

## Application of Genetic Algorithm on Heat Exchanger Network Optimization

<sup>1</sup>Shahram Ghanizadeh and <sup>2</sup>Mojtaba Sedigh Fazli

<sup>1</sup>RIPI (Research Institute of Petroleum Industry), P.O. Box 14665-137, Tehran, Iran

<sup>2</sup>Department of Computer, Islamic Azad University, Damavand Branch, Damavand, Iran

**Abstract:** Synthesis of Heat Exchanger Networks (HENs) is inherently a Mixed Integer and Nonlinear Programming (MINLP) problem. Solving such problems leads to difficulties in optimization of continuous and binary variables. This study presents a new efficient and robust method in which structural parameters are optimized by G.A. and continuous variables are handled due to a modified objective function for maximum energy recovery. Node representation is used for addressing the exchangers and networks considered as a sequence of genes. Each gene consists of nodes for generating different structures within a network. Results show this method may find new or near optimal solutions with less than 2% increase in HENs annual cost.

**Keywords:** Genetic algorithm, heat exchanger network, mixed integer and nonlinear programming problem, optimization

### INTRODUCTION

There are three major methods for heat exchanger network synthesis. The first is pinch technology and is based on thermodynamic concepts that have been introduced by Linnhoff and Flower (1978) and Linnhoff and Hindmarsh (1983). A good review on this technology is a book of Shenoy (1995). The second one belongs to optimization methods and minimization of total annual cost of networks by mathematical programming that has been introduced by Ciric and Floudas (1991) and Yee and Grossmann (1990). Summary of history of these methods has been collected by Floudas (1995). In the recent years trends in mathematical methods were to simplify in MINLP formulation and global optimization as the MILP synthesis method proposed by Barbaro and Begajewicz (2005), using graph theory and NLP method by Shirvakumar and Narasimhan (2002) and global optimization by Bjork and Westerland (2002). The last one is the methods that combine the above concepts proposed by Zhu and Nie (2002).

In mathematical programming methods, the problem is defined as an MINLP problem and is solved by deterministic, stochastic or coupling of them. Deterministic methods like GBD, OA and etc failed to converge due to mixed nature of binary and continuous variables.

Stochastic methods like Simulated Annealing have been applied (S.A.) by Athier *et al.* (1996, 1997), Genetic Algorithm (G.A.) by Lewin *et al.* (1998), Tabu Search (T.S.) by Lin and miller (2003).

Athier *et al.* (1996, 1997) has used a coupled S.A.-NLP method in which S.A. controls structural

optimization and SQP optimizes continuous parameters. Besides S.A. cannot reach to global structure and writers had problems in convergence of their NLP.A coupled method of G.A. and S.A. has been used for optimization of both binary and continuous variables by Yu *et al.* (2000) to avoid trapping in local optima. As it is known handling the constraints in G.A. needs special aspects and convergence in continuous parameters leads to many iterations. Therefore because of discrete nature of G.A., it seems that G.A. is very useful and efficient for structural optimization.

In this study G.A. is used for structural parameters while the fitness of each structure is specified by a modified NLP formulation that is based on Maximum Energy Recovery (MER) which has been described by Lewin *et al.* (1998).

In the following sections the new representation of HEN structure is mentioned and G.A. operators and NLP formulation are presented. Then some case studies are studied and results are compared with those reported in literature.

**Problem definition:** The problem can be defined as follows.

A set of hot and cold streams with their inlet and outlet temperatures are given. Also heat capacity flow rates and heat transfer coefficients of the streams are known. Hot and cold utility are available as an external sources for cooling and heating of the process streams. The goal is to design a network that has minimum total annual cost which is determined by summation of area and utility costs.

