

Rolling Bearing Failure Feature Extraction Based on Hilbert Transform and Stochastic Resonance

Zengqing Ma, Yingna Yang and Jianhua Liang

School of Electrical and Electronics Engineering, Shijiazhuang Tiedao University, China

Abstract: Based on the generate mechanism of rolling bearing fault signal and its modulation model in the process of spreading, an improved method that combining Hilbert transformation and Stochastic Resonance (SR) is proposed for rolling bearing fault features extraction. Subsequently, the method is used to extract fault signal features from three kinds of typical faults, the surface damage of the inner ring, outer ring stripping injury and roller electrical erosion. First, low frequency envelope components are acquired from rolling bearing vibration signals through Hilbert transformation. Then, depending on the advantage of SR that SR is immune to noise and sensitive to periodic signal, cyclical faults signal of the low frequency envelope is highlighted by using the variable step size solution that can overcome adiabatic condition limitation of SR system. The experimental results show that the algorithm can extract the fault feature and identify the fault type effectively.

Keywords: Envelope detection, hilbert transform, rolling bearing, stochastic resonance

INTRODUCTION

Rolling bearings as one of the most commonly used general spare parts in all kinds of rotating machinery, on-line monitoring and fault diagnosis always drew much attention in the field of engineering technology at home and abroad. Wenhu *et al.* (1999) summarized that failure feature information extraction was one of the key problems in the rolling bearing fault diagnosis. Traditional diagnosis technology always based on the time domain or frequency domain characteristics of vibration signals to extract feature vector and identify fault type. At present, commonly used methods include wavelet packet technique, Empirical Mode Decomposition (EMD) and Support Vector Machine (SVM), etc. However, wavelet transform is essentially adjustable window of Fourier transform and wavelet length was limited, so that produced energy leakage and is not self-adapting in signal analysis. For the above problem, corresponding proofs in different angles were given by Rong *et al.* (2006) and Zhihua *et al.* (2006). Huang *et al.* (1998) and Zhigang *et al.* (2012) studied that EMD method was adaptive and suitable for analyzing nonlinear and non-stationary signal, whose principle was for non-stationary signal smooth processing and then gradually decomposed the signal in different fluctuations or trend scales. Lei *et al.* (2009) considered that SVM was a new machine learning method based on statistical

learning theory, which showed unique advantages and good applicative prospect in the small sample problem solving and has good generalization ability. Methods above were all based on the data results to analyze the fault features and achieve fault diagnosis results.

This study analyzes rolling bearing fault signal production mechanism and puts forward a fault feature extraction algorithm integrating Hilbert transformation with Stochastic Resonance (SR) in accordance with the modulation model in spreading. Hilbert transform is actually a phase shifter, which is adaptable to extract signal envelope and doesn't bring in new component in signal processing. Deli and Yanbin (2012) improved Hilbert-Huang transform in the electric power harmonic application. SR is a kind of nonlinear phenomena and its principle is: inputting signal and noise to the nonlinear system, there is the synergy, that noise energy transfer to signal energy and the system products the familiar resonant output similar in mechanics. Wentao (2011) presented rolling bearing surface damage fault feature extraction and diagnosis method with SR. More, taking the variable step size solution, the SR system can also have a good resonance output under the condition of big parameters. Inner ring surface damage, outer ring stripping injury and roller electrical erosion are three typical rolling bearing faults. Adopting the above algorithm, the experiments pick up characteristic information from those faults, which of results show that the algorithm can effectively meet the target of feature extraction and fault identification.

METHODOLOGY

Rolling bearing fault signal modulation model:

Rolling bearing can generate two vibrations in the normal operation. One is normal vibration caused by bearing element material properties, for instance unevenness, roughness and striation; the other is natural vibration, which belongs to forced vibration, induced by the crash of rolling element with inner and outer ring. According to the book written by Liangju (2003), natural vibration has a large natural frequency at 1~20 KHz and sometimes up to 80 KHz.

When some local damage appear in rolling bearing and other components in contact surface occur periodic collision, which is called through vibration. The vibration frequency is also called fault characteristic frequency which can be counted through the rotating speed and the size of the bearing.

