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Covariance Intersection Kalman Fuser for Two-sensor System with Time-delayed Measurements

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Abstract: For a two-sensor linear discrete time-invariant stochastic system with time-delayed measurements, by the measurement transformation method, an equivalent system without measurement delays is obtained and then using the Covariance Intersection (CI) fusion method, the covariance intersection steady-state Kalman fuser is presented. It can handle the estimation fusion problem between local estimation errors for the system with unknown cross-covariances and avoid a large computed burden and computational complexity of cross-covariances. It is proved that its accuracy is higher than that of each local estimator and is lower than that of optimal Kalman fuser weighted by matrices with known cross-covariances. A Monte-Carlo simulation example shows the above accuracy relation and indicates that its actual accuracy is close to that of the Kalman fuser weighted by matrices, hence it has good performances.

Keywords: Consistency, covariance intersection fusion, information fusion kalman fuser, time-delayed measurement, unknown cross-covariance

INTRODUCTION

Multisensor information has been applied to many fields, such as Guidance, defense, robotics, tracking, signal processing (Hall and Llinas, 1997; Li *et al*., 2003; Bar-Shalom and Li, 2001). There are two kinds of optimal information fusion methods: the centralized and distributed information fusion, where the distributed weighted fusion method applies widely because of its less computational burden. Even if, to compute the optimal distributed weighted fusion Kalman estimators (Sun and Deng, 2004; Bar-Shalom and Campo, 1986), it is required to compute the cross-covariances among local Kalman estimator errors, which needs a large computational burden, because these cross-covariances are generally unknown (Deng *et al*., 2008) or their computation is very complex (Sun and Deng, 2009; Sun *et al*., 2009; Liggins *et al*., 2009). In order to overcome this limitation, a Covariance Intersection (CI) fusion method was presented and developed in Julier and Uhlmann (1997), Chen *et al*. (2002), Julier and Uhlmann (1996), Julier and Uhlmann (2009) and Arambel *et al*. (2001), which can solve the fused filtering problems with unknown cross-covariances and has the consistency and robustness (Julier and Uhlmann, 1997). It has wide applications (Julier and Uhlmann, 2007; Guo *et al*., 2010; Bolzani *et al*., 2007). Besides that, the systems with time-delayed measurement exist widely in control and estimation fields, which results that it is more difficult to compute the cross-covariances. Now there are augmented state method (Kailath *et al*., 2000) and non-augmented state method (Sun and Deng, 2009; Zhang and Xie, 2007) to convert the system with time delays into that without time delays.

In this study, for the two-sensor system with measurement delays, a measurement transformation approach is presented, by which the fused estimation problem is converted into that for systems without measurement delays (Sun and Deng, 2009; Sun *et al*., 2009; Liggins *et al*., 2009). And, to overcome the difficulty and complexity of computing cross-covariances, using the CI method (Julier and Uhlmann, 1996, 1997, 2009; Chen *et al*., 2002; Guo *et al*., 2010; Bolzani *et al*., 2007), a CI fusion steady-state Kalman fuser is presented, which avoids to find the cross-covariances and can reduce the computational burden. It can handle the system with unknown cross-covariances and it has higher accuracy comparing with local estimators. It is rigorously proved that the accuracy of CI fuser is higher than that of each local filter and is lower than that of optimal fuser weighted by matrices. The CI fuser is consistent and the good performance.

PROBLEM FORMULATION

Consider the two-sensor linear discrete time-invariant stochastic system:

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$$x(t+1) = \Phi x(t) + \Gamma w(t) \tag{1}$$

$$z_i(t) = H_i x(t-\tau_i) + \xi_i(t),\ i = 1,2 \tag{2}$$

where, t is the discrete time, τ_i is the measurement delay, $x(t) \epsilon R^n$ is the state, $z_i(t) \epsilon R^{m_i}$ is the measurement, w(t) ϵR^r and $\xi_i \epsilon(t) R^{m_i}$ are uncorrelated white noises with zero mean and variances Q_w and $Q_{\xi i}$, respectively.

The objectives are to find the local steady-state optimal Kalman estimators $\hat{x}_i^z(t|t+N)$, (N<0, N = 0, N>0), i = 1, 2, the Kalman fusers $\hat{x}_m^z(t|t+N$, *xdztt+N, xsztt+N* weighted by matrices, diagonal matrices and scalars, the CI fusion Kalman fuser $\hat{x}_{CI}^z(t|t+N)$ and compare their accuracy relations.

