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An Efficient Novel Compensatory Multi-attribute Control Chart for Correlated Multinomial Processes

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Abstract: Monitoring multi-attribute processes is an important issue in many quality control environments. Almost all the priory proposed control charts utilize equal weights for each Attribute Quality Characteristics (AQCs). In such condition, there is no priority among AQCs. But in real-world, compensatory may exist. Hence due to some applied reasons such as function or efficiency, unequal weights for each AQC are possible. This study proposed a novel efficient control chart for simultaneous monitoring of weighted AQC when data expressed by linguistic terms. Correspondingly a new procedure to interpret out-of-control signals is presented. Performance and comparison advantage of the proposed control chart is measured in terms of Average Run Length (ARL) using a real case which priory was expressed. Consequences displayed that considering weight could efficiently extend the prior research for practical circumstancese.

Keywords: Average Run Length (ARL), multi-attribute control chart, Statistical Process Control (SPC), weighted quality characteristics

INTRODUCTION

Statistical Process Control (SPC) is an outstanding process monitoring tools, which quantitatively can be used to measure the quality variables. Nowadays SPC is powerful tool for process monitoring and continuous quality improvement. It is a set of several analytical tools on which the control chart as a graphical display on process stability over time is the most important one. Control charts have been commonly used to monitor process stability and capability (Engin *et al*., 2008). Control charts to monitor and detect shifts in a process generally designed based on the nature of data gathered for quantifying one or several quality-related characteristics of the product or service. If quality characteristics are measurable on a continuous scale, then variable or multivariate control charts are used. In the case that quality-related characteristics cannot be easily represented in numerical form or articulated by a discrete scale then attribute or multi attribute control charts are worthwhile.

Although much research has been done on monitoring multivariate processes (Montgomery, 2003; Niaki and Abbasi, 2005), little work has been done to address monitoring multi-attribute processes. An attribute quality characteristic may be measured using discrete levels in some situations. For example, a glass vessel quality may be expressed into one of the three groups called “conforming”, “marginal” or “nonconforming”, depends on its bursting-strength and surface-finish defects. The problem of SPC with some discrete-level classification schemes rather than a variables or an attributes measure have been discussed by several researchers. The early works for time dependent and time independent large samples, proposed by Patel (1973) based on a Hotelling T^2 control chart to monitor multi-binomial or multi-poisson process. Discrete-level classification schemes rather than a variable or an attributes measure later have been discussed by Marcucci (1985) and Shapiro and Zahedi (1990). Based on the Duncan (1950) method on obtaining approximate percentage points in term of the Chi square distribution delivered a multinomial control chart to monitor AQCs. Through a series of articles and by using fuzzy set theory, linguistic terms came to construct individual attribute control charts. In order to analyze the ambiguity of linguistic terms fuzzy set theory applied in these research studies (Raz and Wang, 1988, 1990a, b; Ali, 2011).

Lu *et al*. (1998) proposed a way to statistically design of Mnp chart as a multi-attribute Shewhart control chart based on an X statistic. Latter Jolayemi (2000) developed an optimal economical design for uncorrelated multi-attribute control charts when multiple assignable causes exist. Larpkiattaworn (2003) by using neural network proposed a bivariate Binomial control chart for the case of two-attribute using some assumptions such as positive correlation and existence of large enough sample size.

Cassady and Nachlas (2006) proposed a three-level classification scheme which classifies the quality of a product into one of the three categories called

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"conforming", "marginal" or "nonconforming". The problems of constructing acceptance sampling plans and Shewhart control charts based on the three-level classification scheme have been discussed by Cassady and Nachlas (2006).

Gadre and Rattihalli (2008) used a MP-test for multinomial distributed processes to determine any changes on the parameters value of the underlying distribution. Niaki and Abbasi (2006) offered a novel way to control multi-attribute processes based on the removing of the correlation among variables. Later they proposed a multi attribute control chart for automated high yield manufacturing (high-quality process) based on generalized Poisson distribution (Niaki and Abbasi, 2007a). Because of exist modern data-acquisition equipment and on-line computers; it is now common practice to monitor several QCs simultaneously rather than a single one.

