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Dynamic Responses of Truss Spar Due to Wave Actions

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Abstract: Spar platforms have been used for drilling, production and storage of oil and gas in the offshore deepwater region. The structure is installed at the deepwater locations in the sea and is exposed to continuous action of wind, wave, current and other environmental forces. Wave force constitutes about 70% of the total environmental force and could be considered as the most significant force affecting the dynamic responses needed for the design of these structures. In this study, the dynamic responses of the truss spar due to wave actions including the wave force theories and wave propagation directions are investigated. Numerical simulations are developed to investigate the accuracy of the wave force theories i.e., Morison equation and Diffraction theory, for large structure such as truss spar. The investigation is further expanded to study responses of the truss spar due to variations directions of the wave propagated. The truss spar is modelled as a rigid body with three degrees of freedom restrained by mooring lines. In the simulation, the mass, damping and stiffness matrices are evaluated at every time step. The equations of motion are formulated for the platform dynamic equilibrium and solved by using Newmark Beta method. To compute the wave force for truss spar, which is large compared to the wave length, Diffraction theory was found to be more appropriate. The Morison equation was found applicable only at the high frequency range. Short crested waves resulted in smaller responses in all the motions than that for long crested waves. Hence, it would be appropriate to consider the short crested wave statistics for the optimum design.

Keywords: Diffraction theory, long crested waves, morison equation, short crested waves

INTRODUCTION

The resources in deep and ultra deepwater regions are focused by the oil and gas industry now due to the limited hydrocarbon resources within the onshore and shallow water regions. Deepwater exploration has begun since 1980s with the installation of the third generation semi-submersible in a water depth of 500 m. In deep and ultra deepwater regions, where the water depth is greater than 300 m, floating offshore structures are found to be more cost effective compared to the fixed one. Spar, semi-submersible, tension leg platform and Floating Production Storage and Offloading vessel (FPSO) are commonly installed for the purpose of oil and gas explorations. Spar generally consist of a closed and water tight vertical circular deep draft hull where the structure weight is balanced by buoyancy provided by it. The centre of gravity for it always remains below the centre of buoyancy and that stabilizes the spar against overturning. It is normally held in place by station-keeping mooring line system. Spar concept was developed from classic spars through truss spar to cell spar as shown in Fig. 1. Wave force was found to be the most significant force that affect the dynamic responses of this structure, contribute about 70% of the total environmental force. Hence the computations of the wave force were highlighted in the design of the structures. There are 3 theories used to calculate the wave force for the offshore structures i.e., the Morison equation, Froude Krylov theory and Diffraction theory. Morison equation takes the wave forces as a product of linear summation of both inertia and drag force. This method is applicable for small structures compared to the wave length. Also, Froude Krylov theory is applicable to small structures. In Froude Krylov theory, the wave force is predominant by the inertia force rather than the drag force which is relatively small. Diffraction theory is applicable for structures that are large compared to water wave length (Chakrabarti, 2001). However, in many studies Morison equation was used to compute the wave force for large structures. The reason being is because of the ease use of the Morison equation compared to the Diffraction theory that is more complicated. This is justifiable because for the waves of large frequency, the ratio of size to the wave length will still fall in the Morison regime. Mekha *et al.* (1995, 1996) studied the behaviour of spar in deep water by time domain method. They were using Morison equation to compute the wave force by considering the second order effects and wave kinematics. In the study they as well investigated the effects of neglecting the hydrodynamic force

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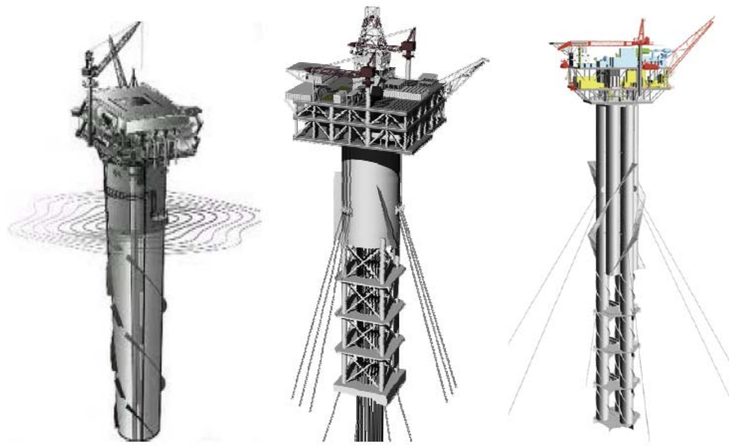