THE STEADY-STATE OPTIMAL WEIGHTED KALMAN FUSER

Introducing the new measurement $y_i(t)$ and measurement noise $v_i(t)$:

$$y_i(t) = z_i(t+\tau_i) \tag{3}$$

$$v_i(t) = \xi_i(t+\tau_i) \tag{4}$$

From (2), we have the new measurement equation without measurement delays.

$$y_i(t) = H_i x(t) + v_i(t),\ i = 1,2 \tag{5}$$

where, the new measurement noise $v_i(t)$ is white noise with zero mean and variance $Q_{vi} = Q_{\zeta i}$.

Denoting the linear space spanned by the stochastic variables (z_i (*t*+*N*), z_i (*t*+*N*-1), …) as *L* (z_i (*t*+*N*), z_i (*t*+*N*-1), …) and the linear space spanned by the stochastic variables $(y_i(t+N-\tau_i), y_i(t+N-\tau_i-1), \cdots)$ as $L(y_i(t+N-\tau_i), y_i(t+N-\tau_i-1), \cdots)$, we have the relation between the two linear measurement spaces as:

$$\begin{aligned} &L(z_i(t+N), z_i(t+N-1), \cdots) \\ &= L(y_i(t+N-\tau_i), y_i(t+N-\tau_i-1), \cdots) \end{aligned} \tag{6}$$

So we have the relation as:

$$\hat{x}_i^z(t \mid t+N) = x_i(t \mid t+N-\tau_i) \tag{7}$$

where $\hat{x}_i(t|t+N-\tau_i)$ is the linear minimum variance filters of x(t) based on (y_i (*t*+*N*-τ_i), y_i (*t*+*N*-1-τ_i), …) and $\hat{x}_i^z(t|t+N)$ is the linear minimum variance filters based on *L*(z_i (*t*+*N*), z_i (*t*+*N*-1), …). So the relation between the estimation errors $\hat{x}_i^z(t|t+N)$ and $\hat{x}_i(t|t+N$ is:

$$\tilde{x}_i^z(t \mid t+N) = \tilde{x}_i(t \mid t+N-\tau_i),\ i = 1,2 \tag{8}$$

where, $\hat{x}_i^z(t|t+N) = x(t) - \hat{x}_i^z(t|t+N)$, $\hat{x}_i(t|t+N) = x(t) - \hat{x}_i(t|t+N)$.

Defining the steady-state estimator error variances and cross-covariances are $p_{ii}^z(N) = E[\tilde{x}_i^z(t|t+N)\tilde{x}_i^{zT}(t|t+N)]$ and $p_{ii}^z(N) = E[\tilde{x}_i^z(t|t+N)\tilde{x}_i^{zT}(t|t+N)]$, respectively. So we have obtained

$$P_{ij}^z(N) = P_{ij}(N-\tau_i, N-\tau_j),\ i = 1,2 \tag{9}$$

where, P_{ij} (*N*-τ_i, *N*-τ_j) = E[$\hat{x}_i(t|t+N-\tau_i)\tilde{x}_i^T(t|t+N-\tau_i$

From (7) we can see that the problem of finding the local steady-state optimal Kalman estimators $\tilde{x}_i^z(t|t+N$ based on the measurement (z_i (*t*+*N*), z_i (*t*+*N*-1), …) is converted t that of finding the optimal steady-state Kalman estimators $\hat{x}_i(t|t+N-\tau_i)$ based on the measurement (y_i (*t*+*N*-τ_i), y_i (*t*+*N*-τ_i-1), …), For $N > \tau_i$, $N = \tau_i$, $N > \tau_i$, the estimators are called the smoothers, filters and predictors, respectively.

Lemma 1: (Sun and Deng, 2009) For the two-sensor system (1) and (5), the local steady-state Kalman predictor $\hat{x}_i(t+1|t)$ is given by:

$$\hat{x}_i(t+1 \mid t) = \Psi_{pi} x_i(t \mid t-1) + K_{pi} y_i(t),\ i = 1,2 \tag{10}$$

where,

$$\Psi_{pi} = \Phi - K_{pi} H_i \tag{11}$$

$$K_{pi} = \Phi \Sigma_i H_i^{\mathrm{T}} (H_i \Sigma_i H_i^{\mathrm{T}} + Q_{vi})^{-1} \tag{12}$$

The prediction error variance matrix Σ_i satisfies the steady-state Riccati equation:

$$\Sigma_i = \Phi[\Sigma_i - \Sigma_i H_i^{\mathrm{T}} (H_i \Sigma_i H_i^{\mathrm{T}} + Q_{vi})^{-1} H_i \Sigma_i] \Phi^{\mathrm{T}} + \Gamma Q_w \Gamma^{\mathrm{T}} \tag{13}$$

and the cross-covariance matrix $\sum_{ij} = E[\hat{x}_i(t+$ *1txiTt+1t* between local prediction errors satisfies the Lyapunov equation:

$$\Sigma_{ij} = \Psi_{pi} \Sigma_{ij} \Psi_{pj}^{\mathrm{T}} + \Gamma Q_w \Gamma^{\mathrm{T}},\ i, j = 1,2,\ i \neq j \tag{14}$$