Devoid of any discussion on the out of control ARL, Lu *et al*. (1998) proposed a multi-attribute np-control chart when correlation is significant. In their model a product unit can be expressed only as either conforming or nonconforming by each of the monitored AQCs. In the case of independence of attributes, Jolayemi (2000) delivered an optimal method for economical design of multi-attribute control chart for processes with multiple assignable causes.

In many researches AQCs are commonly represented by binary random variables which express conforming or non-conforming parts. This manner generally may not be appropriate, because quality characteristic of product does not change suddenly from conforming to non-conforming or vice versa. To express AQCs using linguistic variables may compensate this weakness. Fuzzy set theory may be an appropriate alternative to analyze the ambiguity of linguistic terms. Taleb and Limam (2002) proposed a fuzzy probabilistic control chart for each AQS expressed via lingual terms in more than two categories. Yu *et al*. (2003) used of linguistic variables for expressing AQCs and a Sequential Probability Ratio Test (SPRT) scheme constructed based on the estimated probability function of the linguistic data. A set of inspection method using multi-attribute control charts to identify process deterioration introduced by Gadre and Rattihalli (2008).They used MP-test to determine a change on the parameters value of the underlying distribution when multinomial distribution for multi attribute processes exists as a necessary assumption.

Taleb *et al*. (2006) used a bootstrap re-sampling method to estimate the empirical distribution of the plotted statistics derived after applying transformation on fuzzy observations. They also extended another approaches to deal with multinomial AQCs based on a linear combination of a Chi-square statistics which its distribution was approximated using Satterthwaite's method. Their approach presented for multi-attributes processes when products are classified by each AQC into more than two categories.

By using bootstrap approach, Niaki and Abbasi (2007b) developed a methodology to design multi-Poisson control chart. They also calculated the average run length by using simulation method and also compared their bootstrap method with the T^2 control chart for attributes. Interested person could refer to El-Shehawy (2008), Zadkarami (2008), Abdullah and Green (2011) and Midi and Zamzuri (2010) for details on bootstrap methods in re-sampling.

On a comprehensive other sense Mukhopadhyay (2008) proposed a multi-attribute control chart using the Mahalanobis D^2statistic. Taleb (2009) based on probability and fuzzy set theories proposed two procedures to design multi-AQCs when data gathered in a few linguistic categories. He presented the performance of his control charts based on some shifts on the categories. Recently a control chart for auto-correlated attributes presented by using the modified Elman neural network capabilities (Niaki, 2010). He applied simulated annealing as an alternative training technique instead of back propagation. He also compared the performance of the proposed method with the other control methods of multi-attribute processes.

A new method based on Statistical Cumulative Sum (CUSUM) control chart to detect small drifts on multi-binomial and multi-Poisson control charts presented by Yanting and Fugee (2012). They also investigate their control chart performance by using simulation method.

In all the mentioned research activities each AQC has equal importance. It is necessary to note that in many practical situations process controller may meet situation in which a product has different unequally weighted quality characteristics. Hence they may be on the view of customer that high importance of a quality characteristic (i.e., due to its performance or function) relative to one quality characteristic can at least partially compensate for low performance relative to another one. In decision making, methods that incorporate tradeoffs between high and low performance into the analysis are termed "compensatory" (Kahraman, 2008).

In the present research, designing a multi-attribute control chart subject to different categorical quality characteristic is on the main focus. Hence by applying a weight vector for AQCs, we proposed a novel compensatory multi-attribute control chart to retain preferences. Finally efficiency of the proposed method compared with Taleb *et al*. (2006) control chart in terms of out of control average run length.