Fig.1: Spar platform generations

on mooring line by modelling it as nonlinear spring. The behaviour of the spar subjected to regular, bichromatic and random was studied. Mansouri and Hadidi (2009) investigated the linear hydrodynamic analysis of truss spar by frequency domain. They estimated the wave force on truss section by modified Morison equation and Diffraction theory on hard tank and determined the dynamic responses due to random waves. Boccotti *et al.* (2012) performed an experimental study to investigate the accuracy of Morison equation for wave force acting on cylinders. The forces computed by using Morison equation were compared with the experimentally measured results. A frequency domain cumulated spectral analysis method was developed by Zheng and Liaw (2004) to estimate the higher order statistic of the linear oscillator responses driven by Morison wave force. Besides the wave force theories, the wave propagation direction is also another issue that affects the dynamic responses of the truss spar. According to the directions of the propagations, waves are categorized as long crested and short crested. Long crested waves are defined as waves propagated from uni-direction, while the short crested waves are defined as the linear summation of various long crested waves propagated from different directions. The short crested waves could be propagated from two directions i.e., the bi-directional waves or from more than two directions i.e., the multi-directional waves. For the realistic wind-generated sea state, the short crested waves would provide a better accuracy compared to the long crested waves. Furthermore, short crested wave has different properties compared to long crested wave (Jian *et al.*, 2008) and the appearances of these waves are three dimensional, complex and short crested (Chakrabarti, 2001). Even though many studies were published focusing on long crested waves, the occurrence of such waves are rare in the real sea. Research focusing on short crested waves has been performed since 1970s. However, the scopes are mainly focused on directional wave force, directional wave spectrum, wave kinematics and vertical circular cylinder. Zhu (1993) presented a solution for the diffraction of short crested wave incident on a circular cylinder. It was shown that the wave loading obtained by using plane incident waves would be overestimated when the incident waves are short crested. A solution in closed form for the velocity of the nonlinear short crested waves being diffracted by vertical cylinder was presented by Zhu and Satravaha (1995). Zhu's theory was extended by Jian *et al*. (2008) to include the effect of a uniform current for different incident angles. They derived an analytical solution for the diffraction of short crested incident wave along positive x-axis direction on a large circular cylinder with current. Aage (1990) discussed the safety of using the reduced 3D loads in practical offshore design. The study proved that wave loads on cylinders in 3D wave are significantly smaller than that for 2D waves with identical spectra or even identical wave elevation time series. The wave induced forces due to short crested waves on vertical cylinders with circular, elliptical and square cross sections were discussed by Zhu and Moule (1994). Huntington and Thompson (1976) extended the linear diffraction theory used for large structures in regular long crested waves to short crested multi-directional random waves. The extended theory was applied to a large surface-piercing cylinder numerically and the results were compared with the experimental measured results.

The effectiveness of using short crested wave statistics for the design of offshore structures has been investigated since the beginning of the research focusing on short crested waves began. Besides studying the accuracy of Morison equation for wave force of spar, in this study, also a numerical comparative study was performed by computing the wave force of a truss spar by diffraction theory subjected to long crested waves and short crested waves to investigate the responses of the structures due to the waves.