The -k_i steps steady-state Kalman predictor $\hat{x}_i(t|t+k_i)$ is given by:

$$\hat{x}_i(t \mid t+k_i) = \Phi^{-k_i-1} x_i(t+k_i+1 \mid t+k_i),\ k_i \leq -2 \tag{15}$$

The -k_i steps Kalman prediction error variance $P_i(k_i, k_i) = E[\tilde{x}_i(t|t+k_i)\tilde{x}_i^T(t|t+k_i)]$ and cross-covariance P_{ij} (k_i, k_j) = E$\left[\hat{x}_i(t|t+k_i)\tilde{x}_i^T\left(t|t|t+k_j\right)\right]$ are given by:

$$\begin{aligned} &P_i(k_i, k_i) = \Phi^{-k_i-1} \Sigma_i \Phi^{(-k_i-1)\mathrm{T}} \\ &+ \sum_{j=2}^{-k_i} \Phi^{-k_i-j} \Gamma Q_w \Gamma^{\mathrm{T}} \Phi^{(-k_i-j)\mathrm{T}},\ k_i \leq -2 \end{aligned} \tag{16}$$

$$P_{ij}(k_i,k_j)=\Phi^{-k_i-1}\Sigma_{ij}\Phi^{(-k_i-1)\mathrm{T}}+\sum_{j=0}^{-\max(k_i,k_j)-2}\Phi^{j}\Gamma Q_w\Gamma^{\mathrm{T}}\Phi^{j\mathrm{T}},\ k_i,k_j\le -2 \tag{17}$$

The steady-state Kalman filter $\hat{x}_i(t|t)$ is as following:

$$\hat{\tilde{x}}_i(t\mid t)=\Psi_{fi}x_i(t-1\mid t-1)+K_{fi}y_i(t),\ i=1,2 \tag{18}$$

where,

$$\Psi_{fi}=(I_n-K_{fi}H_i)\Phi \tag{19}$$

$$K_{fi}=\Sigma_i H_i^{\mathrm{T}}(H_i\Sigma_i H_i^{\mathrm{T}}+Q_{vi})^{-1} \tag{20}$$

The corresponding steady-state Kalman filtering error variance $P_{ii}=E[\tilde{x}_i(t|t)\tilde{x}_i^T(t|t)]$ and cross-covariance $P_{ij}=E[\tilde{x}_i(t|t)\tilde{x}_i^T(t|t)]$ are given by:

$$P_{ii}=(I_n-K_{fi}H_i)\Sigma_i \tag{21}$$

$$P_{ij}=\Psi_{fi}P_{ij}\Psi_{fj}^{\mathrm{T}}+(I_n-K_{fi}H_i)\Gamma Q_w\Gamma^{\mathrm{T}}(I_n-K_{fj}H_j)^{\mathrm{T}}\quad i,j=1,2,\ i\ne j \tag{22}$$

The steady-state Kalman smoother $\hat{x}_i(t|t+k_i)$, (k_i>0) is given by:

$$\hat{\tilde{x}}_i(t\mid t+k_i)=x_i(t\mid t-1)+\sum_{r=0}^{k_i}K_i(r)\varepsilon_i(t+r)\quad i=1,2 \tag{23}$$

where,

$$K_i(r)=\Sigma_i\Psi_{pi}^{\mathrm{T}r}H_i^{\mathrm{T}}(H_i\Sigma_i H_i^{\mathrm{T}}+Q_{vi})^{-1} \tag{24}$$

$$\varepsilon_i(t)=y_i(t)-H_i\hat{x}_i(t\mid t-1) \tag{25}$$

The Kalman smoothing error variance $P_i(k_i,k_i)=E[\hat{x}_i(t|t+k_i)\tilde{x}_i^T(t|t+k_i)]$ and cross-covariance $P_{ij}(k_i,k_j)=E[\hat{x}_i(t|t+k_i)\tilde{x}_i^T(t|t+k_j)]$ are given by:

$$P_i(k_i,k_i)=\Sigma_i-\sum_{r=0}^{k_i}K_i(r)(H\Sigma_i H^{\mathrm{T}}+Q_{vi})K_i^{\mathrm{T}}(r),\ k_i>0 \tag{26}$$

$$P_{ij}(k_i,k_j)=\Sigma_{ij}-\sum_{r=0}^{\min(k_i,k_j)}K_i(r)H\Sigma_{ij}H^{\mathrm{T}}K_j^{\mathrm{T}}(r),\ k_i,k_j>0 \tag{27}$$