MATERIALS AND METHODS

Let k denotes the number of AQCs in a multi-attribute process, where each pth QC; $(p = 1, \dots, k)$ has l_p linguistic variable. Also let $\pi_p = \left(\pi_1, \dots, \pi_j, \dots, \pi_{l_p}\right)$

denotes the probability that a randomly selected product units has linguistic variable of j for the pth QC, here $(j = 1, \dots, l_p)$. We assume that $X_p = \left(X_1, \dots, X_{l_p}\right)$ has a multinomial distribution with probability of π_p. If such probability (π_p) is unknown for all j, a common statistical task is on homogeneity testing of proportions between the base period and each monitoring period (Duncan, 1950). Hence the agreement between the observed and hypothetical distribution for the pth AQC can be tested by the test statistic expressed in Eq. (1):

$$Z_{p,i}^2 = n_i n_o \sum_{j=1}^{l_p} \frac{\left(p_{ij} - p_{oj}\right)^2}{X_{ij} + X_{oj}} \quad (1)$$

where, X_{ij} and X_{oj} denote the number of observations of linguistic variable j in the period i and o (base period), respectively. Sample size used the periods i and o respectively denotes by n_i and n_o. Also $p_{ij} = \frac{X_{ij}}{n_i}$ and $p_{oj} = \frac{X_{oj}}{n_o}$ are expected proportion linguistic variable j in the period i and o, correspondingly. Here we defined S_i in Eq. (2) as a weighted statistic which expresses the relative importance of all k AQCs during the monitoring period of i:

$$S_i = \sum_{p=1}^{k} W_p Z_{p,i}^2 \quad (2)$$

In the test statistic W_p shows the relative weights of pth AQC. Deriving such test statistic distribution is not an easy task due to their unequally weights. So, based on the following steps, a re-sampling percentile bootstrap method proposed to estimate upper control limit for the S_i statistic. Efron and Tibshirani (1986) indicated that a rough minimum of 1000 bootstrap samples is usually sufficient to compute reasonably accurate confidence interval estimates:

Step 1: Set α; a false alarm rate, W_p; the quality characteristic weights for $p = 1, \dots, k$ on the desired values.

Step 2: By using any efficient multinomial random number generators, generate more than 1000 values of X_{ij} based on its probability distributions in base period.

Step 3: Compute $Z_{p,i}^2; (p = 1, \dots, k)$ values from Eq. (1) and S_i from Eq. (2) by means of the generated values for $Z_{p,i}^2$; $p = 1, \dots, k$ and values of W_p; $p = 1, \dots, k$.

Step 4: Sort computed values for the S_i in ascending order.

Step 5: The upper control limit for the S_i statistic could be estimated by percentile bootstrap confidence interval which is the value on which $(1 - \alpha)\%$ of the computed values is less than it.

Interpretation of the out-of-control signal is the most important step in multivariate quality control charts. When the control chart detects any out-of-control signal, then it is necessary to find which variable is responsible for that shift. In order to distinguish contribution of the pth AQC in the process shift, Eq. (3) proposed:

$$C^{(p)} = \frac{W_p Z_{p,i}^2}{S_i} \quad (3)$$

If the shift happens, multivariate control chart should be able to detect the out-of-control state after a minimum number of samples taken that is average run length; ARL must be the smallest. Applying the following proposed method could estimate the out of control ARL for the proposed control chart:

Step 1: Generate large samples; say 10,000 based on the shifted proportions and compute values of S_i for each sample.

Step 2: Find proportion of samples plotting outside the correspondent multi attribute control chart, this proportion is denoted by p.

Step 3: Estimate the amount of out of control average run length by $ARL = \frac{1}{p}$.

RESULTS OVER AN ILLUSTRATED EXAMPLE

A numerical example is given from Taleb *et al.* (2006) to illustrate the application of the proposed weighted multi attribute control charts and the interpretation of its out-of-control signal. Suppose quality frozen food in a food process industry is expressed via three distinct AQC such as appearance, color and taste of that should be jointly monitored. The food appearance could be classified by an expert by term set 1 of linguistic variables as $\mathrm{T}(\mathrm{c}_1) = \{\mathrm{c}_{11}, \mathrm{c}_{12}, \mathrm{c}_{13}\} = \{\mathrm{good}, \mathrm{medium}, \mathrm{poor}\}$, also the food color categorized by $\mathrm{T}(\mathrm{c}_2) = \{\mathrm{c}_{21}, \mathrm{c}_{22}, \mathrm{c}_{23}\} = \{\mathrm{standard}, \mathrm{acceptable}, \mathrm{rejected}\}$. In addition the taste of a product unit is classified as $\mathrm{T}(\mathrm{c}_3) = \{\mathrm{c}_{31}, \mathrm{c}_{32}, \mathrm{c}_{33}, \mathrm{c}_{34}\} = \{\mathrm{good}, \mathrm{medium}, \mathrm{poor}\}$.