Rolling bearing is a kind of high precision parts. Once there is a local damage, normal vibration and natural vibration must become more intensive as with material asymmetry. So when faults happen, the frequency of vibration also contains other frequency in addition to the fault characteristic frequency and the natural frequency. Various kinds of vibratory signals of rolling bearing are relatively independent, which are just recombined through multiplication or plus in the process of vibration transmission. Its mechanism is similar to amplitude modulation. Because the natural vibration is the fiercest and natural frequency is the biggest in all of the vibration, it's main that natural vibration modulates other vibration. With the influence of the noise, it is reduced to the under model Eq. (1) from the study of Zhiyang (2011):

$$s(t) = A(1 + \sum_{i=1}^k r_i \sin(2\pi f_i t + \theta_i)) \cdot \sin(2\pi f_r t + \phi) + n(t) \tag{1}$$

where,

f_i = The fault characteristic frequency or other ingredient

f_r = The natural frequency

It can be seen from the model that $r_i \sin(2\pi f_i t + \theta_i)$ is a modulation signal and $A \sin(2\pi f_r t + \phi)$ is high frequency natural oscillation carrier signal.

From the above-mentioned model, it's concerned that low frequency which contains all kinds of fault information in rolling bearing fault diagnosis. If high frequency carrier item can be eliminated and low frequency item can be demodulated from the model, then whether the corresponding fault happened can be judged by detecting whether the demodulation signal

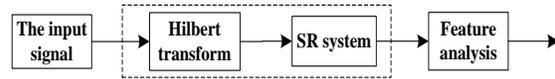


Fig. 1: Algorithmic schematic block diagram of the fault feature extraction

contains obvious fault features. Figure 1 is a block diagram of fault feature extraction for the modulation. And the part in dotted line box is the algorithm that this study puts forward.

- **Hilbert principle:** Hilbert transform has the characteristic of excellent envelope detection and good adaptability. Even if high frequency carrier signal is unknown, it can also achieve the modulation signal envelope.
- **Hilbert algorithm basic principle:** For a real signal $x(t)$, its Hilbert transform is recorded to $\hat{x}(t)$ as the following type (2):

$$\hat{x}(t) = \frac{1}{\pi t} * x(t) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{x(\tau)}{t - \tau} d\tau \tag{2}$$

Fourier transform of formula (2) is:

$$\hat{x}(f) = X(f) \times F\left(\frac{1}{\pi t}\right) = X(f) \times [-j \operatorname{sgn}(f)] \tag{3}$$

Formula (3) illustrates that $\hat{x}(f)$ is equal to the phase-shifting of $X(f)$ in frequency domain, phase delays $\pi/2$ in positive frequency domain and phase leads $\pi/2$ in negative frequency domain. In fact, Hilbert is a phase shifter. $\hat{x}(t)$ and $x(t)$ is orthogonal. An analytical function for $x(t)$ as follow type (4):

$$\tilde{x}(t) = x(t) + j\hat{x}(t) \tag{4}$$

The amplitude and phase of the analytical function are formula (5) and (6):

$$|\tilde{x}(t)| = \sqrt{x(t)^2 + \hat{x}(t)^2} \tag{5}$$

$$\theta(t) = \arctan\left(\frac{\hat{x}(t)}{x(t)}\right) \tag{6}$$

- **Envelope detection principle:** Suppose $x(t) = b(t)g(t)$, $g(t)$ is the high frequency signal and $b(t)$ is the low frequency signal, the Hilbert transfer of $x(t)$ is as flow. The following type (7) has a

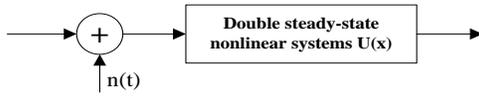


Fig. 2: Double steady-state nonlinear stochastic resonance system

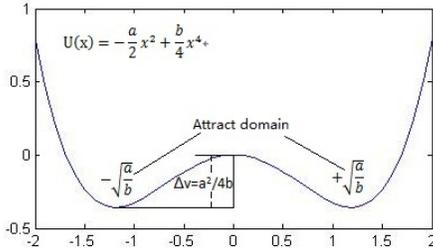


Fig. 3: Potential function diagram

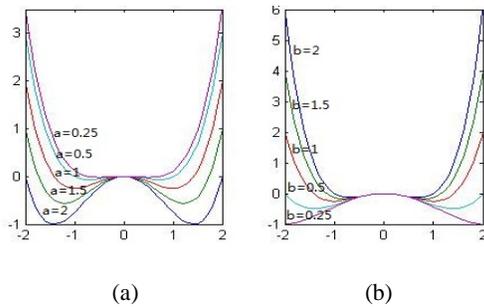


Fig. 4: Potential function diagrams under different values of a and b

detailed proof process in the book of Qiansheng (2010):

$$\tilde{x}(t) = b(t)g(t) \tag{7}$$

The above formulation accounts for that if a low frequency signal multiplies a high frequency signal and the Hilbert transformation of the result depends on the latter one.

