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Review of Models Introduced to Estimate the Distribution of Longitudinal Velocity in Open Channels based on Navier-stokes Equations

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Abstract: Determination of the velocity profile in open channels (particularly in narrow channels) with turbulent flows has always been the subject of attention and studies of researchers. This is due to the fact that knowing the velocity distribution it is possible to calculate the distributions of shear stress and the discharge in the channels. However, due to the complexity of conditions governing the flow, which are the consequences of the non-isotropic state of turbulence and the existence of Prandtl type 2 secondary flows; it is difficult to introduce a model that would be capable of properly defining the velocity distribution in narrow open channels. Many researchers have tried to modify the famous logarithmic law, which responds well in the case of wide open channels, to suit the narrow channels as well, thereby allowing the modified model to describe the negative gradient of velocity beneath the free surface of water (known as dip phenomenon). However, in this context, they have been forced to set limitations on their models, which have in turn limited their applications. For this reason by using the laws of mathematics and simplifying the Navier-Stokes equations, some researchers have presented models that are capable of calculating the velocity profile in narrow channels in a satisfactory manner with the least constraint for application. Two models that are derived from Navier-Stokes are introduced in this study and their results are compared with experimental data.

Keywords: Dip phenomena, maximum velocity, narrow channel, Navier-stokes, turbulent flow, velocity distribution

INTRODUCTION

Estimation of the velocity distribution has always been an important issue for researchers. As the most famous law for estimation of velocity in open channels, logarithmic has for years been used by engineers and studied by various researchers. This law is quite suitable for describing the velocity distribution in the inner region, but for the outer region it is only responsive in wide open channels, which have a width to depth ratio of over 5. It cannot estimate the velocity distribution in narrow channels where there are secondary flows of Prandtl type 2, which create the dip phenomena (negative velocity gradient close to the surface of water) (Bonakdari, 2006). The reason for this is the occurrence of maximum velocity below the free surface of water resulting in a 1 to 1 relation between the component of velocity and the component of the vertical location.

In the recent decade, researchers such as Coles (1956), Chow (1959), Henderson (1966), Sarma *et al*. (2000), Shiono and Feng (2003), Yang *et al*. (2004), Absi (2011) and Bonakdari (2006) have tried to modify the logarithm law or present new models on the basis of conditions governing the outer region of narrow channels (B/h<5) to obtain the velocity distribution in this region. By addition of corrective expressions to the logarithm law, Coles (1956) and some other researchers as Chow (1959) and Henderson (1966), studied more comprehensive the velocity distribution. These laws present a positive gradient of the velocity in the flow depth and the maximum velocity occurs at the free surface; then, Sarma *et al*. (2000) suggested some other laws by dividing cross section into four regions and determining velocity distribution equations in each region to show longitudinal velocity profile over whole of cross section. In compund sections, Shiono and Feng (2003), represented a velocity distribution which showed the effects of steady slope of channel and secondary flow; its maximum velocity occurred in 0.5 to 0.7 y_{max} (maximum level of water surface). Yang *et al*. (2004) have tried to improve the results of the logarithm law in the outer region.

Bonakdari (2006) and Absi (2011) have also used the Navier-Stokes equations to propose models, which have yielded much better results than others. Since these two models are based on mathematical relations and their simplification, they can readily be used for natural and artificial channels. In particular these two models can be applied for wastewater channels to calculate the flow. This study reviews these two models and compares their results with the actual and lab data.

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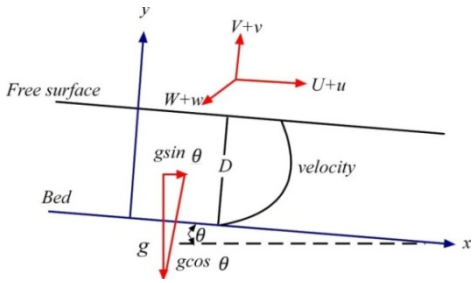


Fig. 1: Selected specifications for models introduced (Bonakdari, 2006)

BONAKDARI'S MODEL

Bonakdari (2006) conducted a number of simplifications on the Navier-Stokes to propose a model for velocity distribution in broad and narrow open channels with turbulent flows. Taking into consideration the assumptions described below, in this model the momentum relation, which has been obtained on the basis of Navier-Stokes equations, has been simplified.