As extension to the referred example suppose all AQCs have unequally weights. Such as that example, Table 1 gives $m = 17$ samples of size $n = 220$.

In our simulation program, we set false alarm rate on $\alpha = 0.05$ and the AQCs weights as $W_1 = 1$, $W_2 = 2$ and $W_3 = 3$. By using percentile bootstrap method, 10,000 values for $Z_p^2; (p = 1, \dots, 3)$ calculated from Eq. (1) and formerly for each re-samples, S_i

Table 1: Frozen food attributes quality levels

k	c_{11}	c_{12}	c_{13}	c_{21}	c_{22}	c_{23}	c_{31}	c_{32}	c_{33}	c_{34}
1	210	7	3	206	9	5	167	48	3	2
2	211	6	3	207	8	5	176	42	2	0
3	206	5	9	202	12	6	163	55	2	0
4	211	5	4	207	8	5	163	51	5	1
5	210	6	4	206	9	5	174	44	1	1
6	208	7	5	204	9	7	174	40	5	1
7	207	7	6	204	9	7	169	46	3	2
8	206	7	7	202	9	9	169	48	2	1
9	203	12	5	200	13	7	167	44	9	0
10	203	9	8	198	11	11	174	42	3	1
11	202	9	9	198	11	11	174	40	6	0
12	209	6	5	207	9	4	172	42	5	1
13	210	3	7	205	5	10	172	44	4	0
14	205	11	4	201	13	6	172	45	2	1
15	210	6	4	206	8	6	169	48	2	1
16	206	10	4	203	13	4	172	46	0	2
17	206	12	2	202	14	4	169	46	5	0

Table 2: Additional sample from the frozen food process

m	c_{11}	c_{12}	c_{13}	c_{21}	c_{22}	c_{23}	c_{31}	c_{32}	c_{33}	c_{34}
18	200	15	5	195	15	10	160	50	5	5

Table 3: Proportion of frozen food quality levels before and after shifts

Condition	c_{11}	c_{12}	c_{13}	c_{21}	c_{22}	c_{23}	c_{31}	c_{32}	c_{33}	c_{34}
In control	0.942	0.035	0.023	0.925	0.045	0.030	0.774	0.206	0.016	0.004
Out of control	0.942	0.035	0.023	0.905	0.065	0.030	0.674	0.306	0.016	0.004

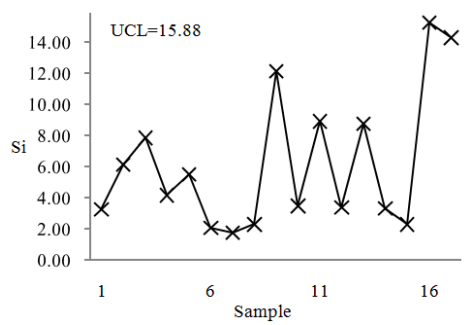

Fig. 1: Multi attribute control chart for frozen food example

Table 4: Performance comparison of control charts in term of ARL

Control charts	ARL
Proposed method	1.24
Taleb *et al*. (2006)	1.44

derived from Eq. (2). Therefore, all S_i ordered ascending from the smallest to the largest. Here the upper control limit is 15.88 on which 95% of the computed S_i values is less than it. The calculated result of control chart is shown on Fig. 1.

It is evident that the designed control chart should be in-control under the first 17 preliminary samples. Performance of the control chart may be measured using the out of control ARL when any drifts occurred on the process parameters. Hence Table 2 demonstrates a typical sample is taken from the existing process when quality engineer judged to be out-of-control. Correspondingly, the hypothesis of the engineer might be examined statistically.