Assume that non-noise part in rolling bearing vibration model is $s_1(t)$:

$$s_1(t) = A \left(1 + \sum_{i=1}^k r_i \sin(2\pi f_i t + \theta_i) \right) \cdot \sin(2\pi f_c t + \varphi) \tag{8}$$

Depending on formulation (7), the Hilbert transformation of formulation (8) is:

$$\hat{s}_1(t) = -A \left(1 + \sum_{i=1}^k r_i \sin(2\pi f_i t + \theta_i) \right) \cdot \cos(2\pi f_c t + \varphi) \tag{9}$$

According to formulation (5) and formulation (9):

$$\begin{aligned} |\tilde{s}_1(t)| &= \sqrt{s_1(t)^2 + \hat{s}_1(t)^2} \\ &= \left| A \left(1 + \sum_{i=1}^k r_i \sin(2\pi f_i t + \theta_i) \right) \right| \end{aligned} \tag{10}$$

As seen, $|\tilde{s}_1(t)|$ in type (10) is the signal envelop which is the concerned low frequency part containing all kinds of fault message. So like that high frequency part can be eliminated from the modulated model through Hilbert transformation and the low frequency envelope message containing fault message can be separated without damaging the original signal. Of course, this is the ideal result without noises. In fact the signal frequency spectrum after Hilbert transformation still contains other frequency components and the features of fault are not evident yet. So it is needed to use SR to making a further operating and extracting the fault characteristic.

- **SR principle:** Noise is often considered a nuisance in signal analysis, whose existence lessens the signal-to-noise ratio and goes against the useful information extraction. However, inputting periodic signal and a certain amount of noise in nonlinear system, the SNR of the system output will be greatly improve in a "resonance" point, so that the original signal submerged by strong noise becomes evident. This is the phenomenon of Stochastic Resonance (SR). But the application of SR system were limited in the adiabatic condition (low frequency small parameter), the strict definition of which were given by Gang (1992). But the reality value of the frequency is much bigger than the constraints. The variable step length and parameters adjustment can break through the limitation of SR adiabatic condition, so that obvious phenomenon of SR can also emerge in large frequency.

- **Basic principle of SR:** Double steady-state system is the research basis of SR, its structure as shown in Fig. 2. The Langevin Eq. (11) with double potential wells is typical model to describe nonlinear double steady-state nonlinear system:

$$\frac{dx}{dt} = -\frac{dU(x)}{dx} + s(t) + n(t) \tag{11}$$

where,

$x(t)$ = The system output

$s(t)$ = Input signal of the nonlinear systems

$n(t)$ = Random noise signal

$U(x)$ (Fig. 3) is symmetrical potential function of double steady-state nonlinear system. a and b are the real numbers greater than zero, which are the shape parameters of the potential well. Without the influence of modulation and noise, the system has two same

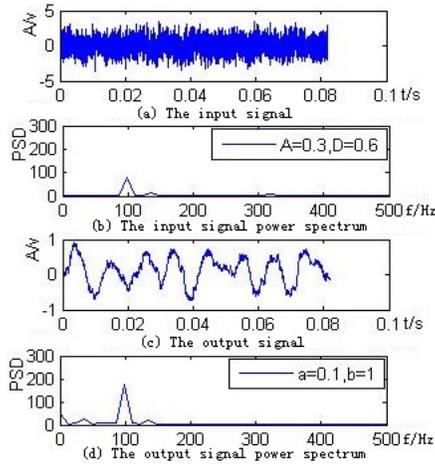


Fig. 5: Frequency of 100 Hz signal variable step stochastic resonance result

wells; the bottom of the well is located in the place $x = \pm\sqrt{a/b}$; and potential barrier height is $\Delta v = a^2 / 4b$. The high degree of the system potential barrier is the key factor influencing the resonance. The higher the potential barrier is the larger requirement of signal energy is. What can be clearly seen from the Fig. 4 is the law that potential function changes following a and b. With the increase of a, potential barrier height increases and the distance between two attractive regions decreases. With the increase of b, potential barrier height decreases and the distance between two attractive regions decreases.