NUMERICAL FORMULATIONS

Wave kinematics: In Linear Airy wave theory, the wave height is assumed to be small relative to the wave length or water depth, where the free surface boundary conditions are linearly made by dropping the wave height terms beyond the first order that satisfied the mean water level rather than the oscillating free surface. The theory expressed the first order velocity potential as:

$$\Phi = \frac{gH}{2\omega}\frac{\cosh k(z+d)}{\cosh(kd)}\sin\Theta \quad (1)$$

where,
g = The gravity acceleration
H = The wave height
$\omega = 2\pi/T$ wave frequency
$k = 2\pi/L$ wave number
T = The wave period
L = The wave length

The water particle velocities and accelerations in x and z direction is given as:

$$u = \frac{\pi H}{T}\frac{\cosh k(z+d)}{\sinh(kd)}\cos\Theta \quad (2)$$

$$v = \frac{\pi H}{T}\frac{\sinh k(z+d)}{\sinh(kd)}\sin\Theta \quad (3)$$

$$\frac{\partial u}{\partial t} = \frac{2\pi^2 H}{T^2}\frac{\cosh k(z+d)}{\sinh(kd)}\sin\Theta \quad (4)$$

$$\frac{\partial v}{\partial t} = -\frac{2\pi^2 H}{T^2}\frac{\sinh k(z+d)}{\sinh(kd)}\cos\Theta \quad (5)$$

where, z is the vertical coordinate and $\Theta = kx\text{-}\omega t$.

Morison equation wave forces due to long crested waves: To study the accuracy of the wave force theories, the wave force of truss spar prototype was first calculated by Morison equation, which incorporated the Linear Airy wave theory in the numerical simulation. The velocity components for x and z directions are given as Chakrabarti (2001):

$$u_x = u - C_x(C_x u + C_y v) \quad (6)$$

$$u_z = -C_z(C_z u + C_y v) \quad (7)$$

where, C_x and C_z are the x and z components of the unit vector C that act along the cylinder axis directed up or down.

The surge force is given as:

$$F_x = C_M\rho\frac{\pi}{4}D^2\frac{\partial u}{\partial t} + 0.5C_D\rho D|w|u_x \quad (8)$$

where,
C_M & C_D : The inertia and drag coefficients
ρ : The mass density of water
D : The water density and spar hull diameter

The same was applied to compute the surge force for soft tank, where the D was modified as the soft tank diameter.

The heave force is computed by carrying out a double integration of the dynamic pressure on the bottom surface of the spar hull, b, which is derived from the Bernoulli equation and the potential velocity:

$$F_z = \iint\Big[\rho g\frac{H}{2}\frac{\cosh k(z+d)}{\cosh kd}\cos\Theta + 0.75\rho g\frac{\pi H^2}{L}\frac{1}{\sinh 2kd}\left(\frac{\cosh 2k(z+d)}{\sinh^2 kd} - \frac{1}{3}\right)\cos 2\Theta - 0.25\rho g\frac{\pi H^2}{L}\frac{1}{\sinh 2kd}(\cosh 2k(z+d) - 1)\Big]\partial b \quad (9)$$

Similar expression was implemented to estimate the heave force for heave plates and soft tank. In the cases, b and z will be replaced by the respective surface elevations and vertical coordinates considered.

Wave diffraction forces due to long crested waves: In the second case, the structure is assumed to be a large structure, where the diffraction effect caused by the existence of the structure is taken into account. The spar hull is predominated by inertia force and drag force could be neglected. The surge force is given as Berthelsen (2000):

$$F_{x\,hull} = \frac{2\rho gH}{k}A(kr)\int_{-h}^{0} e^{ky}\cos(\alpha_1 - \omega t) \quad (10)$$

In complex notation, the force becomes:

$$F_{x\,hull} = \frac{2\rho gH}{k^2}A(kr)\left[1 - e^{-kh}\right]e^{i\alpha_1}e^{-i\omega t} \quad (11)$$

where,

$$A(kr) = \frac{1}{\sqrt{J_1'(kr)^2 + Y_1'(kr)^2}} \quad (12)$$

$$\tan \alpha_1 = \frac{J_1'(kr)}{Y_1'(kr)} \quad (13)$$

where,

J'