Lemma 2: (Sun and Deng, 2009) For the two-sensor system (1) and (5), the steady-state Kalman filtering error variance $P_{ij}(k_i,k_i)=E[\hat{x}_i(t|t+k_i)\tilde{x}_i^T(t|t+k_i)]$ and cross-covariance $P_{ij}(k_i,k_j)=E[\hat{x}_i(t|t+kjxiTtt+ki,(k_i=N-\tau_i,k_j=N-\tau_i)$ are given by:

Case 1: when $k_i=0$, $k_j=0$

$$P_i(k_i,k_i)=P_{ii} \tag{28}$$

$$P_{ij}(k_i,k_j)=P_{ij} \tag{29}$$

Case 2: when k_i<0, k_j<0

$$P_i(k_i,k_i)=\Sigma_i,(k_i=-1) \tag{30}$$

$$P_{ij}(k_i,k_j)=\Sigma_{ij},(k_i=-1,k_j=-1) \tag{31}$$

$$P_i(k_i,k_i)=\Phi^{-k_i-1}\Sigma_i\Phi^{(-k_i-1)\mathrm{T}}+\sum_{j=0}^{-k_i-2}\Phi^{j}\Gamma Q_w\Gamma^{\mathrm{T}}\Phi^{j\mathrm{T}},\quad (k_i\le -2) \tag{32}$$

$$\begin{aligned}P_{ij}(k_i,k_j)&=\Phi^{-k_i-1}\Psi_{pi}^{k_i-k_j}\Sigma_{ij}\Phi^{(-k_j-1)\mathrm{T}}\\&+\sum_{r=-k_i-1}^{-k_j-2}\Phi^{-k_i-1}\Psi_{pi}^{k_i+r+1}\Gamma Q_w\Gamma^{\mathrm{T}}\Phi^{r\mathrm{T}}+\\&\sum_{r=0}^{-k_j-2}\Phi^{r}\Gamma Q_w\Gamma^{\mathrm{T}}\Phi^{r\mathrm{T}},(k_j\le k_i\le -2)\end{aligned} \tag{33}$$

$$\begin{aligned}P_{ij}(k_i,k_j)&=\Phi^{-k_i-1}\Sigma_{ij}\Psi_{pi}^{(k_j-k_i)\mathrm{T}}\Phi^{(-k_j-1)\mathrm{T}}\\&+\sum_{r=-k_j-1}^{-k_i-2}\Phi^{r}\Gamma Q_w\Gamma^{\mathrm{T}}\Psi_{pj}^{k_j+r+1}\Phi^{(-1-k_j)\mathrm{T}}\\&+\sum_{r=-k_j-1}^{-k_i-2}\Phi^{r}\Gamma Q_w\Gamma^{\mathrm{T}}\Psi_{pj}^{k_j+r+1}\Phi^{(-1-k_j)\mathrm{T}}+\\&\sum_{r=0}^{-k_j-2}\Phi^{r}\Gamma Q_w\Gamma^{\mathrm{T}}\Phi^{r\mathrm{T}},(k_i\le k_j\le -2)\end{aligned} \tag{34}$$

$$\begin{aligned}P_{ij}(k_i,k_j)&=\Psi_{pi}^{-k_j-1}\Sigma_{ij}\Phi^{(-k_j-1)\mathrm{T}}\\&+\sum_{r=0}^{-k_j-2}\Psi_{pi}^{r}\Gamma Q_w\Gamma^{\mathrm{T}}\Phi^{r\mathrm{T}},(k_i=-1,k_j\le -2)\end{aligned} \tag{35}$$

$$\begin{aligned}P_{ij}(k_i,k_j)&=\Phi^{-k_i-1}\Sigma_{ij}\Psi_{pi}^{(-k_i-1)\mathrm{T}}+\\&\sum_{r=0}^{-k_i-2}\Phi^{r}\Gamma Q_w\Gamma^{\mathrm{T}}\Psi_{pj}^{r\mathrm{T}},(k_i\le -2,k_j=-1)\end{aligned} \tag{36}$$

Case 3: when k_i>0, k_j>0

$$P_{ii}(k_i,k_i)=\Sigma_i-\sum_{r=0}^{k_i}K_i(r)[H_i\Sigma_i H_i^{\mathrm{T}}+Q_{vi}]K_i^{\mathrm{T}}(r),(k_i>0) \tag{37}$$

$$\begin{aligned}P_{ij}(k_i,k_j)&=\sum_{r=0}^{k_i}[I_n\delta_{r0}-K_i(r)H_i\Psi_{pi}^{r}]\Sigma_{ij}\sum_{s=0}^{k_j}[I_n\delta_{s0}-K_j(s)Hj\Psi_{pj}^{s}]^{\mathrm{T}}\\&+\sum_{r=0}^{\min(k_i,k_j)}\sum_{u=0}^{r-1}K_i(r)H_i\Psi_{pi}^{u\mathrm{T}}\Gamma Q_w\Gamma^{\mathrm{T}}\Psi_{pj}^{u\mathrm{T}}H_j^{\mathrm{T}}K_j^{\mathrm{T}}(r),(i\ne j)\end{aligned} \tag{38}$$