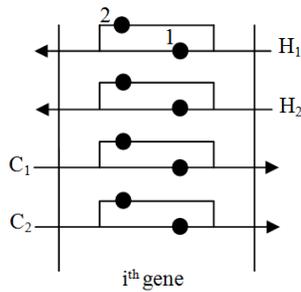


Fig. 1: Nodes in each gene with 2 branches in each splitter

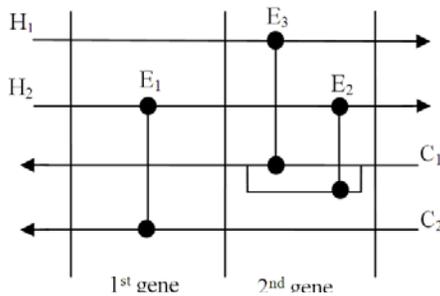


Fig. 2: A HEN with three exchangers

### REPRESENTATION OF NETWORK

In present method a HEN is treated as a chromosome and exchangers within it are considered as a sequence of genes. Therefore each gene includes the address of a heat exchanger. For addressing the location of exchangers the node representation is used like Fig. 1 in which the number of splitters and their branches can be set manually in each gene. This kind of addressing is usual and has been used by some researchers like Bochenek and Jezowski (2006) and Zhu and Asante (1999).

When two nodes are selected, a heat exchanger is defined between those nodes. The number of genes in each network depends on the size of the problem and varies in different case studies. Consider a HEN shown in Fig. 2 with three exchangers.

An Exchanger Address Matrix (EAM) is defined for showing the location of exchangers in the network like the following matrix for Fig. 2. In which each row is an address of an heat exchanger:

$$EAM = \begin{bmatrix} E_1 : 2 & 1 & 2 & 1 & 1 \\ E_2 : 2 & 1 & 1 & 2 & 2 \\ E_3 : 1 & 1 & 1 & 1 & 2 \end{bmatrix}$$

If splitting occurs in a gene, nodes of the splitter are numbered from 1 to the number of branches; otherwise number of each node will be 1. In EAM the

first column is the number of hot streams, the 2<sup>nd</sup> is node number of hot streams and the 3<sup>rd</sup> and the 4<sup>th</sup> are cold streams addresses. The 5<sup>th</sup> column represents the number of genes.

This representation is especially suited for G.A. operators in order to create feasible structures which will be described in the next section.

**G.A. operators:** Three operators are used for this evolutionary algorithm, reproduction, crossover and mutation:

- **Reproduction:** In each population some structures are exactly copied to the next generation due to the survival rate which is set to 40-50%. Selection of these chromosomes is determined by their relative fitness and roulette wheel. For more robustness of the algorithm elitism is added to save the best solution in each population. The number of structures in initial population depends on the size of problem. In small scale examples 20 chromosomes are sufficient to reach the best solution. Note that the initial population is produced randomly. It is clear that the number of iterations is proportional to the size of the initial population and chromosome length. So 40-50 iterations are employed to reach stopping criteria of the algorithm.
- **Crossover:** For combining the genetic materials, single point crossover is used. Two parents are selected by roulette wheel and a random gene number is generated to decompose the parents to four parts. These parts can combine together to produce offspring. In this study the rate of crossover is 50-60%.
- **Mutation:** The definition of mutation and its rate plays an important role in convergence of the algorithm. Because the NLP formulation is able to set the heat load of some exchangers to zero, mutation does not eliminate exchangers from the network and only changes the address of exchangers in genes. This operator removes the whole exchanger in gene and considers new random addresses within the gene. In this way a splitter may be removed and an exchanger may be replaced or vice versa. The best mutation rate for this definition of mutation is 2-4%.

**NLP formulation:** Lewin *et al.* (1998) have used an NLP formulation for Maximum Energy Recovery (MER) and the present formulation is based on their method. Although this approach is efficient, they have not considered a search for minimum approach temperature in the network. Thus in this work a search

was added to find the optimum  $\Delta T_{\min}$ . In this method area calculations are not considered explicitly in the formulation and a penalty term was added to reduce costs as much as possible. In fact this term modifies the objective function and relaxes some exchangers from pinching at  $\Delta T_{\min}$ . This penalty is very important part of the formulation. The objective function is:

Maximize:

$$\sum_{i=1}^{\text{no. of exch.}} X_i + \left( \sum_{i=1}^{2(\text{no. of exch.})} \Delta T \right) / \text{S.F.} \quad (1)$$

where,

$X_i$  = The load of exchangers

$\Delta T$  = Approach temperatures in hot or cold end of exchangers

S.F. = A scaling factor and must be large enough to ensure that the penalty term do not affect the main objective which is maximum energy recovery

Constraints of this NLP are:

- Energy balance for each exchanger on hot and cold streams (nonlinear if splitting occurs)
- Energy balance for hot and cold utilities. Heaters and coolers are included in the formulation and if they are not needed, the optimization sets their loads to zero (Linear)
- Mass balance for splitters (Linear)
- Monotonic decrease of temperatures on streams (Linear)
- Hot and cold end approach temperatures must be equal or greater than  $\Delta T_{\min}$  in each exchanger including utility exchangers (Linear)

- Energy balance at mixing points (Linear)  
For example the constraints for Fig. 3 are:

- Exchangers energy balance:

$$E_1: T_{2, \text{in}}^H - X_1/F_2^H = T_{21}^H, \\ T_{22}^C + X_1/F_2^C = T_{21}^C$$

$$E_2: T_{21}^H - X_2/F_2^H = T_{22}^H, \\ T_{1, \text{in}}^C + X_2/(y_1 F_1^C) = T_{12}^C, S1_{12}$$

$$E_3: T_{11}^H - X_3/F_1^H = T_{12}^H, \\ T_{1, \text{in}}^C + X_3/(y_2 F_1^C) = T_{12}^C, S2_{12}$$

- Utilities energy balance:

$$CU_1: T_{12}^H - C_1/F_1^H = T_{1, \text{out}}^H, \\ CU_2: T_{22}^H - C_2/F_2^H = T_{2, \text{out}}^H$$

$$HU_1: T_{11}^C + H_1/F_1^C = T_{1, \text{out}}^C, \\ HU_2: T_{22}^C + H_2/F_2^C = T_{2, \text{out}}^C$$

- Mass balance on splitters:

$$y_1 + y_2 = 1$$

- Monotonic decrease in temperatures:

$$H_1: T_{1, \text{in}}^H > T_{11}^H > T_{12}^H > T_{1, \text{out}}^H$$

For other streams same relations must written.

- Minimum approach temperatures:

$$E_1: T_{2, \text{in}}^H - T_{21}^C \geq \Delta T_{\min}, \\ T_{21}^H - T_{22}^C \geq \Delta T_{\min}$$

The same inequalities exist for  $E_2, E_3, CU_1, CU_2, HU_1$  and  $HU_2$

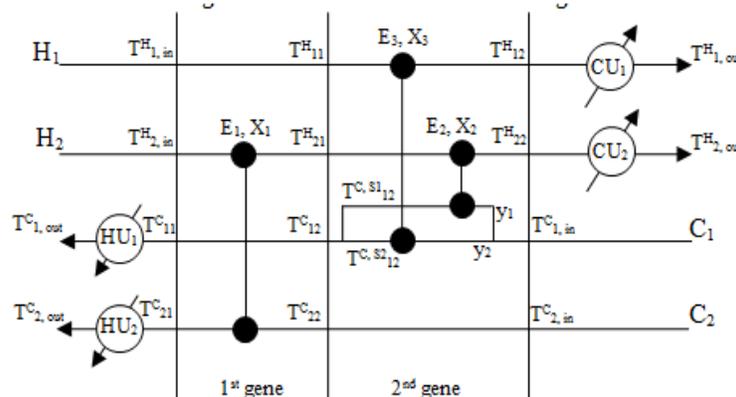


Fig. 3: A network with three exchangers

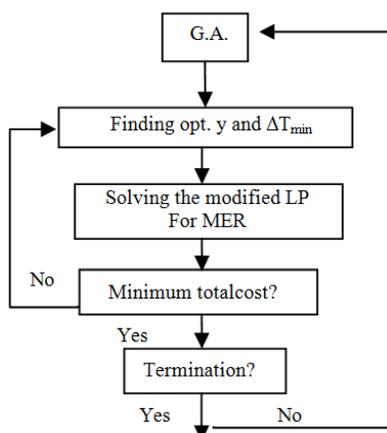


Fig. 4: The algorithm for HEN synthesis

- Mixing point energy balance:

$$T_{12}^C = y_1 T_{12}^{C, s1} + y_2 T_{12}^{C, s2}$$

In the above equations  $T_{ij}$  shows the temperature of  $i^{\text{th}}$  stream that exists at  $j^{\text{th}}$  gene and  $F$  is heat capacity flow rate. According to the above, the overall algorithm is shown in Fig. 4. In this figure, G.A. produces different networks in each population and then each network is optimized by finding the related minimum annual cost which is the summation of area and utility costs. In this algorithm split ratios and minimum approach temperatures are not optimized simultaneously with exchanger heat loads. Instead the inner loop is utilized for finding the best  $y$  and  $\Delta T_{\min}$ . In this loop the problem is converted to a modified linear programming for finding MER so this LP is really the heart of the algorithm. Therefore the NLP is converted to a search for  $y$  and  $\Delta T_{\min}$  and a LP for MER because the nonlinear terms in constraint (a) are eliminated by known split ratios.

Care must be taken for small split ratios because they import ill conditioning to the LP. So split ratios are bounded from 0.1-0.9 and a post analysis needed for removing by-passes as they impose additional area to the network. Also the search range for  $\Delta T_{\min}$  is set to [0.1, 20]. At last the fitness of each network is determined due to the best cost found by inner loop.

In summary this study has the following differences with the method of Lewin *et al.* (1998):

- Representation of network
- Definition of mutation operator
- Modification in LP by introducing a penalty term
- Searching for  $\Delta T_{\min}$
- Elimination of by-passes

These modifications help the algorithm to find better and promising solutions and converge in relatively little iteration.

On the other hand this method is not based on pressure drop aspects and the constant heat transfer coefficients are assumed for synthesis. In the rest of this study some case studies are presented.

## CASE STUDIES

Three case studies are solved by MATLAB codes. The first is an example solved by Yee and Grossmann (1990) and Shirvakumar and Narasimhan (2002). The second and third belong to Lin and Miller (2003). For all these problems counter current heat exchangers were considered. The mutation and crossover rates are set to 3 and 60% respectively and the number of chromosome in each population is 20-30. To control the size of the problem only two branches are allowed in splitters.

**Case 1:** This case has two hot streams and two cold streams with power cost function and originally analyzed by Yee and Grossmann (1990). The data of this example are given in Table 1. Five genes are put into each chromosome for synthesis of this network.

The final network has six exchangers and is shown in Fig. 5 in which the value of  $y_1$  and  $\Delta T_{\min}$  are 0.33 and 4.7 K, respectively. In this figure underlined numbers are heat loads. The total cost of this network is 80250 \$/year while Yee and Grossmann (1990) reported a value of 80274 \$/year and the solution of Shirvakumar and Narasimhan (2002) gives a value of 80851 \$/year.

One of the advantages of G.A. is that it does not produce only one solution. The second best network has six exchangers and one by-pass with total cost of 81987 \$/year. By removing this by-pass the cost decreases to 81631 \$/year.

The same solution may be achieved by superstructures of Yee and Grossmann (1990) and the number of stages more than  $\text{Max}\{\text{NH}, \text{NC}\}$  which is two in this problem.

**Case 2:** This example has been studied by Lin and Miller (2003) with the optimal solution of 154997 \$/year and six exchangers. Table 2 includes stream and cost data for this case and Fig. 6 shows the best solution found by this coupled method in which each chromosome has five genes.