J'_n : The derivative of the Bessel function of the first kind of order n

H'_n : The derivative of the Hankel function of the first kind of order *n*

The surge force for soft tank was modified as:

$$F_{x\,soft\,tank} = \frac{2\rho g H}{k} A(kr) \int_{stb}^{stt} e^{ky} \cos(\alpha_1 - \omega t) \quad (14)$$

where,

stt : The soft tank top elevation

stb : The soft tank bottom elevation

The heave diffraction force was given as the product of diffraction coefficient *(1-0.5 sin (kR))* and the Froude-Krylov force. Hence the force acting on the bottom of the spar hull could be written as:

$$F_z = \left[1 - \frac{1}{2}\sin(kr)\right] \rho g H \pi r^2 \left(\frac{J_1(kr)}{kr}\right) e^{-kh} \cos(\alpha_2 - \omega t) \quad (15)$$

Or in complex notation as:

$$F_z = \left[1 - \frac{1}{2}\sin(kr)\right] \rho g H \pi r^2 \left(\frac{J_1(kr)}{kr}\right) e^{-kh + i\alpha_2} e^{-i\omega t} \quad (16)$$

The same expression was applied to estimate the heave diffraction force for heave plates and soft tank, where *h* was replaced by the respective surface elevations. The pitch moment was determined by integrating the product of the surge force as in Eq. (11) and the lever arm along the hull axis. The moment for spar hull and soft tank were given as:

$$M_{y\,hull} = \int_{-h}^{0} (z + d) F_{x\,hull} dz \quad (17)$$

$$M_{y\,soft\,tank} = \int_{stb}^{stt} (z + d) F_{x\,soft\,tank} dz \quad (18)$$

Wave diffraction forces due to short crested waves: To consider the real sea conditions, the numerical simulation for wave diffraction forces of the classic spar subjected to short crested waves was developed in the second case. The surge diffraction force of spar hull due to short crested waves was given as Zhu (1993):

$$F_{x\,hull} = \int_{-h}^{0} \frac{dFz}{dz} dz \quad (19)$$

or

$$F_{x\,hull} = \int_{-h}^{0} -2\pi\rho g r H \frac{\cosh k(z+d)}{\cosh kd} e^{-i\omega t} R(k_x, k_y, k, r) dz \quad (20)$$

where,

$$R(k_x, k_y, k, r) = i\left[R_0(k_x, k_y, k, r) + \sum_{n=1}^{\infty} R_n(k_x, k_y, k, r)\right] \quad (21)$$

$$R_0(k_x, k_y, k, r) = J_1(k_x r) J_0(k_y r) - \frac{k_x J_1'(k_x r) J_0(k_y r) + k_x J_1(k_x r) J_0'(k_y r)}{k H_1'(kr)} H_1(kr) \quad (22)$$

$$R_n(k_x, k_y, k, r) = i^{2n}\left\{\left[J_{2n+1}(k_x r) J_{2n}(k_y r) - B_{2n+1} H_1(kr)\right] - \left[J_{2n-1}(k_x r) J_{2n}(k_y r) - B_{2n-1} H_1(kr)\right]\right\} \quad (23)$$

The surge diffraction force for soft tank was given as:

$$F_{x\,soft\,tank} = \int_{stb}^{stt} -2\pi\rho g r H \frac{\cosh k(z+d)}{\cosh kd} e^{-i\omega t} R(k_x, k_y, k, r) dz \quad (24)$$

The heave component was estimated by carrying out a double integration of the dynamic pressure on the bottom surface of the spar hull, *b* that was derived from Bernoulli equation and the potential velocity. The heave diffraction force was given as:

$$F_x = \iint \left[\rho g r \frac{\cosh k(z+d)}{\cosh kd} \cos\theta + \frac{3}{4}\rho g \frac{\pi H^2}{L} \frac{1}{\sinh 2kd} \left(\frac{\cosh 2k(z+d)}{\sinh^2 kd} - \frac{1}{3}\right) \cos 2\theta - \frac{1}{4}\rho g \frac{\pi H^2}{L} \frac{1}{\sinh 2ks} (\cosh 2k(z+d) - 1)\right] \partial b \quad (25)$$