Case 4: when k_i<0, k_j<0≥

$$\begin{aligned}P_{ij}(k_i,k_j)&=\Phi^{-k_i-1}\Sigma_{ij}\Psi_{pj}^{(-1-k_j)\mathrm{T}}-\\&\sum_{s=0}^{k_j}\Phi^{-k_i-1}\Sigma_{ij}\Psi_{pj}^{(s-k_i-1)\mathrm{T}}H_j^{\mathrm{T}}K_j^{\mathrm{T}}(s)+\\&\sum_{r=0}^{-k_i-2}\Phi^{r}\Gamma Q_w\Gamma^{\mathrm{T}}\Psi_{pj}^{r\mathrm{T}}+\\&\sum_{s=0}^{k_j}\sum_{r=0}^{-k_i-2}\Phi^{r}\Gamma Q_w\Gamma^{\mathrm{T}}\Psi_{pj}^{(s+r)\mathrm{T}}H_j^{\mathrm{T}}K_j^{\mathrm{T}}(s)\end{aligned} \tag{39}$$

Case 5: when $k_i \geq 0$, $k_j < 0$

$$\begin{aligned} P_{ij}(k_i,k_j) &= \Psi_{pj}^{-k_j-1}\Sigma_{ij}\Phi^{(-k_i-1)\mathrm{T}} - \\ &\sum_{s=0}^{k_i} K_i(s)H_i\Psi_{pi}^{(s-k_j-1)}\Sigma_{ij}\Phi^{(-k_j-1)\mathrm{T}} \\ &+ \sum_{r=0}^{-k_j-2}\Psi_{pi}^{r}\Gamma Q_w\Gamma^{\mathrm{T}}\Phi^{r\mathrm{T}} + \sum_{s=0}^{k_i}\sum_{r=0}^{-k_j-2} K_i(s)H_j\Psi_{pi}^{s+r}\Gamma Q_w\Gamma^{\mathrm{T}}\Phi^{r\mathrm{T}} \end{aligned} \tag{40}$$

Lemma 3: (Sun and Deng, 2004) For the two-sensor system (1) and (5), the optimal Kalman fuser weighted by matrices is given by:

$$\begin{aligned} \hat{x}_m^z(t\mid t+N) &= \Omega_1 x_1^z(t\mid t+N) + \Omega_2 x_2^z(t\mid t+N) \\ &= \Omega_1\hat{x}_1^z(t\mid t+N-\tau_1) + \Omega_2 x_2(t\mid t+N-\tau_2) \end{aligned} \tag{41}$$

where the weighted matrix is given by:

$$[\Omega_1,\Omega_2] = (e^{\mathrm{T}}P^{-1}(k_1,k_2)e)^{-1}e^{\mathrm{T}}P^{-1}(k_1,k_2) \tag{42}$$

$$P(k_i,k_j) = (P_{ij}(k_i,k_j))_{NL\times NL}, (i,j=1,2) \tag{43}$$

where, $e^{\mathrm{T}} = [\mathrm{I}_n, \mathrm{I}_n]$ and the fused error variance matrix P_m is given as:

$$P_m = [e^{\mathrm{T}}P^{-1}(k_i,k_j)e]^{-1} \tag{44}$$

The optimal Kalman fuser $\hat{x}_s^z(t|t+N)$ weighted by scalars is given by:

$$\begin{aligned} \hat{x}_s^z(t\mid t+N) &= a_1 x_1^z(t\mid t+N) + a_2 x_2^z(t\mid t+N) \\ &= a_1\hat{x}_1^z(t\mid t+N-\tau_1) + a_2 x_2(t\mid t+N-\tau_2) \end{aligned} \tag{45}$$

where the optimal weighted coefficients $a = [a_1, a_2]$ is given as:

$$a = \frac{e_s^{\mathrm{T}}P_{tr}^{-1}}{e_s^{\mathrm{T}}P_{tr}^{-1}e_s} \tag{46}$$

where,

$$e_s^{\mathrm{T}} = [1 \quad 1] \tag{47}$$

$$P_{tr} = \begin{bmatrix} \mathrm{tr}P_1(k_1,k_1) & \mathrm{tr}P_{12}(k_1,k_2) \\ \mathrm{tr}P_{21}(k_2,k_1) & \mathrm{tr}P_2(k_2,k_2) \end{bmatrix} \tag{48}$$