The flow has been assumed to be continuous, steady and fully developed ($\frac{\partial}{\partial x} = \frac{\partial}{\partial t} = 0$), the selected specific system is according to Fig. 1 and the relation of the momentum for the longitudinal component of the flow, i.e., x has been obtained as below:

$$\frac{\partial U}{\partial t} + V\frac{\partial U}{\partial y} + W\frac{\partial U}{\partial z} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + v\frac{\partial^2 U}{\partial y^2} + v\frac{\partial^2 U}{\partial z^2} - \frac{\partial \overline{uv}}{\partial y} - \frac{\partial \overline{uw}}{\partial z} + gsin \quad (1)$$

Since the flow with the free surface ($\frac{\partial P}{\partial x} = 0$) and in the central region of the channel the level of changes of velocity i.e. the gradient of the vertical component of the velocity is higher than its horizontal component, ($\frac{\partial .}{\partial y} \gg \frac{\partial .}{\partial z}$) (Rodi, 1993) and further since the outer region is considered to be $0.2D \leq y \leq D$ (Bonakdari, 2006), after integration in the outer region i.e., $0.2D \leq y \leq D$ relation 1 will be obtained as follows:

$$VU_{(D)} - VU_{(y)} = \left[\left(v\frac{dU}{dy}\right)_{(D)} - \left(v\frac{dU}{dy}\right)_{(y)}\right] - \left[(\overline{uv})_{(D)} - (\overline{uv})_{(y)}\right] + gsin\,\theta\,(D - y) \quad (2)$$

On the other hand in the free surface of water, the vertical component of velocity (V), the component of fluctuation of the vertical velocity (v) and the amount of shear velocity i.e., $\frac{dU}{dy}$ are all equal to zero ($V \approx v \approx \frac{dU}{dy} \approx 0$) (Bonakdari, 2006).

Therefore, relation 2 has been simplified as follows:

$$VU_{(y)} = -\left(v\frac{dU}{dy}\right)_{(y)} + (\overline{uv})_{(y)} + gsin\theta(D - y) \quad (3)$$

The first expression to the right of relation 3 reflects the effect of viscosity in the central region of the channel. In the outer region the effect of this expression is negligible and can be ignored ($\left(v\frac{dU}{dy}\right)_{(y)} \cong 0$). Moreover the second expression to the right of the above relation represents the Reynolds stresses, which, assuming the Boussinesq's Hypothesis would be equal to:

$$-(\overline{uv}) = v_t\frac{dU}{dy} \quad (4)$$

This relation has been proposed by Rodi (1993), in which v_t is the vortex viscosity. Nezu and Nakagawa (1993) have proposed the following relation for the vortex viscosity:

$$\frac{v_t}{u_*} = \kappa y(1 - \xi) \quad (5)$$

In which, u_* is the shear velocity and equal to $u_* = \sqrt{\tau_b/\rho}$, τ_b is the local shearing stress and $\frac{y}{D} = \xi$. κ is also the Von Karman constant assumed to be equal to 0.41. By placing relation 5 in Eq. (4) related to Reynolds stresses, the following relation is obtained:

$$-(\overline{uv}) = u_*\kappa y\,(1 - \xi)\frac{dU}{dy} \quad (6)$$

By placing in Eq. (6) in (3):

$$VU_{(y)} = \kappa u_* y(1 - \xi)\frac{dU}{dy} - gDsin\theta(1 - \xi) \quad (7)$$

Bonakdari (2006) have considered the value of V/u_* as equal to the following relation:

$$\frac{V}{u_*} = -k\frac{\xi(1-\xi^2)}{\frac{1}{2}\xi^2 + \xi + C_{Ar}} \quad (8)$$

