Schemes of Repeater Optimizing Distribution based on the MLC Application and CBLRD Simulation

1, 2Qian Qiuye, 1Zhang Yuanbiao, 1Wang Guizhou and 1Chen Ziyue
1Mathematical Modeling Innovation and Practice Basement, Jinan University, Zhuhai Campus, 519070, Guangdong, China
2International Business School, Jinan University, Zhuhai Campus, 519070, Guangdong, China

Abstract: The widely use of repeaters raises concern about their coordination among the public. Since repeaters may suffer interaction and limitation bearing capacity, designing a reasonable repeaters coordination method is of great significance. This study address the problem if repeater coordination in a circular flat area with minimal number of repeaters with seamless coverage theory, system simulation method. With 1,000 users, this study model the coverage, getting the minimal number of repeaters of different coverage radius based on extensive used regular hexagon coverage theory. A numerical example was given in this case. When the number of users increases to 10,000, this study simulate to get the signal density across the area according to the consideration of repeaters and the different distribution of users, which are divided into uniform distribution, linear distribution, normal distribution and lognormal distribution. Then, Multi-Layer Coverage (MLC) and Coverage by Link Rate Density (CBLRD) are created as the distribution scheme on the area where repeat service demand is large. Moreover, for solution on the distribution of the repeaters with barriers, distribution schemes are given considering the transmission of VHF spectrums and the distribution of users around the barrier. Additionally, Spring Comfortable Degree (SCD) is used for evaluation of the results and the developing tends are given to improve the model. Due to the reasonable assumption, the solution of repeater distribution is of pivotal reference value based on the reasonable results.

Keywords: Coverage by Link Rate Density (CBLRD), Multi-Layer Coverage (MLC), repeater optimizing distribution, simulation

INTRODUCTION

Repeaters are widely used in Communication Industry due to the booming development of communication technology. However, there are several of problems remains to be solved owing to the shortcomings of repeaters. It is the task of the top priority to determine the distribution and the minimal number of repeaters.

In fact, quantities of research were done on the distribution of repeaters before. The problem was abstracted to the coverage of an area with circles and the geometric model of how to splice these circles. Tutschku (1997), Natfall (2002), Chi-Fu and Yu-Chee (2005) and Wang et al. (2006) Regular hexagon proves to be the best through all the no-gap coverage schemes, due to smaller overlapping area and less demand of repeaters on research compared with other schemes. Shi-Jun and Zhao-Hui (2010) and Chungen et al. (2011) Distribution of repeaters in a specific area was figured out with simulated annealing algorithm on Wei (2011).

BACKGROUND

As shortcomings of the VHF spectrum due to the line-of-sight transmission and reception could be offset by repeaters, which can pick up weak singles, amplify them and retransmit them on a different frequency. On the contrary, it is the weakness of repeaters that they would interfere with one another except enough geographical distance, sufficiently separated frequency or CTCSS used in transmission.

Under the condition that the available spectrum varies from 145 to 148 MHz with 54 PL available and the difference of receiving and transmission of a due
repeater is larger than 600 kHz, this study figure out the minimal number of repeaters offering repeat service to 1000 users within a 40 m circular area. Moreover, the change of the solution is found considering more users after that. Additionally, this study will make comments on the distribution of repeaters in specific terrain, such as mountains where line-of-sight propagation would suffer obstruction.

Assumption:

- **Omni-directional antenna is used in all repeaters, leading to the circular coverage region:** Antenna should be divided into Omni-directional antenna and directional antenna. The coverage of directional antenna presents to be an irregular shape due to the transition within a specific angle in horizontal direction. However, Signals are transmitted evenly in horizontal direction by Omni-directional antenna, which results in circular coverage of repeaters. As a result, we assumed that all antenna used is Omni-directional for an effective solution considering coverage on circle flat area.