The corresponding optimal fused estimation error variance P_s is given by:

$$P_s = \sum_{i=1}^{2}\sum_{j=1}^{2} a_i a_j P_{ij}(k_i,k_j) \tag{49}$$

The optimal Kalman fuser $\hat{x}_d^z(t|t|t+N)$ weighted by diagonal matrices is given by:

$$\begin{aligned} \hat{x}_d^z(t\mid t+N) &= A_1 x_1^z(t\mid t+N) + A_2 x_2^z(t\mid t+N) \\ &= A_1\hat{x}_1^z(t\mid t+N-\tau_1) + A_2 x_2(t\mid t+N-\tau_2) \end{aligned} \tag{50}$$

where the optimal weighted diagonal matrices A_j is given as:

$$A_j = \begin{bmatrix} a_{j1} & & & \\ & a_{j2} & & \\ & & \ddots & \\ & & & a_{jn} \end{bmatrix}, \quad j=1,2 \tag{51}$$

The component expression of $\hat{x}_j^z$ and $\hat{x}_d^z$ are given by:

$$\hat{x}_j^z = \begin{bmatrix} \hat{x}_{j1}^z \\ \vdots \\ \hat{x}_{jn}^z \end{bmatrix}, x_d^z = \begin{bmatrix} x_{d1}^z \\ \vdots \\ x_{dn}^z \end{bmatrix} \tag{52}$$

So the equation (50) is equivalent with:

$$\hat{x}_{dl}^z(t\mid t+N) = \sum_{j=1}^{2} a_{jl}x_{jl}^z(t\mid t+N), l=1,\cdots,n \tag{53}$$

where the optimal weighted coefficient vector is:

$$a_l = [a_{1l} \quad a_{2l}] = \frac{e_d^{\mathrm{T}}(P^{ll})^{-1}}{e_d^{\mathrm{T}}(P^{ll})^{-1}e_d} \tag{54}$$

where $e_d^T = [1 \ldots 1]$ and defining:

$$P^{ll} = \begin{bmatrix} P_1^{(ll)} & P_{12}^{(ll)} \\ P_{21}^{(ll)} & P_2^{(ll)} \end{bmatrix} \tag{55}$$

where P_{ij}^{ll} is the (l, l)th diagonal component of P_{ij} (k_i, k_j) and the optimal component fused error variance P_{dl} and the optimal fused error variance P_d are given by:

$$\begin{aligned} P_{dl} &= [e_d^{\mathrm{T}}(P^{ll})^{-1}e_d]^{-1}, l=1,\cdots,n \\ P_d &= \sum_{i=1}^{2}\sum_{j=1}^{2} A_i P_{ij} A_j^{\mathrm{T}} \end{aligned} \tag{56}$$

THE STEADY-STATE CI KALMAN FUSER

From the proposed interpretation, it is clear that the computation of getting the cross-covariances between local filtering is very complex and the computed burden is very large. So a CI fusion method was presented (Julier and Uhlmann, 1997; Chen *et al*., 2002).

Consider a two-sensor system (1) (2) and (5), when P_1 and P_2 are known, but the cross-covariance P_{12} is unknown, the CI Kalman fuser without P_{12} is given by:

$$\begin{aligned} \hat{x}_{CI}^z(t\mid t+N) &= P_{CI}[\omega P_1^{-1}x_1^z(t\mid t+N) + \\ &(1-\omega)P_2^{-1}\hat{x}_2^z(t\mid t+N)] \end{aligned} \tag{57}$$

where the CI fusion error variance matrix P_{CI} is given by:

$$P_{CI} = [\omega P_1^{-1} + (1-\omega)P_2^{-1}]^{-1} \tag{58}$$

where, $P_i \square P_i(k_i,k_i), i=1,2$, $P_{12} \square P_{12}(k_1,k_2)$, $P_{21} \square P_{21}(k_2,k_1)$ and the weighted coefficient $\omega\epsilon[0,1]$ and minimizes the performance index:

$$J = \min_{\omega} \mathrm{tr}P_{CI} = \min_{\omega\in[0,1]} \mathrm{tr}\{[\omega P_1^{-1} + (1-\omega)P_2^{-1}]^{-1}\} \quad (59)$$

where the notation tr denotes the trace of matrix. The optimal weighting coefficient ω can be solved by the gold section method or Fibonacci method (Julier and Uhlmann, 1996).