The model proposed by Bonakdari (2006) is the following relation, which is obtained by placing relation 8 in Eq. (7) and taking their integral:

$$\frac{U(\xi)}{u_*} = \left(\frac{\frac{1}{2}\xi_i^2 + \xi_i + C_{Ar}}{\frac{1}{2}\xi^2 + \xi + C_{Ar}}\right) \times \left[\left(\frac{\left(\frac{1}{4}\xi^2 + \xi + C_{Ar}\,ln\xi\right) - \left(\frac{1}{4}\xi_i^2 + \xi_i + C_{Ar}\,\ln(\xi_i)\right)}{\frac{1}{2}\xi_i^2 + \xi_i + C_{Ar}}\right)\right] \times \frac{\alpha}{\kappa} + \frac{U(\xi_i)}{u_*} \quad (9)$$

In which k is the Von Karman constant, $\alpha = (gDsin\theta)/u_*^2$ has a physical significance calculated on the basis of the specifications of the channel and the hydraulic conditions of the flow and ξ_i represents a specific situation in vertical axis.

Table 1: Similarities Bonakdari's and Absi's models

Reynolds stresses	Viscosity in outer region	The limits of outer region	Flow condition in the outer region			Dominant equations	Model name
Boussinesq's hypothesis	$\left(v\frac{dU}{dy}\right) \cong 0$	0.2≤y≤D	$\frac{\partial.}{\partial y} \gg \frac{\partial.}{\partial z}$	Free surface flow	Continuous and steady flow	Navier-stokes equations	Bonakdari model
Boussinesq's hypothesis	$\left(v\frac{dU}{dy}\right) \cong 0$	0.2≤y≤D	$\frac{\partial.}{\partial y} \gg \frac{\partial.}{\partial z}$	Free surface flow	Continuous and steady flow	Navier-stokes equations	Absi model

Table 2: Differences Bonakdari's and Absi's model

Vortex viscosity	Parameter α	The condition of the width component of velocity	Velocity distribution in the inner region	Model name
Relation (5)	$\alpha = (gsin\theta)/u_*^2$	Relation (8)	Logarithm law is used	Bonakdari model
Relation (15)	$\alpha = \alpha_1 + \alpha_2$	Yang hypothesis $\frac{UV}{u_*^2} \approx -\alpha_2 \frac{y}{h}$	The proposed model is responsive	Absi model

The final form of the Bonakdari's model will be as below, in which logarithm is used for the inner region (relation 10) with the assumption that the length of this layer is 0.2 h, while providing Eq. (11) for the outer region, which includes regions above the border layer to the surface of open water:

$$\frac{U(\xi_i)}{u_*} = \frac{1}{k}\, ln\left(\frac{0.2h}{k_s}\right) + B_s \tag{10}$$

$$\frac{U(\xi)}{u_*} = \left(\frac{\frac{1}{2}\xi_i^2+\xi_i+C_{Ar}}{\frac{1}{2}\xi^2+\xi+C_{Ar}}\right) \times \left[\left(\frac{\left(\frac{1}{4}\xi^2+\xi+C_{Ar}\, ln\xi\right)-\left(\frac{1}{4}\xi_i^2+\xi_i+C_{Ar}\ln(\xi_i)\right)}{\frac{1}{2}\xi_i^2+\xi_i+C_{Ar}}\right) \times \alpha\kappa + 1\kappa ln0.2h\kappa s + Bs\, u* \tag{11}$$

Researchers have reported differing values of 5.1 to 5.5 for the parameter B_s (Nezu and Rodi, 1986; Cardoso *et al*., 1989; Kirkgoz and Ardiclioglu, 1997). The advantage of the model lies in the fact that it can be applied in all channels whether broad or narrow and its parameters can be easily calculated on the basis of geometric and hydraulic specifications of the channel and the flow.

ABSI'S MODEL

Absi (2011) also presented his model by simplifying the Navier-Stokes equations. This model and its underlying assumptions will be explained in the following sections. Absi has also considered ($\frac{\partial.}{\partial y} \gg \frac{\partial.}{\partial z}$) for the central region of the channel. He has also considered the value of expression $\left(v\frac{dU}{dy}\right)$ as negligible and the relation 1 in the following form Absi (2011):

$$\frac{dUV}{dy} + \frac{d\overline{uv}}{dy} = gsin\theta \tag{12}$$

By taking the integral of the above relation, substituting $\alpha_1 = (gDsin\theta)/u_*^2 - 1$ and using the hypothesis proposed by Yang for the expression $\frac{UV}{u_*^2}$ (i.e., $\frac{UV}{u_*^2} \approx -\alpha_2 \frac{y}{h}$) relation 12 is changed as follows:

$$-\frac{\overline{uv}}{u_*^2} = (1-\xi) - \alpha\xi \tag{13}$$

In which, $\alpha = \alpha_1 + \alpha_2$. Absi has applied the Boussinesq's hypothesis, i.e., Eq. (4) for the expression $\overline{uv}$:

$$\frac{dU}{dy} = \frac{u_*^2}{v_t}\left[(1-\xi) - \alpha\xi\right] \tag{14}$$

Instead of using the equation proposed by Nezu and Nakagawa (1993) for vortex velocity Eq. (5), Absi has proposed the following relation (Nezu and Rodi, 1986):

$$\frac{v_t}{u_* h} = \kappa(1-\xi)\left[\frac{1}{\xi} + \pi\Pi sin(\pi\xi)\right]^{-1} \tag{15}$$

By placing relation 15 in Eq. (14), the following relation is obtained:

$$\frac{dU_a}{d\xi} = \frac{1}{\kappa}\left(1 - \alpha\frac{\xi}{1-\xi}\right)\left[\frac{1}{\xi} + \pi\Pi sin(\pi\xi)\right] \tag{16}$$

In this relation $U_a = \frac{U}{u_*}$ and for $\alpha = 0$ and $\Pi = 0$, the integral of the above relation is the logarithm law. Finally by taking the integral of relation 16, Absi (2011) has proposed his model, which consists of 4 expressions:

$$U_a = \frac{1}{\kappa} ln\left(\frac{\xi}{\xi_0}\right) + \frac{2\Pi}{\kappa} sin^2\left(\frac{\pi}{2}\xi\right) + \frac{\alpha}{\kappa} ln(1-\xi) - \frac{\alpha\pi\Pi}{\kappa}\int_{\xi 0}^{\xi} \frac{\xi}{1-\xi} sin(\pi\xi) d\xi \tag{17}$$

The first three expressions of this model are in order the logarithm law, the Coles function and the modified expression added by Yang to the logarithm law. However the fourth expression, which is in the form of a specific integral, is the expression added by Absi, which is the distinguishing parameter of this model compared to the previous modified models.

Differences and similarities of models: There are differences and similarities between the assumptions made by these two researchers as shown in Table 1 and 2. The similarities are mainly related to the general

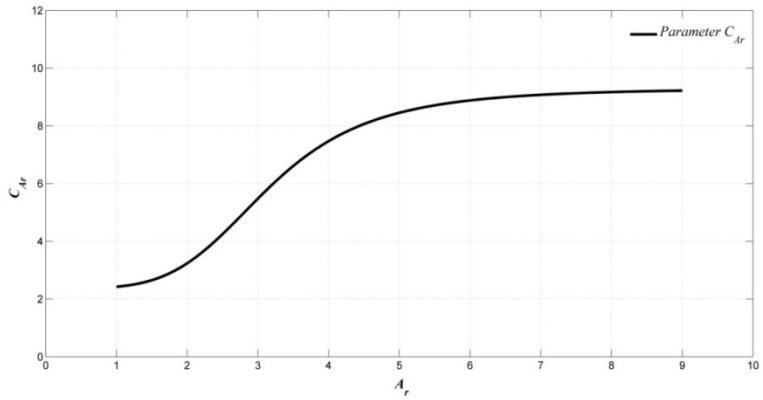


Fig. 2: C_{Ar} based on different A_r (Bonakdari, 2006)

Table 3: The different values proposed for parameter Π

Dependent conditions	Π Value	Researcher
Dependent on reynolds number	0.55	Coles (1956)
At high reynolds values for level bed	0.55	Cebeci and Smith (1974)
Dependent on reynolds shearing value for level bed	0~0.20	Nezu and Rodi (1986)
For steady flow and level bed	0.08	Cardoso *et al*. (1989)
For level bed	0.10	Kirkgoz and Ardiclioglu (1997)
For froude number of greater than 1 and reynolds number of greater than 10^5	0.30	Li *et al*. (1995)
To better demonstrate the dip phenomena	0.45	Absi (2011)

condition of the flow in open channels, but the differences are related to the condition of applying secondary flow, which is an essential and determining parameter in the accuracy and performance of the models, to the velocity distribution. The effect of this important parameter and the method of applying it on the models is clearly seen in the results of these two models.