- **Principle of variable step SR algorithm:** In practical application, $s(t)$ and $n(t)$ in the type of (11) general don't have the exact analytical expression, so the output $x(t)$ also has no analytical solution and we cannot directly solve the differential equation. Qiang *et al.* (2006) applied one variable step SR algorithm. Use for reference, this study gives a difference four order Runge-Kutta solution algorithm to get $x(t)$ and its specific process such as Eq. (12):

$$\begin{cases} x_{n+1} = x_n + \frac{k_1}{6} + \frac{k_2}{3} + \frac{k_3}{3} + \frac{k_4}{6} \\ k_1 = h[a x_n - b x_n^3 + s_{n+1}] \\ k_2 = h[a(x_n + \frac{k_1}{2}) - b(x_n + \frac{k_1}{2})^3 + s_{n+1}] \\ k_3 = h[a(x_n + \frac{k_2}{2}) - b(x_n + \frac{k_2}{2})^3 + s_{n+1}] \\ k_4 = h[a(x_n + \frac{k_3}{2}) - b(x_n + \frac{k_3}{2})^3 + s_{n+1}] \end{cases} \quad (12)$$

In the above type, $s_n(t) = A \cos(2\pi f_0 t) + n(t)$; $n(t)$ is the wideband noise. x_n and S_n is the n th sampling values of the system output(t) and input $S_n(t)$. h is the

integral step, that determined by the largest frequency largest frequency f_s ($f_s \geq 50 f_0$) of the detected signal and $h = 1/f_s$.

But the larger the frequency is, the denser the numerical point is, the smaller the computation step h is and the greater the clutter energy in calculation results is. So to meet the need, h is taken a little larger value in operation. And in the below experiments, h is greater than the reciprocal value of the frequency.

In the variable step algorithm realization, the synergy effect can reach the optimal result, combining parameters change rules in Fig. 4. At the same time, it has improved testing frequency f_0 a lot. In the following Fig. 5, $A = 0.3$, $D = 0.6$, $f_0 = 100\text{Hz}$, $f_s = 50\text{KHz}$, $h = 0.02$. Figure 5 proves that obvious phenomenon of SR can also emerge in large frequency through the adjustment of variable step and parameters.

RESULTS AND DISCUSSION

The data source: The fault data is about the 352226×2-2RZ rolling bearing. Its main structural parameters are listed in Table 1. According parameters in Table 1, theoretical value of the fault characteristic frequency for 35222×62-2RZ rolling bearing can be gotten.

In the experiment, three typical faults, the surface damage of the inner ring, outer ring stripping injury and roller electrical erosion, are chosen to extract fault features by using the proposed algorithm. There are three pictures about those three faults in Fig. 6. Figure 6a shows that a pit appears in the rolling bearing inner ring and its diameter size is about 18 mm, which belongs to the inner ring surface damages. Figure 6b shows that one of rolling bearing rollers surface has a 5 mm strip corrosion, which belongs to the roller electrical erosions. Figure 6c shows that there are a few paint bodies spall on the outer ring of rolling bearing, which belongs to the outer ring detachment faults.

The result of experiment: In the experiment, rolling bearing only have one type fault and other parts are normal; the rotation speed is 467r/s, the sampling frequency is 5120 Hz and the sampling time is 20s. In order to observe the data clearly, the data were divided in small sections for 4096 length to deal with. Firstly, choose arbitrary sample data, respectively calculate power spectrum, envelope spectrum of the sample data; secondly, use the proposed method to processing the sample data, then compute power spectrum of processing results; lastly, will get experimental results as the Fig. 7 to 9.

In Fig. 7: from the original signal in Fig. 7a, any characteristics are seen; Fig. 7b is the power spectrum of original signal, which show that energy disperses in the frequency domain and fault characteristic can't be seen yet; after the Hilbert transform of original signal

The middle diameter D/mm	The diameter of roller D/mm	Contact angle $\alpha/^\circ$	The number of rollers z
176.29	24.74	8.833	20

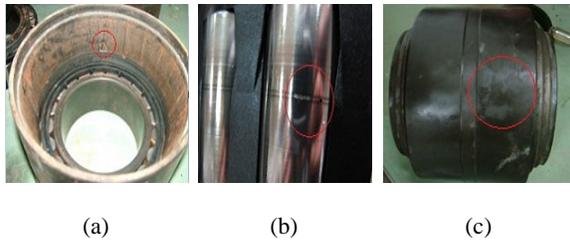


Fig. 6: Three kinds of typical damage photos of roller bearings: (a) Surface damage of the inner ring; (b) Roller electrical erosion; (c) Outer ring stripping injury

