

Cosine Similarity Measure between Vague Sets and Its Application of Fault Diagnosis

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Abstract: In order to propose a novel cosine similarity measure between vague sets and to apply it to the fault diagnosis of turbine, a new similarity measure value between a testing sample and the knowledge of system faults is evaluated in the vibration fault diagnosis of turbine. The testing sample is near to a type of fault knowledge if the measure value is big. Thus, the type of vibration fault is determined according to the maximum measure value (more than some threshold). The fault-diagnosis problems of the turbine are investigated by use of the proposed cosine similarity measure. The results demonstrate that the proposed method not only diagnoses the main fault types of the turbine, but also provides useful information for multi-fault analyses and future trends. Therefore, the proposed method is reasonable and effective and provides another useful tool for fault analyses.

Keywords: Cosine similarity measure, fault diagnosis, turbine, vague set, vibration fault

INTRODUCTION

The technique of fault diagnosis has produced the huge economic benefits by scheduling preventive maintenance and preventing extensive downtime periods caused by extensive failure. Therefore, it becomes a research hotspot. In the past, various fault-diagnosis techniques have been proposed, including expert systems (Wang and Yang, 1996), neural networks (Chen *et al.*, 1996). The expert system can take human expertise and has been successfully applied in this field. However, there are some intrinsic shortcomings for the expert system, such as the difficulty of acquiring knowledge and maintaining a database.

These may vary from utility to utility due to the heuristic nature of the method and no general mathematical formulation can be utilized. Neural networks can directly acquire experience from training data and exhibit highly nonlinear input–output relationships. This can overcome some of the shortcomings of the expert system. However, the training data must be sufficient and compatible to ensure proper training. A further limitation of the approach of neural networks is its inability to produce linguistic output, because it is difficult to understand the content of network memory. To overcome above shortcomings, Wang (2004) proposed a vibration fault diagnosis method of generator sets based on extension theory. Then Ye (2006, 2009) proposed the fault diagnosis methods of turbine based on similarity measures between vague sets and fuzzy cross entropy of vague sets.

In fact, the degree of similarity or dissimilarity between the objects under study plays an important role. In vector space, especially, the cosine similarity measure (Salton and McGill, 1987) is often used in information retrieval, citation analysis and automatic classification. However, this similarity measure does not deal with the similarity measures for vague information. Therefore, Ye (2011) proposed a cosine similarity measure between intuitionistic fuzzy sets and applied it to pattern recognition and medical diagnosis. The main purposes of this study are to propose another cosine similarity measure between vague sets in vector space based on the extension of the cosine similarity measure (Salton and McGill, 1987) and to apply it to the fault diagnosis of turbine. Then the feasibility and rationality of the proposed fault diagnosis method is validated by the fault diagnosis example of the turbine.

COSINE SIMILARITY MEASURE

Let $X = (x_1, x_2, \dots, x_n)$ and $Y = (y_1, y_2, \dots, y_n)$ be the two vectors of length n where all the coordinates are positive. The cosine similarity measure between the two vectors (measuring the “similarity” of these vectors) (Salton and McGill, 1987) is defined as:

$$Cos = \frac{X \cdot Y}{\|X\|_2 \|Y\|_2} = \frac{\sum_{i=1}^n x_i y_i}{\sqrt{\sum_{i=1}^n x_i^2} \sqrt{\sum_{i=1}^n y_i^2}} \quad (1)$$

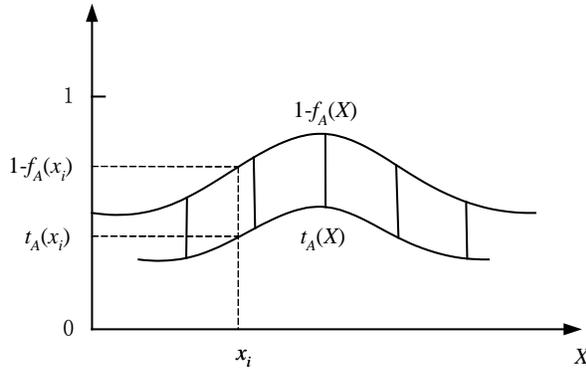


Fig. 1: A vague set

where $X \cdot Y = \sum_{i=1}^n x_i y_i$ is the inner product of the vectors X and Y and where $\|X\|_2 = \sqrt{\sum_{i=1}^n x_i^2}$ and $\|Y\|_2 = \sqrt{\sum_{i=1}^n y_i^2}$ are the Euclidean norms of X and Y (also called the L_2 norms).