Comparison of parameters: The main parameters in Bonakdari's model are α and C_{Ar}. α represents the physical condition of the channel, i.e., dimension, slope and the condition of the channel's bed and its relation is such that it always has a positive value. Parameter C_{Ar}, which represent the location of maximum velocity, is the main parameter in Bonakdari's model. This parameter shows the curve of the velocity profile in the region close to the location of the maximum velocity and is the determining parameter of this curve. The experimental relation, which has been obtained by Bonakdari for C_{Ar} and ξ_{dip} on the basis of lab data, is as under:

$$C_{Ar} = 9.3\,\xi_{dip}^{1.7} \tag{18}$$

The parameter ξ_{dip} has been studied by various researchers and many experimental relations have been proposed for it. Wang *et al*. (2001) and Yang *et al*. (2004) have proposed an experimental relation on the basis of A_r. Bonakdari has also proposed a lab relation on the basis of A_r as below. Figure 2 shows the graph obtained from this experimental relation for different values of A_r (Bonakdari, 2006):

$$\xi_{dip} = \frac{42.4+A_r^{4.2}}{94.7+A_r^{4.2}} \tag{19}$$

The parameters of Absi's model are in order α_1, α_2 and Π. Parameter α_1, is similar to Bonakdari's parameter, with this difference however that its value is always one unit less that the Bonakdari's parameter and describes the physical conditions of the channel. Parameter α_2 was introduced in the year 2004 by Yang *et al*. (2004). It acts as the parameter in Bonakdari's model and describes the curve of velocity profile and the location of maximum velocity in narrow channels. The equation proposed by Yang for this parameter is presented below and depends on the breadth component of the channel, i.e. z and the breadth to depth ratio:

$$\alpha_2 = 1.3\exp\left(-\frac{0.5A_r 2z}{B}\right) \tag{20}$$

Parameter Π was defined used for the first time by Coles (1956). This parameter has been studied by various researchers and different values have been proposed for it as shown in Table 3. As Π increases its accuracy and effect on the curve of velocity profile is quite noticeable and by calibrating this parameter with lab and actual data its impact can be further increased.

Parameter $\alpha = \alpha_1 + \alpha_2$ has also been obtained by Absi (2011) as follows by taking the derivate of Eq. (16) to determine the location of maximum velocity:

$$\alpha = \frac{1}{\xi_{dip}} - 1 \tag{21}$$

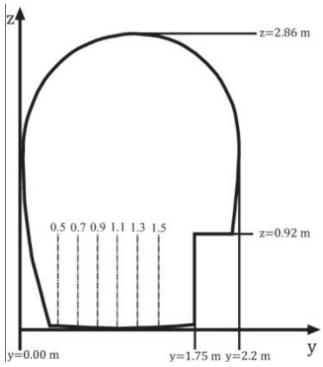


Fig. 3: Cordon bleu wastewater channel (Bonakdari, 2006)

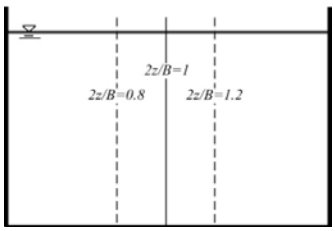


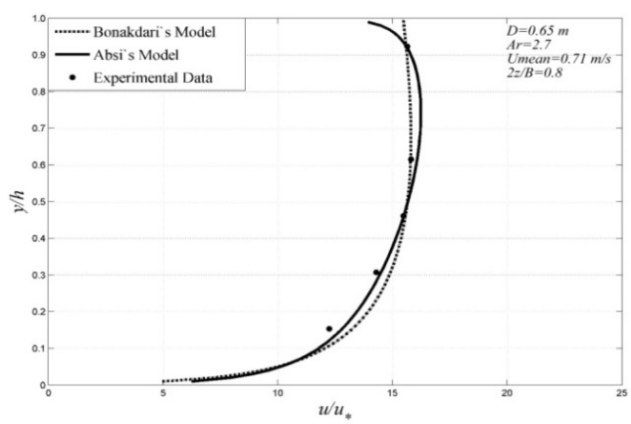
Fig. 4: Rectangular open channel of the lab (Li *et al*., 1995; Chiu and Hsu, 2006)