- **Coverage of repeaters mainly depends on emission power:** According to the theory of electromagnetic wave propagation under the condition that the repeater is of the same height, relation between emission power and coverage is:

  \[ P_R(r_c) = P_T G_T G_R \left( \frac{\lambda}{4\pi r_c} \right)^n \]

  where,
  - \( P_R \) = Received power
  - \( \lambda \) = The wavelength of the laser
  - \( G_T \& G_R \) = The transmission gain of sending and receiving antennas
  - \( n \) = The coefficient of channel fading

  Owing to the approximate physical properties of radio wave, whose frequency varies from 145 to 148 MHz, coverage of repeaters is assumed to be the same whether transmission frequency is considered. Meanwhile, we assume that the transmission gain of antennas of each repeater is the same. To conclude, emission power determines the coverage of repeaters

- **Terrain and weather rarely affect the transition of signals:** In fact, these two factors do have effects, but in order to simplify the model, we ignore the interactions between factors.

### PRIMARY MODEL

**Interfere and ultimate number of repeat service:** There are three ways to prevent interferer of repeaters with VHF spectrum, which, to be specific, are enough geographical distance, sufficiently separated frequencies for transmission and the ‘Continuous Tone-Coded Squelch System’ (CTCSS).

First, interferer between adjacent repeaters cannot be prevented merely by geographical distance, since no-gap coverage is required for the 40-mile-radius circular area with circular coverage of repeaters.

Moreover, interferer can be avoided so long as the difference between transmission frequencies of two adjacent repeaters is larger than 25 kHz according to the technical parameters on FCC. With different frequencies, the ultimate number of repeat service of a due repeater is:

\[ p = \frac{3MHz}{25kHz} = 120 \]

Additionally, 54 different PL tones in CTCSS offer solution of interferer, providing more repeat service. Combining with the method above using different frequencies, the ultimate number of repeated service offered by a repeater is 6480, larger than 1000, the number of users. Thus the number of repeat service is enough under this condition.

**Coverage of repeaters:** According to the networking mechanism (Shi-Jun and Zhao-Hui, 2010), regular polygons, namely equilateral triangle, square and regular hexagon, are used for no-gap coverage on flat area, as is shown on Fig. 1.

Each kind of shape represents the coverage of a repeater. Covering the same area, the more shapes means the more repeaters required. In order to minimize the number of repeaters, regular hexagon is the best choice.

In fact, the coverage with regular hexagon is widely used in the field of region communication network, which is the same as what we assume.

**Number and distribution of repeaters:** We build geometrical model according to the research (Shi-Jun and Zhao-Hui, 2010) and figure out the minimal number of regular hexagons for the complete coverage of the flat area. The model could be described as the following equation where \( n \) the minimal number of the repeater is, \( S_{area} \) is the area requiring coverage and \( r \) is the radius of the regular hexagons:
In contrast, distribution of repeaters on edge of the area is neglected with the equation, which leads to the result that may not be the minimal number of repeater. We simulate the repeater coverage with MATLAB for a result of higher accuracy. That means the minimal number of repeaters in different conditions where the coverage of repeaters varies are calculated. The coverage could be described as Fig. 2. If the radius of coverage is 5 miles, under this condition, the number of repeaters is 94.

In order to find out the relationship between coverage radius and minimal number of repeaters, we simulate the distribution and the minimal number of repeaters while the radius of coverage varies from 1 to 25 miles. Then we get the result as is shown in Table 1.

Meanwhile, the graph of the relationship could be got as is shown in Fig. 3.

As is shown in the figure, the number of repeater decrease with the increase of the radius of coverage except in the condition where the radius changes from 11 to 15 miles, when the number of repeaters almost stay the same.

Further to the Primary model, repeat service demand may exceed the load of repeaters in some areas when the number of users is up to 10000, leading to insufficient repeat service provided. Furthermore, repeat service demands are affected by distribution of the users as well. Therefore, we make the new scheme to accommodate to the changes.

Multi-Layer Coverage (MLC) and Coverage by Link Rate Density (CBLRD) are adopted for meeting the demands in the area where users are serried. Our results depend on the distribution of users, which are distribution and heterogeneous distributions.

Homogeneous distribution of the users: To ensure the efficiency and quality of the repeat service while facing the boost of users, MLC is used to lighten the load of repeaters as is shown in Fig. 4.