Based on the above out of control sample, the S_i is calculated by Eq. (2) and it is equal to 17.73. The contribution indices from Eq. (3) are derived as 0.13, 0.21 and 0.64, respectively for each AQC. It can be concluded that:

- Process is declared to be out-of-control when the 18th sample is taken ($S_i > 15.88$).
- By computing $C^{(p)}$ for all the three AQC and comparing them, he 3th AQC (taste of a product) has the biggest share on process out of control signal.

The efficiency of the proposed control chart compared with the Taleb *et al*. (2006) on term of out of control ARL. Process shifts are chosen such as proportions of c_{31} decreased by 0.1 and proportions of c_{32} on the other hand, increased by 0.1. Table 3 presents the proportion of each AQC levels before and after shifts.

The results of Table 4 show that the proposed method perform moderately better than the previous when the in control ARL value of both control charts was set as the same. Meanwhile our proposed control chart encompasses the weight vector for all quality characteristics. In special case where all AQC has the same importance, one could set all weights equally on 1.

DISCUSSION

Monitoring multi attribute processes, where correlations between attributes exist, is an imperative issue in statistical quality control. There is a quantity of enriched methods to do such monitoring when all AQC

have equal importance weights. It is important to presents ways to develop control charts when compensatory may exist in real world. Rather than most previously published multi-attribute control charts we focused on quality characteristics which have not necessary equal importance. In some practical cases customer may compensate any deficiency on a quality characteristics by the others. For example for assessing the quality of rice, some customer prefers the rice's smell to its color. Here smell and color may have different weights. Through the present study, we promote an existing multi-attribute control charts by enhancing preference weights to each quality characteristics to overcome lack of customer favorites. Hence we employed the perception of simultaneous confidence intervals to derive control limits for several unequally weighted correlated AQC in which a multinomial distribution is involved. To perform this we utilized a percentile bootstrap method in designing control chart to sustain the most well-known performance of 99.73%. Consequently by using simulation, we compared our comprehensive proposed control chart by the corresponding one previously published by Taleb *et al*. (2006) for his problem at hand in terms of out-of-control average-run-length criterion as a well-known control chart performance measure. We showed that the latter does not explain suitable performance when unequal weights for AQCs are presented. The results of the study showed that the proposed method could act more efficiently when weights of AQCs are significantly different. If weights are equal the proposed method could act as previous studies. Future research studies on the subject of this research may consider processes possessing other statistical distributions and also other aggregating method in calculating product score based on weighted AQCs.

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REFERENCES

Abdullah, M.F.L. and R.J. Green, 2011. Noise analysis adjustable bootstrap trans-impedance and voltage feedback receiver amplifier. J. Appl. Sci., 11: 3775-3782.

Ali, H.K., 2011. Fuzzy controller design of servo system. Asian J. Appl. Sci., 4: 403-413.

Cassady, C.R. and J.A. Nachlas, 2006. Evaluating and implementing 3-level control charts. Qual. Eng., 18(3): 285-292.

Duncan, A.J., 1950. A chi-square chart for controlling a set of percentages. Ind. Qual. Control, 7: 11-15.

Efron, B. and R.J. Tibshirani, 1986. Bootstrap methods for standard errors, confidence intervals and other measures of statistical accuracy. Stat. Sci., 1: 54-77.

El-Shehawy, S.A., 2008. Selection of a NLR-model by re-sampling technique. Asian J. Math. Stat., 1: 109-117.

Engin, O., A. Celik and I. Kaya, 2008. A fuzzy approach to define sample size for attributes control chart in multistage processes: An application in engine valve manufacturing process. Appl. Soft Comput., 8: 1654-1663.

Gadre, M.P. and R.N. Rattihalli, 2008. Multi-attribute unit and group runs control chart to identify the process deterioration. J. Stat. Comput. Simul., 7(11): 999-1008.