COMPARISON OF MODELS FOR VELOCITY ESTIMATING

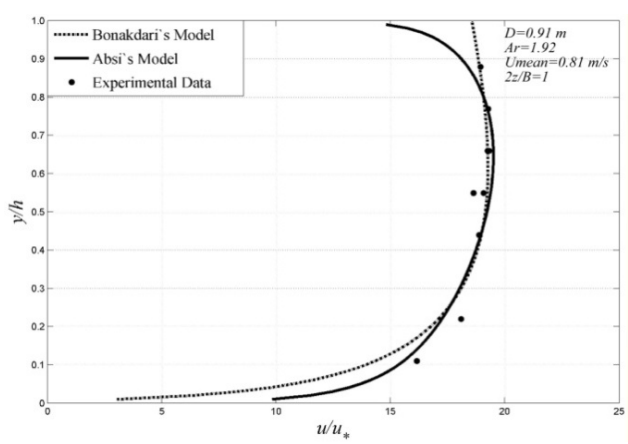
To compare the Bonakdari's model with the Absi's model, lab data collected from a natural channel (Bonakdari, 2006) and two lab channels (Chiu and Hsu, 2006; Tominaga *et al*., 1989), were used. The cross sections of these channels as well as the sections on which velocity was measured are shown in Fig. 3 and 4.

In Fig. 5 the two models introduced by Bonakdari (2006) and the collected data are evaluated. The velocity distributions for depths of 0.65, 091 and 1.19 m in a quasi-rectangular channel have been traced. As observed, the Bonakdari's model has a greater conformity than the Absi's model and the maximum deviation in Bonakdari's model is 3% whereas in Absi's model it can reach up to 5% as well.

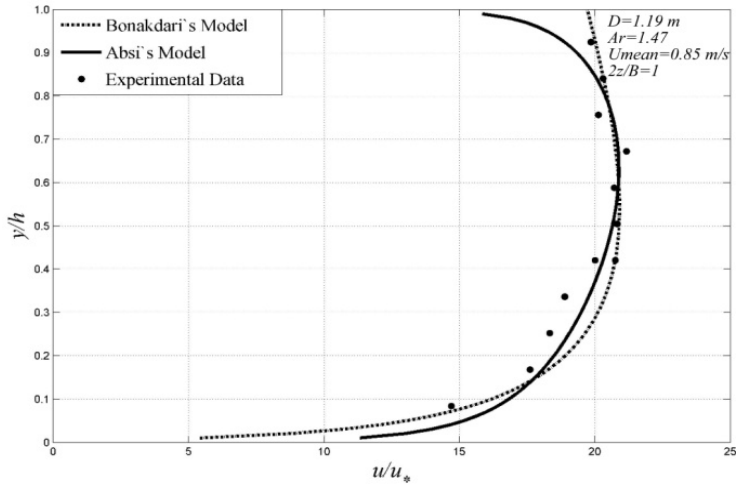
Figure 6 evaluates the Bonakdari's and Absi's models on the basis of data provided by Chiu and Hsu (2006) measured in a lab channel. This figure shows the Absi's model, which is as good as the Bonakdari's model with a deviation of approximately 3% in velocity distribution.