The genetic algorithm finds the same structure as Lin and Miller (2003) and Yee and Grossmann (1990) with the annual cost of 156730 \$/year. As the NLP formulation do not optimize the area of exchangers simultaneously with other parameters, an increase of 1.1% occurs in total annual cost.



when the size of networks increases. But this study is not very sensible to the size of problems and is very robust because the usual NLP is replaced with a LP which is easier to optimize. So this method can be extended to industrial problems in the future studies.

#### REFERENCES

- Athier, G., P. Floquet, L. Pibouleau and S. Domenech, 1996. Optimization of heat exchanger networks by coupled simulated annealing and NLP procedures. *Comput. Chem. Eng.*, 20: 13-18.
- Athier, G., P. Floquet, L. Pibouleau and S. Domenech, 1997. Process optimization by simulated annealing and NLP procedures: Application to heat exchanger network synthesis. *Comput. Chem. Eng.*, 21: 475-480.
- Barbaro, A. and M.J. Begajewicz, 2005. New rigorous one-step MILP formulation for heat exchanger network synthesis. *Comput. Chem. Eng.*, 29: 1945-1976.
- Bjork, K.M. and T. Westerland, 2002. Global optimization of heat exchanger network synthesis problems with and without the isothermal mixing assumption. *Comput. Chem. Eng.*, 26: 1581-1593.
- Bochenek, R. and J.M. Jezowski, 2006. Genetic algorithms approach for retrofitting heat exchanger network with standard heat exchangers. *Proceeding of the 16th European Symposium on Computer aided Process Engineering and 9th International Symposium on Process Systems Engineering*, pp: 871-876.
- Ciric, A.R. and C.A. Floudas, 1991. Heat exchanger network synthesis without decomposition. *Comput. Chem. Eng.*, 15: 385-396.
- Floudas, C.A., 1995. *Nonlinear and Mixed Integer Optimization*. Oxford University Press, New York.
- Lewin, D.R., 1998. A generalized method for HEN synthesis using stochastic optimization-II: The synthesis of cost-optimal networks. *Comput. Chem. Eng.*, 22: 1387-1405.
- Lewin, D.R., H. Wang and O. Shalev, 1998. A generalized method for HEN synthesis using stochastic optimization-I: General framework and MER optimal synthesis. *Comput. Chem. Eng.*, 22: 1503-1513.
- Lin, B. and D.C. Miller, 2003. Solving heat exchanger network synthesis problems with tabu search. *Comput. Chem. Eng.*, 28: 1451-1464.
- Linnhoff, B. and J.R. Flower, 1978. Synthesis of heat exchangers networks, systematic generation of energy optimal networks. *AIChE J.*, 24: 633-642.
- Linnhoff, B. and E. Hindmarsh, 1983. The pinch design method for heat exchangers networks. *Chem. Eng. Sci.*, 38: 745-763.
- Shenoy, U.V., 1995. *Heat Exchanger Network Synthesis: The Pinch Technology Based Approach*. Gulf Publishing Co., Houston.
- Shirvakumar, K. and S. Narasimhan, 2002. A robust and efficient NLP formulation using graph theoretic principles for synthesis of heat exchanger networks. *Comput. Chem. Eng.*, 26: 1517-1532.
- Yee, T.F. and I.E. Grossmann, 1990. Simultaneous optimization models for heat integration-Heat exchanger network synthesis. *Comput. Chem. Eng.*, 14: 1165-1184.
- Yu, H., H. Fang, P. Yao and Y. Yuan, 2000. A combined genetic algorithm/simulated annealing for large system energy integration. *Comput. Chem. Eng.* 24: 2023-2035.
- Zamora, J.M. and I.E. Grossmann, 1998. A global MINLP optimization algorithm for the synthesis of heat exchanger networks with no stream splits. *Comput. Chem. Eng.*, 22: 367-384.
- Zhu, X.X. and N.D. Asante, 1999. Diagnosis and optimization approach for heat exchanger network retrofit. *AIChE J.*, 45: 1488-1503.
- Zhu, X.X. and X.R. Nie, 2002. Pressure drop consideration for heat exchanger network grassroots design. *Comput. Chem. Eng.*, 26: 1661-1676.