Table 1: Properties of the truss spar

Variable	Model	Prototype
Hull		
Diameter, m	0.300	12.000
Total length, m	0.430	17.200
Draft, m	0.225	9.0000
Wall thickness, m	0.002	0.0800
Truss section		
Diameter, m	0.010	0.4000
Diagonal length, m	0.256	10.240
Nos. diagonal member	24.00	24.000
Vertical length, m	0.143	5.7200
Nos. vertical member	12.00	12.000
Wall thickness, m	0.002	0.0800
Soft tank		
Nos. vertical plate	4.000	4.0000
Length, m	0.300	12.000
Depth, m	0.050	2.0000
Nos. horizontal plate	2.000	2.0000
Length, m	0.300	12.000
Depth, m	0.300	12.000
Wall thickness, m	0.002	0.0800
Total weight, kg	80.889	5176925.169

Table 2: Wave properties

Model			Prototype		
Wave period (s)	Wave frequency (Hz)	Wave height (m)	Wave period (s)	Wave frequency (Hz)	Wave height (m)
0.6	1.67	0.04	3.80	0.0417	1.60
0.7	1.43	0.04	4.40	0.0357	1.60
0.8	1.25	0.05	5.10	0.0313	2.00
0.9	1.11	0.05	5.70	0.0278	2.00
1.0	1.00	0.06	6.30	0.0250	2.40
1.2	0.83	0.06	7.60	0.0208	2.40
1.4	0.71	0.07	8.90	0.0179	2.80
1.6	0.63	0.08	10.1	0.0156	3.20
1.8	0.56	0.08	11.4	0.0139	3.20
2.0	0.50	0.08	12.6	0.0125	3.20

The same expression was employed to estimate the heave diffraction force for heave plates and soft tank, where b and z will be replaced by the respective surface elevations and vertical coordinates, respectively. The total moment about the axis parallel to the y-axis passing through the bottom of the cylinder was given as:

$$M_{y\,hull} = \int_{-h}^{0} (z + d) F_{x\,hull} dz \tag{26}$$

$$M_{y\,soft\,tank} = \int_{stb}^{stt} (z + d) F_{x\,soft\,tank} dz \tag{27}$$

NUMERICAL RESULTS AND DISCUSSION

The dynamic responses of the truss spar prototype as shown in Fig. 2 were determined numerically. The simulations considered the truss spar properties as shown in Table 1 and the generated wave characteristics as shown in Table 2 as the input and the results were compared among the wave force theories and wave directions. The prototype was scaled up from an experimental model with a scale 1:40 by Froude Scaling Law.

Figure 2 to 4 presented the dynamic responses of truss spar prototype in terms of Response Amplitude Operator (RAO) by incorporated the Morison equation and Diffraction theory. In general, it could be observed that the trends of the RAOs for surge, heave and pitch motions agree for both the theories. Morison equation is applicable to structure that is small compared to the wave length; it is justifiable for the high frequency range. It is noticeable that the magnitudes agreed only at the high frequency range. The magnitudes of the RAOs were found significantly varied at the low frequency range, where maximum differences of about 89, 71 and 86% were found for surge, heave and pitch RAOs, respectively.

Figure 5 to 8 show the comparison of surge, heave and pitch RAOs of truss spar by diffraction theory subjected to long crested and short crested waves. The trends of the surge heave and pitch RAOs agreed for both the wave propagated directions. For surge RAO comparison, as shown in Fig. 5, short crested waves generally resulted in smaller responses as compared to the long crested waves. However, for frequencies 0.079 and 0.099 Hz, respectively the short crested waves were found 28 and 92% greater than the long crested. Heave and pitch RAOs comparison are presented in Fig. 6 and 7. The responses due to short crested waves were found much smaller than that for long crested waves,