(a) D = 0.65 m

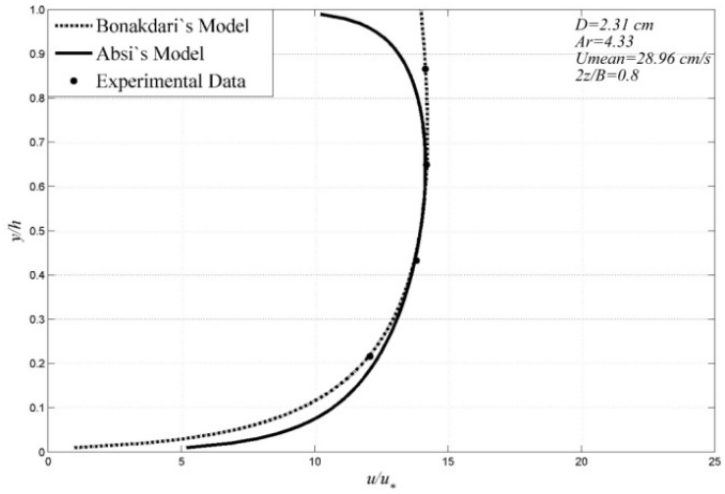


(b) D = 0.91 m

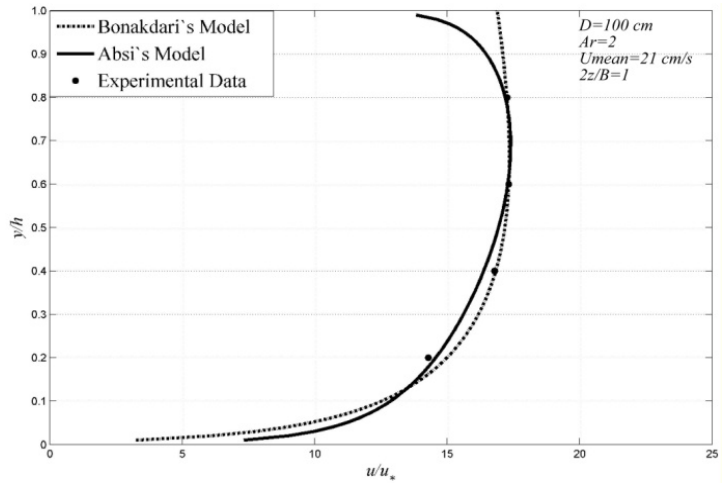


(c) D = 1.19 m

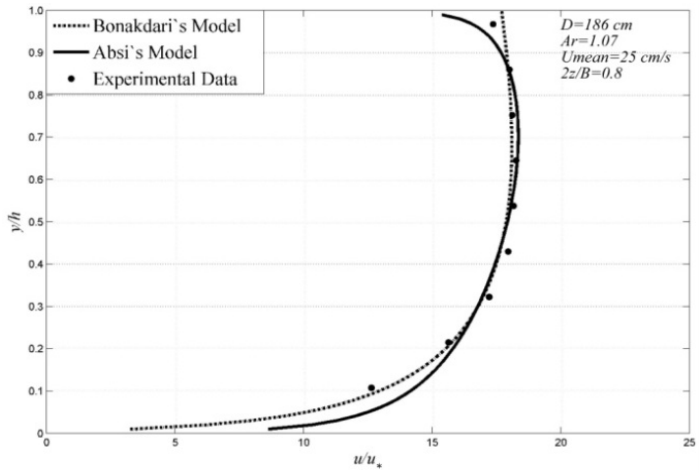
Fig. 5: Profile of longitudinal velocity traced on the basis of data provided by Bonakdari (2006)



