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Depiction of the Automated Fiber Placement Robotic Manipulator's Jacobian Matrix by SOA

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Abstract: In order to simplify depiction of Jacobian matrix, the 7-DOF automation fiber placement robotic manipulator's Jacobian matrix is solved by SOA in surrounding of mathematic comparing the Jacobian matrix solved by SOA to by differential method and vector cross product method, and finding that the Jacobian matrix solved by SOA has simple form and clear physical meaning, and for any structural robotic manipulator the Jacobian matrix solved by SOA has the form of analytical expression. Research shows that the Jacobian matrix solved by SOA is an effective method.

Keywords: Fiber placement robotic manipulator, Jacobian matrix, spatial operator algebra

INTRODUCTION

Jacobian matrix is an important concept in robotics, which reflects the mapping relation between the robotic manipulator's operating space and joint space (Xiong, 2002). The robotic manipulator's many performance indices such as work space analysis, performance measure, singularity analysis and etc., are analyzed and calculated by Jacobian matrix (Guo and Geng, 2008), meanwhile Jacobian matrix is also the foundation of trajectory planning, stiffness and motion accuracy. Traditional method solved Jacobian matrix is the differential to the robotic manipulator direct kinematics, usually the process and the results are more complex and error-prone (Yu *et al.*, 2009). Chen *et al.* (1995) described the robotic manipulator's Jacobian matrix clearly using exponential product formula of screw theory, highlighted the robotic manipulator's geometric characteristics, while avoiding the deficiency of using local parameters in differential method, Whitney proposed vector product method solved Jacobian matrix based on the concept of moving coordinate system, Jain and Rodriguez (1995) developed the vector product further to the Spatial Operator Algebra (SOA) method. The robotic manipulator's Jacobian matrix is described by SOA, the mathematical form is simple (Fang *et al.*, 2008) and each operator has clear physical meaning, the algorithm can be calculated by recursive based on its physical meaning, there is no duplication and no waste, it's an excellent computing. The automatic fiber placement robotic manipulator's Jacobian matrix is studied by SOA in this study.

TOPOLOGY ANALYSIS OF THE AUTOMATED FIBER PLACEMENT ROBOTIC MANIPULATOR

The automated fiber placement robotic manipulator model is shown in Fig. 1 (left). From its structure, there is a robotic manipulator composed by 3 displacements and 3 revolute and a rotational mandrel.

Clearly, two parts of the automated fiber placement robotic manipulator belong to two systems. The following equivalent transformation in analyzing the robotic manipulator will be done: mandrel as stationary and the coordinate system fixed mandrel coincide with the base coordinate system, virtual revolute joints linked the base of the fiber placement robotic manipulator and the mandrel together and the rotational motion of the mandrel is equivalent to the robotic manipulator's rotation around the mandrel. So the fiber placement robotic manipulator with 6-DOF and the mandrel with 1-DOF becomes a 7-DOF redundant robotic manipulator. The shoulder has a revolute joint, the elbow has three displacements joint, wrist has three revolute joint. The three revolute joint axes of the wrist intersect at one point, the automatic fiber placement robotic manipulator's topology after equivalent motion as shown in Fig. 1 (right). Establishing D-H coordinate system and its structural parameters as shown in Table 1.

Definition of Jacobian matrix based on SOA: SOA is new mathematical tool in studying multi-body system, the key innovation is combined several ways which looks like having nothing to do with mechanics:

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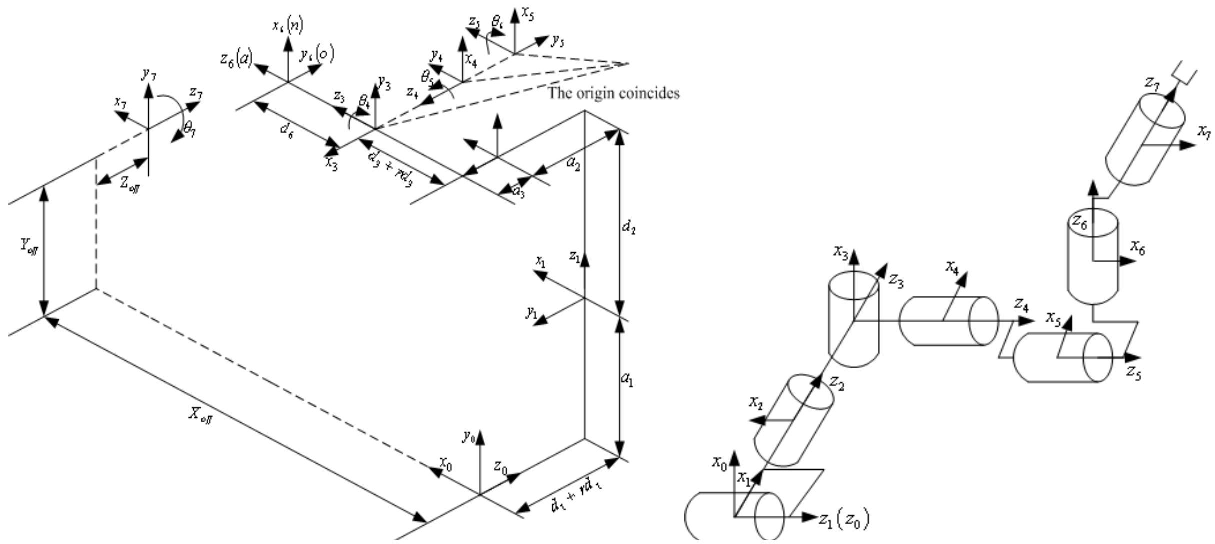

Fig. 1: The automated fiber placement robotic manipulator's structure and topology

Table 1: Links parameters of automated fiber placement robotic manipulator

link i	a_{i-1} (mm)	α_{i-1} (°)	d_i (mm)	θ_i (°)	The scope of the joint variables
1	0	0	d_1	0	d_1: -150-150 mm
2	0	90	d_2	-90	d_2: -110-110 mm
3	α_2	90	d_3	0	d_3: -100-100 mm
4	0	0	c	θ_4	θ_4: -210°-210°
5	0	90	0	θ_5	θ_5: -150°-150°
6	0	-90	0	θ_6	θ_6: -260°-260°

filtering and prediction; function analysis and linear operator method; linear system control theory. Accordingly, Jain and Rodriguez inventor of SOA integrated the Kalman filtering in signal analysis and processing and state equation structures in prediction theory and the internal structure in multi-body system dynamics and established the spatial operator algebra system of multi-body dynamics. SOA has a number of basic operators, each operator achieved by space recursive; the number of its operations is the same order to DOF of systems. With system complexity increasing, computational complexity and computation increase only in linear change, so the computational efficiency achieving the ideal state. In order to facilitate description by SOA, each joint of multi-body systems is needed to number, the numbering is from the end-effector to the base in ascending order (Fig. 2), that is the number of the end-effector is the one rigid body, the number of the base is the N+1 rigid body, the outer of the end-effector defined as 0, which the numbering is opposition to robotics (Fig. 3).