Cosine formula is then defined as the inner product of these two vectors divided by the product of their lengths. This is nothing but the cosine of the angle between the vectors. The cosine similarity measure takes value in the interval $[0, 1]$. It is undefined if $x_i = 0$ or/and $y_i = 0$ ($i = 1, 2, \dots, n$). Then, let the cosine measure value be zero when $x_i = 0$ or/and $y_i = 0$ ($i = 1, 2, \dots, n$).

VAGUE SET AND ITS SIMILARITY MEASURE

Vague set: The vague set, which is a generalization of the concept of a fuzzy set, has been introduced by Gau and Buehrer (1993). Vague set theory is introduced as follows.

A vague set A in the universe of discourse $X = \{x_1, x_2, \dots, x_n\}$ is characterized by a truth-membership function $t_A(x_i)$ and a false-membership function $f_A(x_i)$ for an element $x_i \in X$ ($i = 1, 2, \dots, n$), $t_A(x_i): X \rightarrow [0, 1]$, $f_A(x_i): X \rightarrow [0, 1]$, where $t_A(x_i)$ is a lower bound on the grade of membership of x_i derived from the evidence for x_i , $f_A(x_i)$ is a lower bound on the negation of x_i derived from the evidence against x_i and the functions $t_A(x_i)$ and $f_A(x_i)$ are constrained by the condition $0 \leq t_A(x_i) + f_A(x_i) \leq 1$. The grade of membership of x_i in the vague set A is bounded to a subinterval $[t_A(x_i), 1-f_A(x_i)]$ of $[0, 1]$. The vague value $[t_A(x_i), 1-f_A(x_i)]$ indicates that the exact grade of membership $\mu_A(x_i)$ of x_i may be unknown. But it is bounded by $t_A(x_i) \leq \mu_A(x_i) \leq 1-f_A(x_i)$. Figure 1 shows a vague set in the universe of discourse X .

In the sequel, we will omit the argument x_i of $t_A(x_i)$ and $f_A(x_i)$ throughout unless they are needed for clarity.

Let X be the universe of discourse $X = \{x_1, x_2, \dots, x_n\}$, $x_i \in X$. A vague set A of the universe of discourse X can be represented by:

$$A = \{(x_1, [t_{A1}, 1-f_{A1}]), (x_2, [t_{A2}, 1-f_{A2}]), \dots, (x_n, [t_{An}, 1-f_{An}])\}$$

Let $t_i^* = 1 - f_i$, where $1 \leq i \leq n$. In this case, the vague set A can be rewritten as:

$$A = \{(x_1, [t_{A1}, t_{A1}^*]), (x_2, [t_{A2}, t_{A2}^*]), \dots, (x_n, [t_{An}, t_{An}^*])\}.$$

Cosine similarity measure between vague sets:

Assume that there are two vague sets $A = \{(x_1, [t_{A1}, t_{A1}^*]), (x_2, [t_{A2}, t_{A2}^*]), \dots, (x_n, [t_{An}, t_{An}^*])\}$ and $B = \{(x_1, [t_{B1}, t_{B1}^*]), (x_2, [t_{B2}, t_{B2}^*]), \dots, (x_n, [t_{Bn}, t_{Bn}^*])\}$ in the universe of discourse $X = \{x_1, x_2, \dots, x_n\}$, $x_i \in X$. The parameters (elements) in A and B can be considered as two pairs of vector representations with the length of n elements:

$$T_A = (t_{A1}, t_{A2}, \dots, t_{An}) \text{ and } T_A^* = (t_{A1}^*, t_{A2}^*, \dots, t_{An}^*)$$

$$T_B = (t_{B1}, t_{B2}, \dots, t_{Bn}) \text{ and } T_B^* = (t_{B1}^*, t_{B2}^*, \dots, t_{Bn}^*)$$

Based on the extension of cosine similarity measure (Salton and McGill, 1987), a similarity measure between T_A and T_B is proposed in the vector space as follows:

$$Cos(T_A, T_B) = \frac{\sum_{i=1}^n t_{Ai} t_{Bi}}{\sqrt{\sum_{i=1}^n (t_{Ai})^2} \sqrt{\sum_{i=1}^n (t_{Bi})^2}} \tag{2}$$