(a) D = 2.31 cm

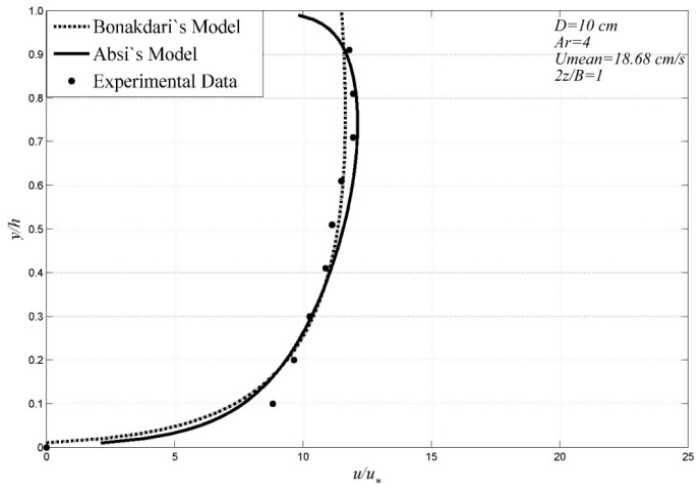


(b) D = 100 cm

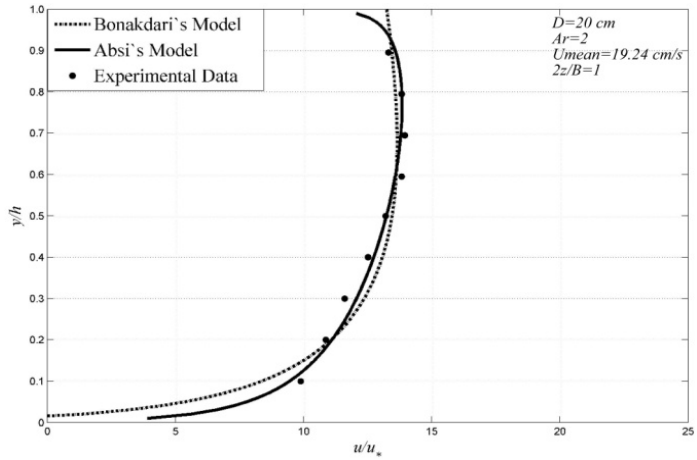


(c) D = 186 cm

Fig. 6: Profile of longitudinal velocity traced on the basis of data provided by Chiu and Hsu (2006)



(a) D = 10 cm



(b) D = 20 cm

Fig. 7: Profile of longitudinal velocity traced on the basis of data provided by Tominaga *et al*. (1989)

Table 4: Values of parameters on the basis of lab data

Models	Bonakdari's parameter		Absi's parameter	
Parameters	α	C_{Ar}	α	Π
D = 0.6500	1.30	4.73	0.35	0.45
D = 0.9100	2	3.10	1.05	0.45
D = 1.1900	2.20	2.60	1.08	0.45
D = 0.0231	1.55	7.87	0.60	0.10
D = 1.8600	1.77	2.44	0.82	0.10
D = 1000	1.70	3.32	0.78	0.10
D = 0.1000	1.35	7.57	0.37	0.10
D = 0.2000	1.85	3.32	0.92	0.10

In Fig. 7, which has been traced on the basis of data measured by Tominaga *et al*. (1989) in a rectangular lab channel, the accuracy of these two models in estimating the longitudinal velocity profile in narrow channels. Table 4 is also drawn on the basis of these data showing the parameters defined in each model for these data.

As observed in Fig. 5 to 7 due to the application of relation 15 instead of 5, the Absi provide a better description of the velocity profile curve in narrow channels and the dip phenomena. The expression $\left[\frac{1}{\xi} + \pi \Pi sin(\pi\xi)\right]^{-1}$ in relation 15 is a trigonometric function dependent on parameter Π and the role of this parameter in describing the curve of velocity profile in narrow channels. This parameter has resulted in better results of Absi's model in describing the dip phenomena at narrow channels in comparison with Bonakdari's model. On the other hand due to a more accurate definition of the breadth component of the velocity, i.e., V/u_* in the central region of the channel, the Bonakdari's model has a better conformity with the lab data (Fig. 5), while close to the channel walls the Absi model has a better conformity with the measured data.

CONCLUSION

Since the logarithm law cannot be applied for narrow channel of breadth to depth ratio of less than 5 and cannot describe the dip phenomena, various researchers have tried to modify the law or present a new model on the basis of law governing open channel. In this study the two models proposed by Bonakdari and Absi on the basis of Navier-Stokes equations to describe the longitudinal velocity profile, have been studied and evaluated according to measured data. The main difference of these models lies in their definition of the two parameters of v_t and V/u_*.

By applying the more accurate relation for v_t, the Absi model describes better the negative gradient of velocity in the proximity of free water surface thereby making it possible to use this model for narrow channels. On the other hand, by considering the effects of secondary flow in his model Bonakdari has also proposed it for use in narrow channels. The advantages of these models are the low number of parameters and the limitations applied due to initial assumptions, which have made the widespread use of these two models both in broad and narrow channels. The easy and simple calculations in these models have facilitated their application in water and river engineering and in estimation of flow, velocity distribution and shear stress. In future by calibrating these two models' parameters and by benefiting from greater number of data it will be possible to improve their accuracy and performance.

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