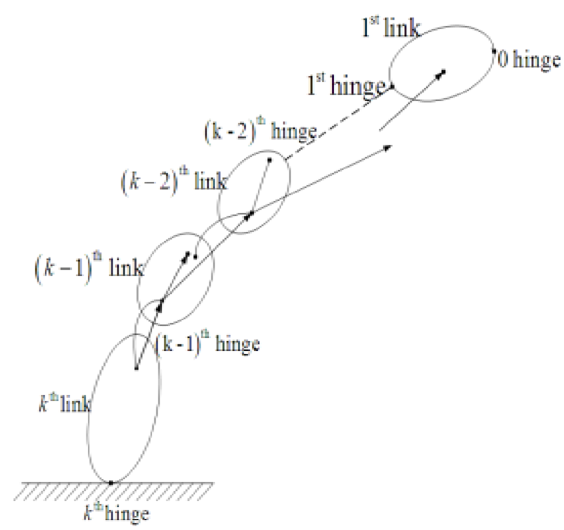

Fig. 2: Series of rigid body numbering

Six dimensional space velocity is defined as V(k) = Col{ω(k), v(k)} in inertial coordinate system according to screw to the typical rigid body B(k), ω(k) denoted angular velocity of the typical rigid body B(k) reference point, v(k) denoted line velocity of the typical rigid body B(k) reference point, the corresponding 6-dimensional space acceleration is the differential of six dimensional space velocity V(k) to the time. Six dimensional force is defined as F(k) = Col{t(k), f(k)}, t(k), f(k) is the force and torque acting on the rigid body B(k). The basic spatial operators associated with Jacobian matrix are the following:

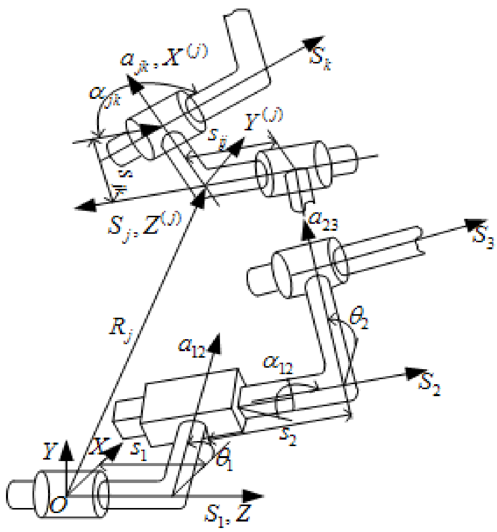

Fig. 3: Robotic manipulator numbering

The rigid force translation operator of system φ, the rigid force translation operator φ (6×6 upper triangular matrix) between arbitrary two points A, B (usually refers to reference point and articulated point of rigid body) is defined and satisfied F (k) = φ (k, k-1) F (k-1). (Down triangle) operator ε_φ can be composed by all φ(k, k-1) according to certain rules and called the rigid body forces recursive translation operator between systems and its function φ = $(I-\varepsilon_\varphi)^{-1}$ called the rigid force translation operator of system, this is an operator which acts and hands force. The rigid force translation operator (6×6 upper triangular matrix) to arbitrary two points A, B (usually refers to reference points of two rigid body) in system is defined as:

$$\phi(k+1,k) = \begin{pmatrix} I & \tilde{l}\,(k+1,k) \\ 0 & I \end{pmatrix} \quad (1)$$

Here l (A, B) is an anti-symmetric matrix which the vector between the two point A, B in inertial coordinate system, I is unit matrix of 3×3. So the rigid force translation operator between two adjacent reference points in serial multi-body system expressed as:

$$\varepsilon_\phi \triangleq \begin{pmatrix} 0 & 0 & L & 0 & 0 \\ \phi(2,1) & 0 & L & 0 & 0 \\ 0 & \phi(3,2) & L & 0 & 0 \\ M & M & O & M & M \\ 0 & 0 & L & \phi(n,n-1) & 0 \end{pmatrix}$$

So the rigid force translation operator of serial multi-body system is:

$$\phi \triangleq \begin{pmatrix} I & 0 & 0 & L & 0 \\ \phi(2,1) & I & 0 & L & 0 \\ M & M & O & M & 0 \\ \phi(n,1) & \phi(n,2) & L & L & I \end{pmatrix}$$

The rigid velocity translation operator of systems φ^*, according to duality principle, there is V (k-1) = φ^*(k, k-1) V(k), so the duality operator called the rigid velocity translation operator, this is an operator which acts and hands velocity.