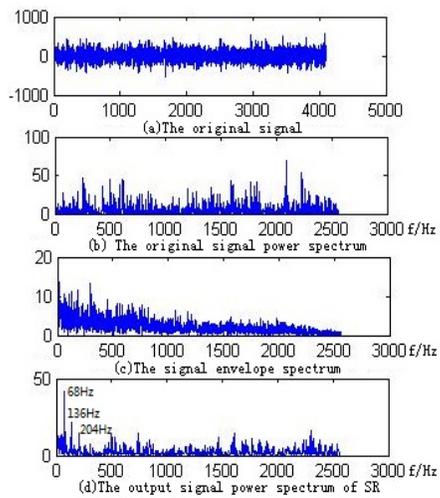


Fig. 7: Signal analysis results about surface damage of the inner ring

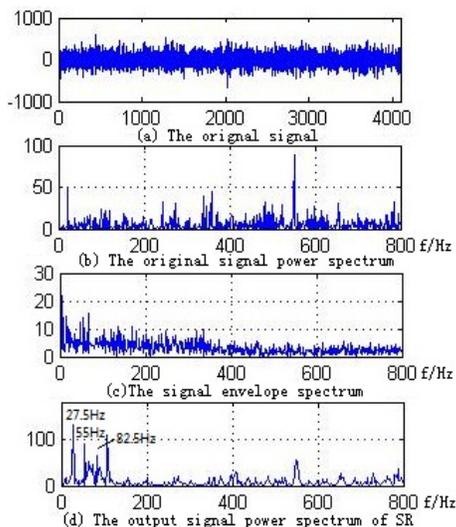


Fig. 8: Signal analysis results about roller electrical erosion

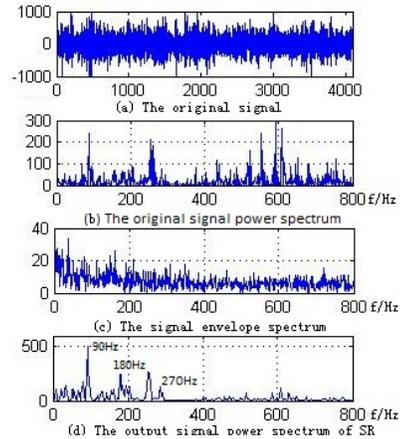


Fig. 9: Signal analysis results about outer ring stripping injury

and then figuring up the envelope spectrum of envelope signal, Fig. 7c is obtained, in what spectrum energy concentrate below 800 Hz ac, which show that the proposed method effectively eliminates the influence of high frequency carrier signal, but the fault characteristic is not obvious; input the envelope signal to SR system, compute power spectrum of the results and Fig. 7d is gotten, in what there are obvious peaks at the frequency of 68 Hz ac, 136 Hz ac and 204 Hz ac, which is said that peaks appear at the two times and three times frequency, so it is known that frequency of 68 Hz is the fault characteristic frequency.

The results of Fig. 8 and 9 are similar to Fig. 7, here no longer to say, but what needs to point out is that in order to facilitate observe, Fig. 8b to d and 9 are only showed within 0~800 Hz ac parts. In Fig. 8d, obvious peaks appear at the frequency of 90 Hz, 180 Hz and 270 Hz ac, so frequency of 90 Hz is the fault characteristic frequency of the electrical erosion fault roller bearing. In Fig. 9d, obvious peaks appear at the frequency of 27.5 Hz, 55 Hz and 82.5 Hz ac, so frequency of 27.5 Hz is the fault characteristic frequency of the outer peel fault roller bearing.

To sum up, through the Hilbert transform and SR system, obvious peak appears at the fault characteristic frequency in the figure of the output signal power spectrum of SR, which verifies the feasibility of the proposed algorithm.

- **Error analysis:** Table 2 is error analysis list of rolling bearing fault characteristic frequency under experimental conditions. Among them, the second column is theoretical value of the fault characteristic frequency, which is calculated by searching relevant manual in Simple vibration diagnosis field practical technology written by Liangju (2003). The third column is the measured value. From Table 2, it is known that the experiment fault characteristic frequency is not completely equal to the theoretical value. There are mainly two aspects reasons: on the one hand, it is the accuracy of bearing itself, such as bearing

Table 2: Error analysis list of roller bearing fault characteristic frequency

Fault type	Theoretical value /Hz	Measured value /Hz	Error (%)
Outer ring stripping injury	27.372	27.5	0.4655
Inner ring surface damage	67.471	68	0.7779
Roller electrical erosion	89.195	90	0.8944

geometry size error, not pure rolling factors and so on; on the other hand, it is outside factors, such as noise and each part calculation error.