THE CONSISTENCY AND ACCURACY OF CI KALMAN FUSER

Theorem1: The actual CI fusion error variance matrix $\bar{P}_{CI}$ is given by:

$$\bar{P}_{CI} = P_{CI}[\omega^2 P_1^{-1} + \omega(1-\omega)P_1^{-1}P_{12}P_2^{-1} + \omega(1-\omega)P_2^{-1}P_{21}P_1^{-1} + (1-\omega)^2 P_2^{-1}]P_{CI} \quad (60)$$

Proof: From (58), we can obtain $P_{CI}[\omega P_1^{-1} + P_2^{-1}] = I$, which yields

$$x(t) = P_{CI}[\omega P_1^{-1}x(t) + (1-\omega)P_2^{-1}x(t)] \quad (61)$$

Subtracting (57) from (61), we get the actual CI fuser error:

$$\tilde{x}_{CI}(t\,|\,t+N) = P_{CI}[\omega P_1^{-1}\tilde{x}_1(t\,|\,t+N) + (1-\omega)P_2^{-1}\tilde{x}_2(t\,|\,t+N)] \quad (62)$$

The actual CI fusion error variance $\bar{P}_{CI} = E[\tilde{x}_{CI}(t|t+N)\tilde{x}_{CI}^T(t|t+N)]$, where E denotes the mathematical expectation, T denotes the transpose. Substituting (62) into $\bar{P}_{CI}$ yields that (60) holds. The proof is completed.

Theorem 2: For local and fused Kalman filters, we have the accuracy relations:

$$P_m \le P_d, P_m \le P_s, P_m \le \bar{P}_{CI}, P_m \le P_i, i=1,2 \quad (63)$$

$$\bar{P}_{CI} \le P_{CI} \quad (64)$$

$$\mathrm{tr}P_m \le \mathrm{tr}P_d \le \mathrm{tr}P_s \le \mathrm{tr}P_i, i=1,2 \quad (65)$$

$$\mathrm{tr}P_m \le \mathrm{tr}\bar{P}_{CI} \le \mathrm{tr}P_{CI} \le \mathrm{tr}P_i, i=1,2 \quad (66)$$

Proof: from (41), $\hat{x}_m^z(t|t+N)$ is the Unbiased Linear Minimum Variance (ULMV) estimate of $x(t)$ based on the linear space $L_m = L_m(\hat{x}_1^z(t|t+N), \hat{x}_2^z(t|t+N))$ spanned by $(\hat{x}_1^z(t|t+N), \hat{x}_2^z(t|t+N))$. From (45), (50), (57), we have $\hat{x}_\theta^z(t|t+N), \epsilon L_m, \theta = s, d, CI, i,$ which yields (63). It has been proven $\bar{P}_{CI} \le \bar{P}_{CI}$ and $\mathrm{trP_m} \le \mathrm{trP_d} \le \mathrm{trP_s} \le \mathrm{trP_i}$, = 1, 2 in the reference (Sun and Deng, 2004; Julier and Uhlmann, 1997; Chen *et al*., 2002), so we have (64) and (65) hold. In the performance index (59), taking ω = 0, we have J = $\mathrm{trP_2}$ and taking ω = 1, we have J = $\mathrm{trP_1}$. Hence the optimal weighting coefficient $\omega\epsilon[0,1]$ yields $\mathrm{trP_{CI}} \le \mathrm{trP_1} \le \mathrm{trP_{CI}} \le \mathrm{trP_2}$, i.e. $\mathrm{trP_{CI}} \le \mathrm{trP_i}$, i = 1, 2. Taking trace operation for (63) and (64), we have (66) holds. The proof is completed.

SIMULATION EXAMPLE

Consider the tracking system with two-sensor and with time-delayed measurements:

$$x(t+1) = \begin{bmatrix} 1 & T_0 \\ 0 & 1 \end{bmatrix} x(t) + \begin{bmatrix} 0.5T_0^2 \\ T_0 \end{bmatrix}$$

$$z_i(t) = H_i x(t-\tau_i) + \xi_i(t), i = 1,2$$

$$H_1 = [1 \quad 0], H_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \tau_1 = 1, \tau_2 = 2$$

where, $x(t)\epsilon R^2$ is the state, $z_i(t)$ is the measurement with delays for sensor i, w(t) and $\zeta_i(t)$ are white noise with zero mean and variances Q_w and $Q_{\zeta i}$, respectively. According to the corresponding transformation, we obtain the measurement $y_i(t)$ without time delays:

$$y_i(t) = z_i(t+\tau_i) = H_i x(t) + v_i(t)$$

where $v_i(t) = \zeta_i(t+\tau_i)$ is the transformed measurement white noise with zero mean and variance $Q_{vi} = Q_{\zeta i}$. Thus the problem of finding the local steady-state optimal Kalman smoother $\hat{x}_i^z(t|t+N), (i=1,2)$ based on the measurement $z_i(t)$ is converted into the problem of finding the local steady-state optimal Kalman filter $\hat{x}_1(t|t)$ and Kalman predictor $\hat{x}_2(t|t-1)$ based on the new measurement $y_i(t)$, respectively. Further, applying the Kalman filtering method yields the steady-state optimal fusion Kalman smoothers $\hat{x}_m^z(t+1, xdztt+1, xsztt+1$ weighted by matrices,