Projection operator H from state space to joint space, its function is that 6-dimensional space force on the joint are projected onto the joint axis, which is projection operator to motion space. If there is only one rotational degrees of freedom in the serial rigid multi-body system, without displacement degree of freedom, then H is a 6×1 column vector, the first three rows is projection of the unit axis vector in the base coordinate, the second three rows is 0.Operator H that is rotated Z is:

$$\boldsymbol{H}(k) = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}^{\mathrm{T}}$$

Analogous, operator H that is displaced Z is:

$$\boldsymbol{H}(k) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}^{\mathrm{T}}$$

Operator H from state space to joint space, expressed as:

$$\boldsymbol{H} = \begin{pmatrix} \boldsymbol{H}(1) & 0 & L & 0 \\ 0 & \boldsymbol{H}(2) & L & 0 \\ L & L & O & M \\ 0 & 0 & L & \boldsymbol{H}(n) \end{pmatrix}$$

The duality operator H^* of projection operator H is projection operator from joint space to state space, it's function is that projected six dimensional space force on joint onto state space.

The space force pick operator B:

$$B = [\phi(1,0) \;\; 0 \;\; L \;\; 0]$$

The space velocity pick operator B^*:

$$B^* = [\phi^*(1,0) \quad 0 \quad L \quad 0]$$

The algorithm of Jacobian matrix by SOA: After defining the space velocity of rigid body $V(k) = Col\{\omega(k), v(k)\}$, angular velocity $\omega = \{\omega(1), \omega(2), \ldots, \omega(n)\}$ and line velocity $v = \{v(1), v(2), \ldots, v(n)\}$ of rigid body recursive as follows:

$$\begin{cases} V(n+1) = 0 \\ V(k) = \phi^*(k+1,k)V(k+1) + H(k)\omega \\ V(0) = \phi^*(1,0)V(1) \\ k = 1, 2, L\ , n \end{cases} \tag{2}$$

Summing up the above recursion is:

$$V(k) = \sum_{i=k}^{n} \phi^*(i,k)H(i)\,\omega(i) \tag{3}$$

So,

$$V(0) = B^*\phi^*H^*\,\dot{\theta} \tag{4}$$

According to meaning of Jacobian matrix, the Jacobian matrix $J \in R^{6\times n}$ is:

$$J = B^*\phi^*H^* \tag{5}$$

Jacobian matrix of serial multi-rigid-body system can be solved by operator B^*, φ^*, H^*, substitute the structural parameters of the automated fiber placement robotic manipulator to formula (5), run the algorithm in mathematic 6.0, Jacobian matrix of the automated fiber placement robotic manipulator can be solved:

$$J = \begin{bmatrix} -\sin(\theta_1)*(c-d_4+a_1)+\cos(\theta_1)*d_3 & 0 & \sin(\theta_1) & -\cos(\theta_1) & 0 & 0 & 0 \\ \cos(\theta_1)*(c-d_4+a_1)+\sin(\theta_1)*d_3 & 0 & -\cos(\theta_1) & -\sin(\theta_1) & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\cos(\theta_1) & -\sin(\theta_1)*\cos(\theta_5) & -\sin(\theta_1)*\sin(\theta_5)*\sin(\theta_6)-\cos(\theta_1)*\cos(\theta_6) \\ 0 & 0 & 0 & 0 & -\sin(\theta_1) & \cos(\theta_1)*\cos(\theta_5) & \cos(\theta_1)*\sin(\theta_5)*\sin(\theta_6)-\sin(\theta_1)*\cos(\theta_6) \\ 1 & 0 & 0 & 0 & 0 & -\sin(\theta_5) & \cos(\theta_5)*\sin(\theta_6) \end{bmatrix} \tag{6}$$

Jacobian matrix of the automated fiber placement robotic manipulator solved by SOA is exactly the same by the differential method and by the vector cross product method through calculation and analysis, thus proving that solving Jacobian matrix of the automated fiber placement robotic manipulator using SOA theory is an effective method.

CONCLUSION

Jacobian matrix of the automated fiber placement robotic manipulator is described by SOA, the mathematical forms are simple, but also for all forms of serial robotic manipulator Jacobian matrices are with analytical forms. Jacobian matrix of serial robotic manipulator described by SOA whose Mathematical forms not only simple, but each operator has clear physical meaning, the algorithm can be calculated by recursiving based on its physical meaning, there is no duplication and no waste, it’s a excellent computing. The calculation quantity is little and easy to implement in software.

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