There are 3 transmission ways of users following according to the figure:

- User-Small Repeater-User
- User-small Repeater-Big Repeater-Small Repeater-User
- User-small Repeater-Several Big Repeaters-Small Repeater-User

It is certain that demand of repeater service is stronger in the center of the area than that on edge in spite of the uniform distribution of the users, which lead to a more complex distribution of repeaters on edge. To figure out the number of the repeaters in a specific area owing to the random communication between users, we simulate the communication between users. Then, signal density distribution of users in the area are shown in Fig. 5, with the circle number of 0.1, 1 and 10 million, respectively.

Longitudinal axis represents the demand of repeat service, which is equal with the number of repeaters demand. The demand is stronger in the higher position of longitudinal axis. Comparing with the 3 graphs, demand of repeaters in center area increase with the
The smoother the curve is, the more stable the result of simulation will be. The solution of the high central demand is CBLRD, which means reduce the radius of the repeaters to meet the demand of repeat service since decreasing of radius leads directly to the load of repeaters.

We combine the two schemes, MLC and CBLRD, together, for a more effective solution of the problem, which is shown in Fig. 6.

The problem could be well solved with the combination of the two schemes, since MLC ensure the efficiency while CBLRD ensure the quality of communication.

**Heterogeneous distributions of the users:** In this section, heterogeneous distribution of users is considered result from the fact according to the research of Fu and Han (2008). Simulation is used here for the communication frequency of users when users present to be linear distribution, normal distribution and lognormal distribution. As a result, the distribution scheme of repeaters can be figured out.

**Linear distribution:** The foundation of linear distribution of the users is:

\[ D(r) = a + br \]
Fig. 7: Density of linear distribution

where,
\[ D(r) : \text{The density of users where the distance from the center of the area is } r \]
\[ a : \text{The density of users in the center of the area} \]
\[ b : \text{The attenuation coefficient of users} \]

We define the point with the largest density of users is 20 miles from the center of the area. Therefore, the user’s density distribution and the signal density distribution of the area could be described as Fig. 7.

Normal distribution: As normal distribution is widely used, we simulate to get the user density distribution and signal density distribution under the condition of normal distribution of users as is shown in Fig. 8.

Lognormal distribution: The foundation of lognormal distribution of users is:

\[ D(r) = ae^{br} \]

where,
\[ D(r) : \text{The density of users where the distance from the center of the area is } r \]
\[ a, b : \text{Parameters} \]
Thus the user’s density distribution and the signal density distribution of the area could be described as Fig. 9.

**ANALYSIS OF THE MODEL**

**Comments on specific terrain:** Due to the physical property of VHF spectrum, reflection and diffraction occurs in the area with barrier such as mountains, which result in the abnormal propagation of signals and even cause signal interference. The condition could be described as Fig. 10.

As is shown in the figure, the mountain reflects the signals transmitted by repeaters. Additionally, if there is no user in the some areas, directional antenna is used for communication in this case.

To find a solution of the problem above, repeaters are usually placed on the highland. We make discussions on the 4 conditions in reality and give solutions in each condition.

**Users around the barrier are sporadic:** A big repeater should be placed on the high land, such as the top of the mountain, as is shown in Fig. 11.

The signals transmitted by small repeaters are received by a big repeater, who transmits the signals to small repeaters in other areas, which realize the normal communication around barrier.
Users around barrier are centralized in some specific areas: Small repeaters with directional antenna are recommended in this case, avoiding the signals being overlapped. The mechanism is further described in Fig. 12.

Users are centralized in the Area-A and Area-B in the figure. Placement of repeaters on the highland makes the communication between Area-A and Area-B possible.

Users are centralized on one side of the barrier: Owing to the reflection of VHC spectrum, interference occurs frequently in this case. To reduce the interference, we suggest a small repeater with directional antenna build at the bottom of the barrier, which is shown in Fig. 13.

Moreover, another solution, placing a big repeater on the barrier, is recommended due to a wider-communication area for users, which is described with Fig. 14.

Users are all on the right side of the barrier and comparatively sporadic according to the figure. The placement of the big repeater on the barrier solves the problem, realizing larger communication coverage.