A cosine similarity measure between T_A^* and T_B^* is proposed in the vector space as follows:

$$Cos^*(T_A^*, T_B^*) = \frac{\sum_{i=1}^n t_{Ai}^* t_{Bi}^*}{\sqrt{\sum_{i=1}^n (t_{Ai}^*)^2} \sqrt{\sum_{i=1}^n (t_{Bi}^*)^2}} \tag{3}$$

Thus, the cosine similarity measure between A and B is proposed in the vector space as follows:

$$C_{vs}(A, B) = \alpha Cos(T_A, T_B) + (1 - \alpha) Cos^*(T_A^*, T_B^*) \tag{4}$$

where, $\alpha \in [0, 1]$ is the weight of similarity measures.

The cosine similarity measure between vague sets A and B satisfies the following properties:

- (P1) $0 \leq C_{vs}(A, B) \leq 1$
- (P2) $C_{vs}(A, B) = C_{vs}(B, A)$
- (P3) $C_{vs}(A, B) = 1$ if and only if $A = B$, i.e., $t_{Ai} = t_{Bi}$ and $t_{Ai}^* = t_{Bi}^*$ for $i = 1, 2, \dots, n$
- (P4) $C_{vs}(A, B) = 0$ if $t_{Ai} = 0$ and $t_{Ai}^* = 0$ or/and $t_{Bi} = 0$ and $t_{Bi}^* = 0$ for $i = 1, 2, \dots, n$

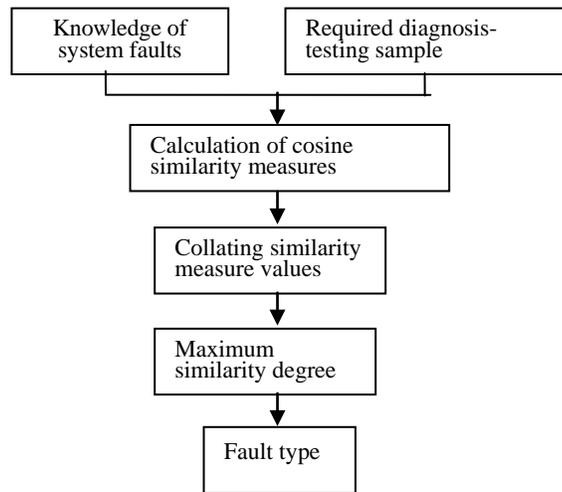


Fig. 2: Block diagram of fault diagnosis using the cosine similarity measure of vague sets

Proof:

- (P1) It is obvious that the property is true according to cosine values for Eq. (2) and (3).
- (P2) It is obvious that the property is true.
- (P3) When $A = B$, there are $t_{Ai} = t_{Bi}$ and $t_{Ai}^* = t_{Bi}^*$ for $i = 1, 2, \dots, n$. So there is $C_{vs}(A, B) = 1$. When $C_{vs}(A, B) = 1$, there are $t_{Ai} = t_{Bi}$ and $t_{Ai}^* = t_{Bi}^*$ for $i = 1, 2, \dots, n$. So there is $A = B$.
- (P4) It is obvious that the property is true.

FAULT DIAGNOSIS BASED ON THE COSINE SIMILARITY MEASURE

In this section, we will apply the cosine similarity measure between vague sets to fault diagnosis. Essentially, the technique of equipment diagnosis is a pattern recognition problem. In other word, the operating status of the machine is divided into normal and abnormal statuses. Further speaking, the signal sample of the abnormality belongs to which type on earth; this is a pattern recognition problem again.

Fault diagnosis principle: Assume that there exist m fault patterns (knowledge of fault samples), which are represented by a vague set F_i ($i = 1, 2, \dots, m$) and there is a testing sample to be recognized which is represented by a vague set F_t . Then diagnosing result F_k should be the nearest one to F_t , i.e:

$$C_{vs}(F_t, F_k) = \text{Max}_{1 \leq i \leq m} \{C_{vs}(F_t, F_i)\}$$

where $C_{vs}(F_t, F_i)$ expresses the similarity measure between the vague sets F_t and F_i . The value of $C_{vs}(F_t, F_i)$ is calculated by using Eq. (2-4). Then, we decide that the testing sample F_t should belong to the fault pattern F_k ,

where, $k = \arg \text{Max}_{1 \leq i \leq m} \{C_{vs}(F_t, F_i)\}$

The cosine similarity measure of vague sets provides the strong means for the fault diagnosis. It can realize the classification and identification of the fault, i.e., we can compare the similarity measure values by calculating the similarity measures between a diagnosing sample and knowledge of system faults and then confirm the fault type according to the maximum similarity degree (more than some threshold).The fault diagnosis process using the cosine similarity measure of vague sets is shown in Fig. 2 (Ye, 2009).