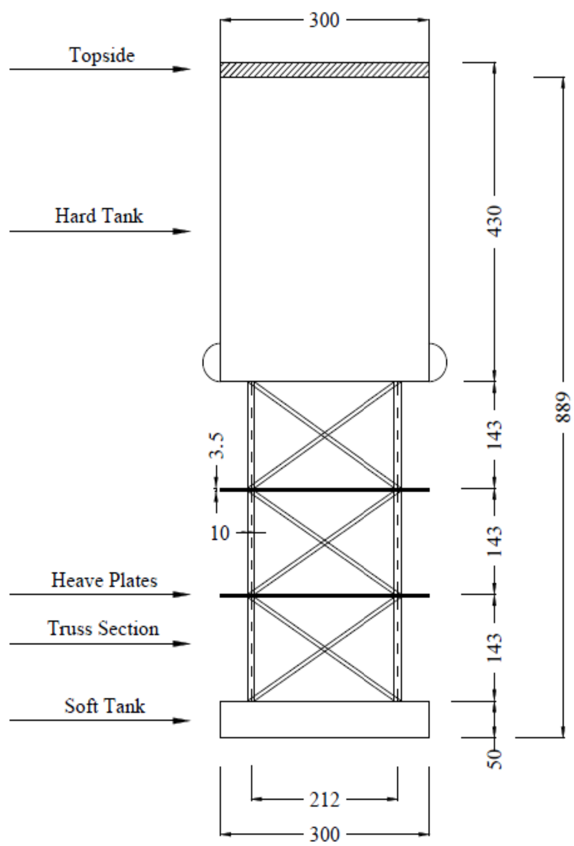

Fig. 2: Truss spar model configurations (in mm)