Spring comfortable degree: Under the condition that the coverage of repeaters stays the same, the communication efficiency will suffer if repeaters are too far from each other. In contrast, the too small distance between repeaters leads to interference. Considering the features above, comfortable degree of spring are used here to characterize the best distance between repeaters.

According to Hooke law followed, $k$ is elastic coefficient while $x_0$ is the length of spring in natural condition:
From the equation of Hooke law, we deduce the following equation where $C$ is the comfortable degree, $x_i$ is the distance between the repeaters, $x_0$ is the best distance between two repeaters and $k$ is the coefficient of comfortable degree:

$$F = k(x - x_0)$$

According to the equation above, the distribution of repeaters is more rational if the comfortable degree increases.

**Model improvement:**

**Precision of the CBLRD model:** MLC and CBLRD are mentioned for solving the coverage problem. We take the method of simulation to find the dense area without accurate data as is shown in Fig. 15.

A new method of coverage is offered for a more accurate result. As is shown in Fig. 16, $r$ is the radius of the repeater coverage. Then the area could be figured out:

$$S = \frac{3\sqrt{3}}{2} r^2$$

We assume that only one piece of signals could be transmitted with the same PL tone. $\bar{\rho}$ is the average density of signals in a repeater while $n$ is the largest load of a repeater. Then we get the following equations:

$$\bar{\rho}S = n$$

$$n = \frac{3\sqrt{3}}{2} \frac{\bar{\rho}r^2}{\bar{\rho}}$$

$$r = \sqrt{\frac{2n}{3\sqrt{3} \bar{\rho}}}$$

The number of repeat service cannot reach 6480, the maxim, since the transmission and receiving between repeaters occupy large quantities of spectrum with the influence of weather terrain and error. The relationship between rate density and radius of coordination covers is described in Fig. 17.

The specific number of radius can be calculated to optimize the coverage model.

**Evaluation of the model:**

**Strength:**

- **Assumptions are reasonable:** Based on the wide-using coverage mechanism with regular hexagons,
the results, which are the minimal numbers of repeaters under the different condition of various coverage radiuses, are reasonable and accurate.

- **The consideration of signal density distribution optimizes the results**: As signal density varies from place to place in the circle area, simulation for signal density distribution makes sense.

- **The solution to avoid overload of repeaters is suggested**: Combining with the simulation, MLC and CBLRD are used when the number of users increased to meet the demands in different conditions more objectively.

- **Simulating the distribution of users with different foundations**: We characterize the different distribution of users with linear distribution model, normal distribution model and lognormal distribution model. Then the signal density distributions related to different user density distributions are figured out, ensuring the integrity of the result.

**Weakness**: For simplicity of the problem, we ignore the geographical influence due to little information of geographical environment given.

Data in reality are not available for model validation due to the lack of accurate distribution data of repeaters.

Distribution model of users that we use differs from that in reality, where the distribution of users is irregular and hard to characterize. The use value of the model is not affected. On the contrary, the practicability of the model would be influenced if the irregular distribution of users were considered.

**CONCLUSION**

We solve the problem of repeater number efficiently by simulation based on the repeater distribution and the coverage theory with regular hexagons.

When the number of user is large enough, MLC and CBLRD are recommended as the coverage scheme. The innovation of our study is the proposal of characterizing the communication frequency with signal density. Meanwhile, the discussion is made on different distribution conditions of users, such as uniform distribution, linear distribution, normal distribution etc. The graphs of communication density are made for a visualized description of the density of the signals in the area.

Moreover, comfortable degree is created for the analysis of the model, which is described with the following equation:

$$C = \sum_{i=1}^{n} \frac{1}{k|x_i - x_0|}$$

In addition, the repeater distribution schemes are suggested due to the different distribution of users with barriers in order to make the distribution schemes of repeaters. We give the results with explanation of schematic diagram.

Finally, we make improvement of the model, which calculated the coverage radius of repeaters with different load of repeaters, where the radius is:

$$r = \sqrt[3]{\frac{2n}{3\sqrt{3}\rho}}$$

Which optimize the model of CBLRD accurately.

**REFERENCES**


