	8	9	-1.000	0.000	1.000	0.030		42	43	1.000	0.000	1.000	0.050
	9	21	1.000	1.000	1.000	2.000		43	6	1.000	0.000	0.000	0.000
	21	22	1.000	1.000	1.000	2.000							
	22	7	20.00	0.000	1.000	0.020							

Table A6: Data of loads

Number of days	Hour of the days																							
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
2	0.92	0.92	0.92	0.92	0.92	0.92	1.03	1.03	1.15	1.10	1.15	0.90	0.92	0.92	1.15	1.15	1.15	0.92	0.92	1.06	1.05	1.05	0.92	0.91
3	0.62	0.62	0.61	0.61	0.61	0.91	0.91	0.91	1.08	1.37	1.67	0.60	0.60	0.91	1.21	1.67	1.67	1.67	0.76	1.21	1.21	0.93	0.91	0.75
4	0.86	1.47	0.86	0.71	1.47	1.11	0.45	0.72	0.72	1.35	1.10	0.67	1.47	1.10	0.88	0.37	1.23	0.98	1.47	0.74	1.10	1.10	1.47	0.60
5	1.47	1.19	1.09	0.52	0.52	0.95	1.30	1.30	0.65	0.65	0.26	0.26	1.35	1.43	1.56	1.56	1.43	1.43	0.26	0.52	0.52	1.30	1.30	1.18
6	0.74	0.30	0.30	0.30	0.30	0.74	0.74	0.89	1.34	1.34	1.64	1.04	1.04	1.04	1.50	1.50	1.04	1.34	0.89	0.89	0.89	1.40	1.40	1.40
7	0.53	0.11	0.11	0.11	0.11	0.36	0.79	0.79	2.15	2.38	1.85	1.06	0.53	1.59	1.86	1.85	1.08	1.38	1.32	1.06	1.06	0.79	0.79	0.34
8	1.03	1.03	0.19	0.19	0.19	0.19	0.60	0.80	0.56	0.56	0.56	1.31	1.31	1.69	1.69	2.00	2.00	1.10	0.50	1.30	1.30	1.30	1.30	1.30
9	1.07	0.53	0.18	0.18	0.18	0.53	0.53	1.07	1.07	2.14	1.74	0.53	0.89	1.07	0.89	0.53	0.55	1.60	1.60	1.07	2.14	1.42	1.41	1.08
10	0.84	0.52	0.52	0.52	0.52	0.85	1.07	1.07	1.93	2.56	1.72	0.67	0.38	0.58	1.07	1.55	1.55	0.65	0.34	1.28	1.93	1.07	0.65	0.20
11	0.52	0.52	0.26	0.26	0.15	0.36	1.29	1.80	0.54	0.52	2.32	0.26	0.26	0.71	1.29	2.32	0.26	0.26	1.29	2.58	2.58	1.80	1.29	0.56
12	0.52	0.52	0.52	0.52	0.52	1.04	2.60	1.82	1.06	0.26	0.26	0.26	0.83	0.68	0.60	0.52	0.52	0.52	1.04	2.87	2.87	1.30	1.30	1.05
13	0.41	0.34	0.57	0.50	0.34	0.54	0.81	1.01	0.68	1.51	1.85	1.49	1.49	1.14	1.34	1.24	0.81	1.01	1.17	1.17	1.68	1.51	0.85	0.54
14	0.24	0.24	0.24	0.24	0.24	0.24	0.24	2.08	2.08	0.73	2.08	0.73	2.08	0.73	2.78	0.73	2.18	0.80	1.20	1.21	0.97	0.73	0.73	0.48
15	0.67	0.67	0.67	0.67	0.67	0.67	0.67	1.33	1.33	1.33	1.33	1.33	1.33	0.67	0.67	1.33	1.33	1.33	1.33	1.33	1.33	0.67	0.67	0.67