In addition, it can be seen from the table that error result of the rolling bearing fault characteristic frequency increases following the frequency. This phenomenon is mainly caused by the influence of SR link. SR theory is put forward under the adiabatic condition that requires input periodic signal frequency is far less than one. In the section Principle of variable step SR algorithm in C. SR principle of II. Methods, the variable step method and parameter adjustment make it produce resonance output in the larger frequency. However, with the frequency increase, resonance effect weaken and the error result extent.

CONCLUSION

According to the simplified modulation model of the rolling bearing fault signal, an improved method with combination of Hilbert transform and SR is put forward and used to analyzing the actual testing data in this study. The experiment results show that the method is effective and its advantages are summarized as follows.

- When the surface damage appears, bearing vibration signal energy may increases in some frequency band, mean that fault signal is modulated on the energy concentration band. It verifies the rationality of the model.
- Through envelope demodulation, low frequency envelope signal with fault information can be obtained. From that its envelope spectrum energy is concentrated in the low frequency region, it can be seen that Hilbert algorithm can effectively eliminate high frequency carrier signal.
- Using the nonlinear characteristics of stochastic resonance makes the cycle fault signal in the low frequency signal enhancing its energy and noise being abated, which improve the Signal-to-Noise Ratio (SNR). By combining the parameter adjustment and variable step algorithm for SR, it has very good effect not only to the simulation data but also to the actual fault periodic signal. It realizes the rolling bearing characteristic parameter extraction.
- Fault characteristic frequency measurement results has a certain deviation from the theoretical value, but remained in a small range.

ACKNOWLEDGMENT

This research is supported by National Natural Science Funds of China Key Program (11227201), National Natural Science Foundation of China (11202141, 11172182, 11202142), Key Program of the Ministry of Railways of China (2011J013-A) and Educational Commission of Hebei Province, China (Z2011228).

REFERENCES

- Deli, L. and Q. Yanbin, 2012. Improved Hilbert-Huang transform in the electric power harmonic application. *J. Power Syst. Protect. Control*, 6: 69-73.
- Gang, H., 1992. Random force and nonlinear system [M]. Shanghai Science and Technology Education Press, China.
- Huang, N.E., Z. Shen, S.R. Long, M.C. Wu, H.H. Shih, Q. Zheng, N.C. Yen, C.C. Tung and H.H. Liu, 1998. The Empirical mode decomposition and the Hilbert spectrum for nonlinear and nonstationary time series analysis. *Proc. R. Soc. London*, 454(12): 903-995.
- Lei, G., Jin Chen, Z. Yi, *et al.*, 2009. Wavelet support vector machine in the application of rolling bearing fault diagnosis. *J. Shanghai Jiaotong Univ.*, 4: 678-682.
- Liangju, Y., 2003. Simple Vibration Diagnosis Field Practical Technology [M]. China Machine Press, China.
- Qiang, L., W. Taiyong, *et al.*, 2006. The weak signal detection technology based on variable step size stochastic resonance. *J. Tianjin Univ.*, 4: 432-437.
- Qiansheng, C., 2010. Digital Signal Processing. 2nd Edn., Peking University Press.
- Rong, J., W. Xiaoyu, *et al.*, 2006. The application of regressive HHT based on the least squares Support Vector Machine (SVM) in hydroelectric generating fault diagnosis. *Proc. CSEE*, 26(22): 128-133.
- Wenhu, H., *et al.*, 1999. Equipment Fault Diagnosis Principle, Technology and Application. Science Press, Beijing.
- Wentao, S., 2011. Rolling bearing surface damage fault feature extraction and diagnosis method [D]. Shandong University, Shandong, China.
- Zhigang, Z., S. Xiaohui, Z. Chen and B. Tang, 2012. Rolling bearing fault feature extraction based on improved EMD and sliding kurtosis algorithm. *J. Vib. Shock.*, 31(22): 81-83.
- Zhijia, H., L. Tian, *et al.*, 2006. Neural network model based on empirical mode decomposition and its application in the rotor system fault diagnosis. *Proc. CSEE*, 26(20): 149-153.
- Zhiyang, W., 2011. Constraint independent component analysis and the application in rolling bearing fault diagnosis [D]. Ph.D. Thesis, Mechanical Design and Theory, Shanghai Jiaotong University.