Fault diagnosis of turbine: An example with the steam turbine-generator set demonstrates the effectiveness of the fault diagnosis of turbine by using the proposed fault diagnosis method. The vibration of huge steam turbine-generator sets suffers the influence of a lot of factors, such as mechanical structure, load, vacuum degree, hot inflation of cylinder body and rotor, fluctuation of network load, temperature of lubricant oil and ground. In generator sets, interaction effects in these factors show the vibration of the generator sets. In the vibration fault diagnosis of the generator sets, we have established the relation between the cause and the fault symptom of the turbine adopted from (Ye, 2006, 2009). Now, we investigate the fault diagnosis problems by means of the cosine similarity measure of vague sets to demonstrate the effectiveness of the fault diagnosis problems of turbine.

The ten kinds of familiar fault types in rotating machines, such as unbalance, offset center and oil-membrane oscillation, are used as the knowledge of fault samples. By use of nine ranges of different frequency spectrum, the power spectrum of the vibration signals from some generator sets is referred to

Table 1: Knowledge of system faults

Fault samples	Frequency range (f: operating frequency)								
	0.01-0.39f	0.40-0.49f	0.50f	0.51-0.99f	f	2f	3-5f	Odd times of f	High freq >5f
Unbalance	(0.00,0.00)	(0.00,0.00)	(0.00,0.00)	(0.00,0.00)	(0.85,1.00)	(0.04,0.06)	(0.04,0.07)	(0.00,0.00)	(0.00,0.00)
Pneumatic force couple	(0.00,0.00)	(0.28,0.31)	(0.09,0.12)	(0.55,0.70)	(0.00,0.00)	(0.00,0.00)	(0.00,0.00)	(0.00,0.00)	(0.08,0.13)
Offset center	(0.00,0.00)	(0.00,0.00)	(0.00,0.00)	(0.00,0.00)	(0.30,0.58)	(0.40,0.62)	(0.08,0.13)	(0.00,0.00)	(0.00,0.00)
Oil-membrane oscillation	(0.09,0.11)	(0.78,0.82)	(0.00,0.00)	(0.08,0.11)	(0.00,0.00)	(0.00,0.00)	(0.00,0.00)	(0.00,0.00)	(0.00,0.00)
Radial impact friction of rotor	(0.09,0.12)	(0.09,0.11)	(0.08,0.12)	(0.09,0.12)	(0.18,0.21)	(0.08,0.13)	(0.08,0.13)	(0.08,0.12)	(0.08,0.12)
Symbiosis looseness	(0.00,0.00)	(0.00,0.00)	(0.00,0.00)	(0.00,0.00)	(0.18,0.22)	(0.12,0.17)	(0.37,0.45)	(0.00,0.00)	(0.22,0.28)
Damage of antithrust bearing	(0.00,0.00)	(0.00,0.00)	(0.08,0.12)	(0.86,0.93)	(0.00,0.00)	(0.00,0.00)	(0.00,0.00)	(0.00,0.00)	(0.00,0.00)
Surge	(0.00,0.00)	(0.27,0.32)	(0.08,0.12)	(0.54,0.62)	(0.00,0.00)	(0.00,0.00)	(0.00,0.00)	(0.00,0.00)	(0.00,0.00)
Looseness of bearing block	(0.85,0.93)	(0.00,0.00)	(0.00,0.00)	(0.00,0.00)	(0.00,0.00)	(0.00,0.00)	(0.00,0.00)	(0.08,0.12)	(0.00,0.00)
Non-uniform bearing stiffness	(0.00,0.00)	(0.00,0.00)	(0.00,0.00)	(0.00,0.00)	(0.00,0.00)	(0.77,0.83)	(0.19,0.23)	(0.00,0.00)	(0.00,0.00)

vague sets. We use the typical fault samples as the fault knowledge as shown in Table 1 (Ye, 2006, 2009).

In the diagnosis process, at first, we establish the knowledge database of fault types and then calculate the cosine similarity measures between a fault-testing sample and fault knowledge samples.