16	1.99	1.99	1.77	1.77	1.77	1.77	1.77	0.44	0.44	1.11	1.11	1.11	0.18	0.18	0.66	0.66	0.66	0.22	0.22	0.66	0.66	0.66	1.10	1.10
17	0.36	0.36	0.36	0.36	0.36	0.36	0.36	0.70	1.96	1.60	1.60	1.24	1.40	1.60	1.96	1.96	1.60	1.60	0.71	0.71	0.89	0.89	0.90	0.16
18	0.37	0.37	0.37	0.37	0.37	0.37	0.37	0.37	0.37	2.22	2.95	1.11	0.37	0.72	1.48	2.57	2.22	1.11	0.74	0.74	1.48	1.48	0.74	0.74
19	0.19	0.19	0.19	0.19	0.29	1.94	1.94	1.94	0.04	0.04	0.04	0.04	1.97	1.97	1.97	1.97	1.97	1.97	1.99	0.79	0.79	0.79	0.39	0.39
20	0.32	0.32	0.32	0.32	0.32	0.94	2.83	1.57	0.64	1.26	2.51	1.89	0.18	0.18	0.18	1.38	1.26	0.18	0.18	0.94	2.83	2.20	0.94	0.32
21	0.21	0.21	0.21	0.21	0.21	0.82	1.23	0.41	0.82	2.26	2.26	0.41	0.41	1.23	2.26	2.26	2.26	1.23	0.41	1.02	1.64	1.42	1.41	0.21
22	0.65	0.43	0.43	0.43	0.43	1.08	1.08	2.16	2.16	2.16	2.16	2.16	2.16	0.23	0.23	1.08	0.22	1.08	1.08	0.65	0.65	0.43	0.43	0.43
23	0.23	0.23	0.23	0.23	0.23	0.91	0.91	1.36	1.36	2.70	2.70	0.40	0.05	0.45	1.58	2.49	1.81	0.68	0.68	1.36	1.36	0.91	0.91	0.23
24	0.28	0.28	0.28	0.28	0.28	0.28	0.60	2.00	1.70	1.72	1.71	1.88	1.87	1.71	1.71	1.71	1.71	0.60	0.60	0.60	0.60	0.60	0.60	0.60
25	0.37	0.37	0.37	0.37	0.37	0.37	0.37	0.37	0.37	1.48	1.48	2.95	2.95	0.37	0.37	0.37	0.48	1.95	2.95	0.37	0.84	1.89	1.85	0.37
26	0.84	0.34	0.34	0.34	0.34	0.84	0.84	1.01	1.51	1.51	1.47	1.18	1.18	1.18	1.40	1.40	1.18	1.51	1.01	1.01	1.01	0.90	0.90	0.79
27	0.67	0.62	0.62	0.67	0.70	0.77	1.03	1.13	1.24	1.24	1.19	1.03	1.13	1.13	1.08	1.13	1.13	1.03	0.98	1.03	1.24	1.29	1.08	0.82
28	0.70	0.62	0.62	0.62	0.70	0.77	0.93	1.08	1.24	1.32	1.24	1.01	0.93	0.93	1.08	1.24	1.24	1.16	0.85	1.08	1.32	1.32	1.16	0.85
29	0.79	0.79	0.68	0.56	0.68	0.68	0.79	1.01	1.35	1.46	1.24	0.79	0.68	0.68	1.24	1.35	1.35	1.24	0.79	0.90	1.46	1.35	1.13	1.01
30	1.01	1.01	1.01	1.01	0.91	0.91	1.01	0.91	1.01	1.01	1.01	0.81	0.91	1.01	1.01	0.91	1.01	1.01	0.91	1.01	1.03	1.03	1.01	0.91
31	1.01	1.01	1.02	1.02	1.02	1.02	1.01	1.00	1.00	0.99	0.98	0.98	0.99	1.00	1.00	0.99	0.99	0.98	0.99	1.00	1.00	1.00	1.00	0.99

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