Jolayemi, J.K., 2000. An optimal design of multi-attribute control charts for processes subject to a multiplicity of assignable causes. Appl. Math. Comput., 114(2-3): 187-203.

Kahraman, C., 2008. Fuzzy Multi-criteria Decision Making: Theory and Applications with Recent Developments. Springer, New York, Vol. 16.

Larpkiattaworn, S., 2003. A neural network approach for multi-attribute process control with comparison of two current techniques and guidelines for practical use. Ph.D. Thesis, University of Pittsburgh, USA.

Lu, X.S., M. Xie, T.N. Goh and C.D. Lai, 1998. Control chart for multivariate attribute processes. Int. J. Prod. Res., 36: 3477-3489.

Marcucci, M., 1985. Monitoring multinomial processes. J. Qual. Technol., 17(2): 86-91.

Midi, H. and Z.H. Zamzuri, 2010. The performance of bootstrapping autoregressive AR (9) process on the Malaysian opening price for second board. J. Appl. Sci., 10: 2101-2107.

Montgomery, D.C., 2003. Introduction to Statistical Quality Control. 5th Edn., Wiley Publisher, NY, USA.

Mukhopadhyay, A.R., 2008. Multivariate attribute control chart using Mahalanobis D^2 statistic. J. Appl. Stat., 35(4): 421-429.

Niaki, S.T.A., 2010. Artificial neural network in applying multi attribute control chart for AR processes. Proceeding of the 2nd International Conference on Computer and Automation Engineering (ICCAE), 5: 216-220.

Niaki, S.T.A. and B. Abbasi, 2005. Fault diagnosis in multivariate control charts using artificial neural networks. Qual. Reliab. Eng. Int., 21(8): 825-840.

Niaki, S.T.A. and B. Abbasi, 2006. A symmetric rectangular region for multi-attributes C control chart. Proceedings of the 36th International Conference on Computers and Industrial Engineering (ICCIE 2006). Taipei, Taiwan, pp: 3100-3108.

Niaki, S.T.A. and B. Abbasi, 2007a. On the monitoring of multi-attributes high-quality production processes, Metrika, 66: 373-388.

Niaki, S.T.A. and B. Abbasi, 2007b. Bootstrap method approach in designing multi-attribute control charts. Int. J. Adv. Manufact. Technol., 35(56): 434-442.

Patel, H.I., 1973. Quality control methods for multivariate binomial and Poisson distributions. Technometrics, 15: 103-112.

Raz, T. and J. Wang, 1988. Applying fuzzy set theory in the development of quality control charts. Proceedings of the International Industrial Engineering Conference. Orlando, FL, pp: 30-36.

Raz, T. and J. Wang, 1990a. Probabilistic and membership approaches in the construction of control charts for linguistic data. Prod. Plan. Control, 1: 147-157.

Raz, T. and J. Wang, 1990b. On the construction of control charts using linguistic variables. Int. J. Prod. Res., 28(3): 477-487.

Shapiro, S. and H. Zahedi, 1990. Bernoulli trials and discrete distributions. J. Qual. Technol., 22(3): 193-205.

Taleb, H., 2009. Control charts applications for multivariate attribute processes. Comput. Ind. Eng., 56: 399-410.

Taleb, H. and M. Limam, 2002. On fuzzy and probabilistic control charts. Int. J. Prod. Res., 40: 2849-2863.

Taleb, H., M. Limam and K. Hirota, 2006. Multivariate fuzzy multinomial control charts. Qual. Technol. Quant. Manag., 3(4): 437-453

Yanting, L. and T. Fugee, 2012. Multiple attribute control charts with false discovery rate control. Qual. Reliab. Eng. Int., 28(8): 857-871.

Yu, F.J., C.Y. Low and S.S. Cheng, 2003. Design for an SPRT control scheme based on linguistic data. Int. J. Prod. Res., 41(6): 1299-1309.

Zadkarami, M.R., 2008. Bootstrapping: A nonparametric approach to identify the effect of sparsity of data in the binary regression models. J. Appl. Sci., 8: 2991-2997.