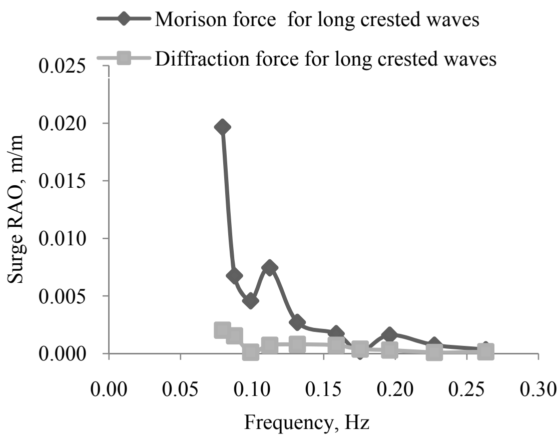

Fig. 3: Surge RAO for wave force theories comparison

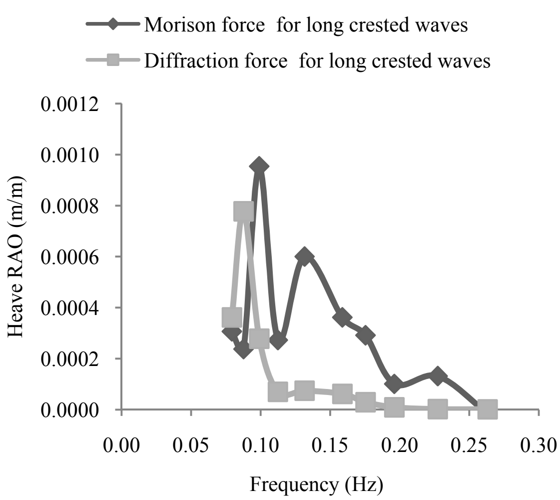

Fig. 4: Heave RAO for wave force theories comparison

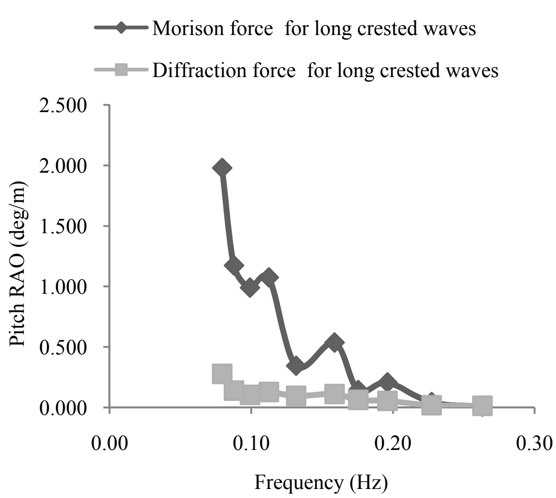

Fig. 5: Pitch RAO for wave force theories comparison

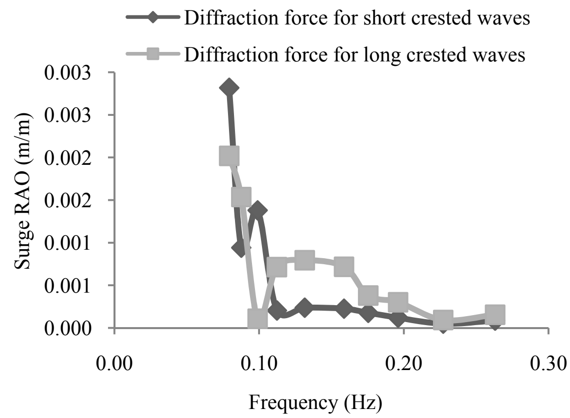

Fig. 6: Surge RAO for wave directions comparison

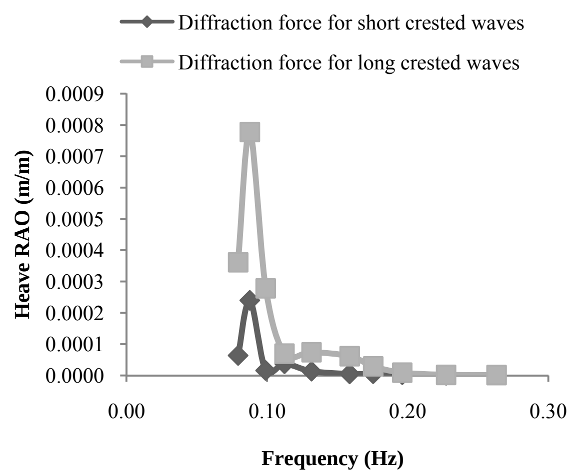

Fig. 7: Heave RAO for wave directions comparison

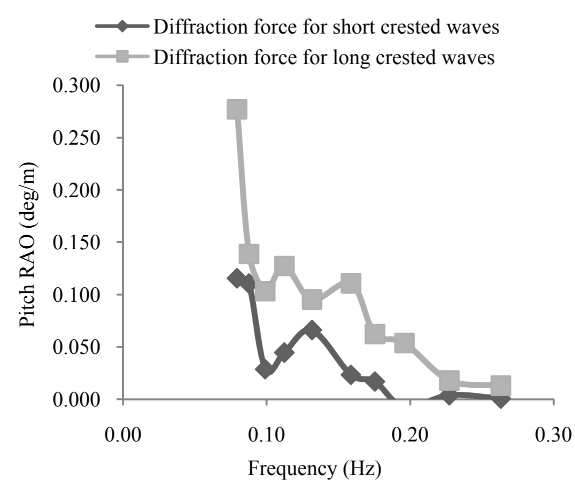

Fig. 8: Pitch RAO for wave directions comparison

where maximum difference of about 69 and 58% were found, respectively.

CONCLUSION

Numerical simulations were performed to investigate the dynamic responses of a truss spar prototype due to wave actions i.e., the wave force theories and the wave propagation directions. From the simulations the following conclusions were drawn:

- **Wave force theories:** To compute the wave force for truss spar, which is large compared to the wave length, Diffraction theory was found to be more appropriate. The Morison equation was found applicable only at the high frequency range. Even though the trend of the responses agreed, the magnitudes significantly varied especially at the low frequency range. Maximum differences of about 89, 71 and 86% were found for surge, heave and pitch RAOs, respectively.
- **Wave propagation directions:** Wave diffraction force subjected to long crested and short crested waves were considered. In general, the short crested waves resulted in smaller responses for all the surge, heave and pitch motions than that for long crested waves. Hence, an optimum design might be developed by considering the short crested waves statistics rather than that for long crested waves in the design of the offshore structures.

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