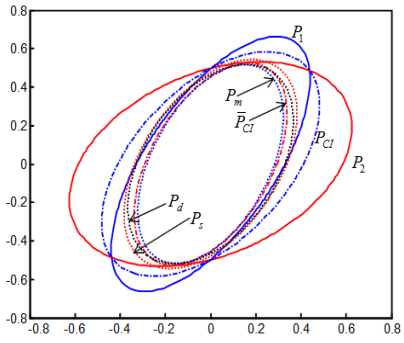

Fig. 1: The accuracy comparison of $\mathrm{P_1}$, $\mathrm{P_2}$, $\mathrm{P_0}$, $\mathrm{P_s}$, $\mathrm{P_{CI}}$ and $\bar{P}_{CI}$

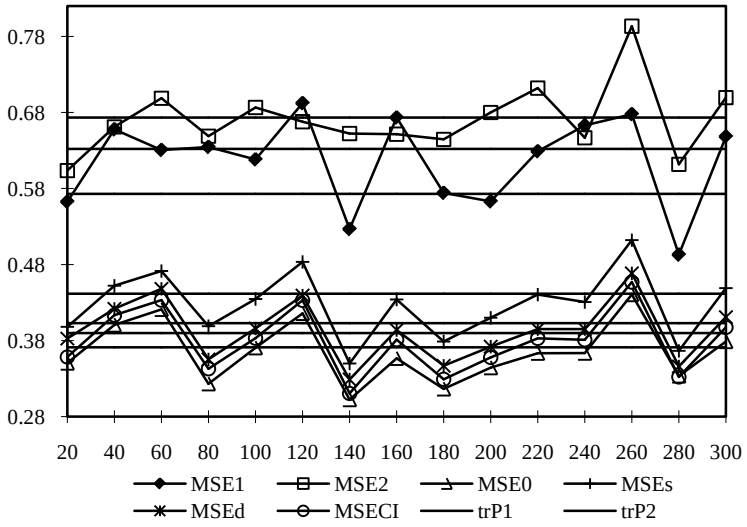

Fig. 2: The curves of MSEi(t)

diagonal matrices and scalars and CI fused Kalman smoother $\hat{x}_{CI}^{z}(t|t+1)$. In simulation, we take

$$T_0 = 0.4, Q_w = 1, Q_{\varsigma 1} = 0.36, Q_{\varsigma 2} = \begin{bmatrix} 4 & 0 \\ 0 & 0.16 \end{bmatrix}$$

In order to give a powerful geometric interpretation with respect to accuracy relations of local and fusers, the covariance ellipse for a covariance matrix P is defined as the locus of points $\{x: x^T P^{-1} x = c\}$ where c is a constant. In the sequel, c = 1 will be assumed without loss of generality. It was proved (Julier and Uhlmann, 1996) that $P_a \leq P_b$ is equivalent to that the ellipse for P_a is enclosed in the ellipse for P_b. The simulation results are shown in Fig. 1.

From Fig. 1, it is clearly shown that the covariance ellipse for P_m lies within the intersection of ellipses for P_1 and P_2, the ellipse for CI fused theoretical covariance P_{CI} encloses the intersection region of ellipses for P_1 and P_2 (Chen *et al*., 2002) and passes through the four points of intersection of ellipses for P_1 and P_2. The ellipse for CI fused actual covariance $\bar{P}_{CI}$ lies within the ellipse for theoretical covariance $\bar{P}_{CI}$.

In order to verify the above theoretical results for accuracy relation, 200 Monte-Carlo runs are performed for t = 1, …, 300 the results are shown in Fig. 2.
where the straight lines denote trP_i, i = 0, 1, 2, CI and $tr\bar{P}_{CI}$, the curves denote the corresponding mean square errors $MSE_i(t)$. We see the CI fuser has good performance, whose accuracy is approximates to the accuracy of the optimal smoother, because $MSE_{CI}(t)$ is approximates to $MSE_m(t)$.

CONCLUSION

For the two-sensor system with time-delayed measurements and with unknown cross-covariance matrices, a CI fusion steady-state Kalman fuser has been presented, which has the advantage that the computation of cross-covariances can be avoided. It is proved that its accuracy is higher than that of each local estimator and is lower than that of optimal fuser weighted by matrices. A two-sensor system Monte-Carlo simulation example shows that its accuracy is approximates to the accuracy of the optimal fuser and is higher than those of the fusers weighted by diagonal matrices and scalars.

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