Ten fault knowledge samples in Table 1 are expressed by the following vague sets:

- $F_1 = \{(x_1, [0.00, 0.00]), (x_2, [0.00, 0.00]), (x_3, [0.00, 0.00]), (x_4, [0.00, 0.00]), (x_5, [0.85, 1.00]), (x_6, [0.04, 0.06]), (x_7, [0.04, 0.07]), (x_8, [0.00, 0.00]), (x_9, [0.00, 0.00])\}$
- $F_2 = \{(x_1, [0.00, 0.00]), (x_2, [0.28, 0.31]), (x_3, [0.09, 0.12]), (x_4, [0.55, 0.70]), (x_5, [0.00, 0.00]), (x_6, [0.00, 0.00]), (x_7, [0.00, 0.00]), (x_8, [0.00, 0.00]), (x_9, [0.08, 0.13])\}$
- $F_3 = \{(x_1, [0.00, 0.00]), (x_2, [0.00, 0.00]), (x_3, [0.00, 0.00]), (x_4, [0.00, 0.00]), (x_5, [0.30, 0.58]), (x_6, [0.40, 0.62]), (x_7, [0.08, 0.13]), (x_8, [0.00, 0.00]), (x_9, [0.00, 0.00])\}$
- $F_4 = \{(x_1, [0.09, 0.11]), (x_2, [0.78, 0.82]), (x_3, [0.00, 0.00]), (x_4, [0.08, 0.11]), (x_5, [0.00, 0.00]), (x_6, [0.00, 0.00]), (x_7, [0.00, 0.00]), (x_8, [0.00, 0.00]), (x_9, [0.00, 0.00])\}$
- $F_5 = \{(x_1, [0.09, 0.12]), (x_2, [0.09, 0.11]), (x_3, [0.08, 0.12]), (x_4, [0.09, 0.12]), (x_5, [0.18, 0.21]), (x_6, [0.08, 0.13]), (x_7, [0.08, 0.13]), (x_8, [0.08, 0.12]), (x_9, [0.08, 0.12])\}$
- $F_6 = \{(x_1, [0.00, 0.00]), (x_2, [0.00, 0.00]), (x_3, [0.00, 0.00]), (x_4, [0.00, 0.00]), (x_5, [0.18, 0.22]), (x_6, [0.12, 0.17]), (x_7, [0.37, 0.45]), (x_8, [0.00, 0.00]), (x_9, [0.22, 0.28])\}$
- $F_7 = \{(x_1, [0.00, 0.00]), (x_2, [0.00, 0.00]), (x_3, [0.08, 0.12]), (x_4, [0.86, 0.93]), (x_5, [0.00, 0.00]), (x_6, [0.00, 0.00]), (x_7, [0.00, 0.00]), (x_8, [0.00, 0.00]), (x_9, [0.00, 0.00])\}$
- $F_8 = \{(x_1, [0.00, 0.00]), (x_2, [0.27, 0.32]), (x_3, [0.08, 0.12]), (x_4, [0.54, 0.62]), (x_5, [0.00, 0.00]), (x_6, [0.00, 0.00]), (x_7, [0.00, 0.00]), (x_8, [0.00, 0.00]), (x_9, [0.00, 0.00])\}$
- $F_9 = \{(x_1, [0.85, 0.93]), (x_2, [0.00, 0.00]), (x_3, [0.00, 0.00]), (x_4, [0.00, 0.00]), (x_5, [0.00, 0.00]), (x_6, [0.00, 0.00]), (x_7, [0.00, 0.00]), (x_8, [0.08, 0.12]), (x_9, [0.00, 0.00])\}$

$$F_{10} = \{(x_1, [0.00, 0.00]), (x_2, [0.00, 0.00]), (x_3, [0.00, 0.00]), (x_4, [0.00, 0.00]), (x_5, [0.00, 0.00]), (x_6, [0.77, 0.83]), (x_7, [0.19, 0.23]), (x_8, [0.00, 0.00]), (x_9, [0.00, 0.00])\}$$

Suppose that the vague sets of two fault-testing samples are as follows:

- $F_{t1} = \{(x_1, [0.00, 0.00]), (x_2, [0.00, 0.00]), (x_3, [0.10, 0.10]), (x_4, [0.90, 0.90]), (x_5, [0.00, 0.00]), (x_6, [0.00, 0.00]), (x_7, [0.00, 0.00]), (x_8, [0.00, 0.00]), (x_9, [0.00, 0.00])\}$
- $F_{t2} = \{(x_1, [0.39, 0.39]), (x_2, [0.07, 0.07]), (x_3, [0.00, 0.00]), (x_4, [0.06, 0.06]), (x_5, [0.00, 0.00]), (x_6, [0.13, 0.13]), (x_7, [0.00, 0.00]), (x_8, [0.00, 0.00]), (x_9, [0.35, 0.35])\}$

Take $\alpha = 0.5$ in general. The similarity measure values of the vague sets are calculated by use of the proposed cosine similarity measure as follows:

$$C_{vs}(F_{t1}, F_1) = 0.0000, C_{vs}(F_{t1}, F_2) = 0.8937, C_{vs}(F_{t1}, F_3) = 0.0000, C_{vs}(F_{t1}, F_4) = 0.1159, C_{vs}(F_{t1}, F_5) = 0.3296, C_{vs}(F_{t1}, F_6) = 0.0000, C_{vs}(F_{t1}, F_7) = 0.9998, C_{vs}(F_{t1}, F_8) = 0.8924, C_{vs}(F_{t1}, F_9) = 0.0000, C_{vs}(F_{t1}, F_{10}) = 0.0000$$

$$C_{vs}(F_{t2}, F_1) = 0.0127, C_{vs}(F_{t2}, F_2) = 0.2439, C_{vs}(F_{t2}, F_3) = 0.1794, C_{vs}(F_{t2}, F_4) = 0.2262, C_{vs}(F_{t2}, F_5) = 0.5347, C_{vs}(F_{t2}, F_6) = 0.3586, C_{vs}(F_{t2}, F_7) = 0.1089, C_{vs}(F_{t2}, F_8) = 0.1537, C_{vs}(F_{t2}, F_9) = 0.7075, C_{vs}(F_{t2}, F_{10}) = 0.1223$$

Diagnosis result 1: have very low possibility owing to the similarity measure value of zero. By actual checking, we discover that one of antithrust bearings is damage. Consequently it causes the violent vibration of the turbine. It is easy to diagnose the fault types of the turbine from the first fault diagnosis results. For the first fault-testing sample, the cosine similarity measure value of the main fault type F_7 is equal to 0.9998 (maximum value), which indicate that the vibration fault of the turbine is firstly resulted from damage of antithrust bearing. Then, because the cosine similarity measure values of the fault types F_2 and F_8 are more than 0.5 (threshold), the fault types of pneumatic force couple and surge have high possibility. While the fault

type F_5 (radial impact friction of rotor) has low possibility owing to small measure value (0.3296). Obviously, the fault types of F_1 , F_3 , F_6 , F_9 and F_{10}

Diagnosis result 2: According to the diagnosis results of the second fault-testing sample, we can think that the vibration fault of the turbine is firstly resulted from looseness of bearing block of the fault type F_9 owing to the maximum value of the similarity measure (0.7075). Then, the fault type F_5 (radial impact friction of rotor) has possibility owing to the measure value of 0.5347. While the fault type F_6 (Symbiosis looseness) has low possibility owing to small measure value (0.3586). Other fault types have very low possibility owing to smaller measure values. By actual checking, we discover the friction between the rotor and cylinder body in the turbine and then the vibration values of four ground bolts of the bearing between the turbine and the gearbox are very difference. We also discover that the gap between the nuts and the bearing block is oversize. Consequently the looseness of the bearing block causes the violent vibration of the turbine.

From the above fault diagnostic results, we can see that the proposed diagnosis method is effectiveness. The diagnosis results of the turbine show that the proposed method not only diagnoses the main fault types of the turbine, but also provides useful information for multi-fault analysis and future trends. Therefore, the method can offer effective and reasonable diagnosis information for multi-fault analyses.

CONCLUSION

In this study, we proposed a novel cosine similarity measure between vague sets and its fault diagnosis and investigated the cosine similarity measure for the fault

diagnoses of turbine. The diagnosis results of the turbine show that the proposed method not only diagnoses the main fault types of the turbine, but also provides useful information for multi-fault analyses and future trends. The proposed methods in this study are effective and reasonable in fault diagnoses. This fault diagnosis method is easier and more practical than other traditional artificial intelligence methods. Furthermore, the proposed method will provide another useful tool for fault analyses.

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