

## Comparison Study of Equivalent Circuit Model of Li-Ion Battery for Electrical Vehicles

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**Abstract:** This study described three types of often used equivalent circuit model of Li-ion battery. The model parameter is identified through least square method by HPPC test. The battery model is built on MATLAB/Simulink. The simulation result is compared with experimental result. The final comparison indicated that Li-ion battery with improved PNGV model has higher accuracy.

**Keywords:** Equivalent circuit model, Li-ion battery, parameter identification

### INTRODUCTION

Battery Management System (BMS) is the key technology of electric vehicles. Vehicle control strategy relies heavily on its rapid, accurate and effective description for the battery external characteristics. Because the battery current, temperature, State Of Charge (SOC) and other factors produce a nonlinear effect on battery characteristics. Accurate battery performance model requires complex mathematical model (Lin *et al.*, 2005). Limited to the constraints of hardware resource of BMS, the battery model couldn't be too complicated. So to gain the simple and effective performance model is currently one of the difficulties of the BMS of electric vehicle. The battery system modeling can be divided into electrochemical models, mathematical models and equivalent circuit model. The electrochemical model structure is too complex to establish. Meanwhile, it takes up a lot of hardware resources. Need to be simplified, the mathematical model has large error and only suitable for some special occasions (Jia *et al.*, 2011). Equivalent circuit model describes the battery operating characteristics using circuit network. The principle is simple and the mathematical models is easy to establish and practical to use. The equivalent circuit model is widely used in electric vehicle BMS at present (Yang *et al.*, 2012).

Typical equivalent circuit model has Rint model, Thevenin model, PNGV model, improved PNGV model (Lin *et al.*, 2005). Rint model make battery equivalent to an ideal voltage source in series with the resistance. It can not reflect the polarization of Li-ion battery. Thus, it seldom used in EV' BMS. Thevenin model is a kind of basic equivalent circuit model of a

battery. So it is widely used in early BMS. The PNGV model and the improved PNGV model is a modification base on Thevenin model with a slight increase in circuit elements (Jia *et al.*, 2011). The EV's BMS need to be capable of real-time and accurate detection of the battery state. The battery model selected for BMS should be simple, easy to handle as well as high precision in order to ensure the accuracy of battery state to be estimated. So the battery model should be comprehensively compared and selected from the following aspects: the model accuracy, model structure, parameter identification, Jia *et al.* (2011).

In this study, Thevenin model, PNGV model and improved PNGV model are studied as equivalence model of Li-ion battery respectively. The model structure characteristics and parameter identification method are discussed. The three kinds of model are built on MATLAB/Simulink. Then the model simulate the process that battery discharge with pulse current according to Hybrid Pulse Power Characterization (HPPC) test. Thus, the optimal model is obtained through the comparison between simulation value and experimental value. This study provides the basis for the selection of Li-ion battery equivalent circuit model.

### MODEL STRUCTURAL ANALYSIS

**Thevenin model:** Thevenin Model circuit structure is shown in Fig. 1. The equivalent voltage source  $E_0$  describes the battery's open circuit voltage. The transition process of charge and discharge of RC circuit, which are composed of capacitance  $C_1$  and resistor  $R_1$ , describes the battery polarization. Resistor  $R_0$  describes battery's internal resistance.  $I(t)$  is the

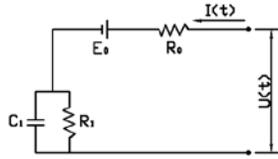


Fig. 1: Thevenin model

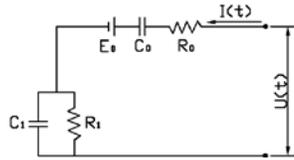


Fig. 2: PNGV model

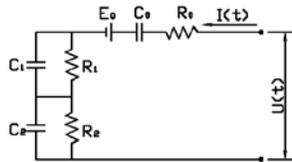


Fig. 3: Improved PNGV model

battery's charge or discharge current and  $U(t)$  is the battery's terminal voltage (Lin *et al.*, 2005).

**PNGV model:** The United States proposed equivalent circuit model of battery in the Partnership for a New Generation of Vehicles (PNGV) in 2001 as shown in Fig. 2.

It describes the internal resistance with ohmic resistance  $R_0$ , the changes of electromotive force with  $C_0$ , battery polarization with  $R_1, C_1$ . The model has clear physical meaning and higher accuracy. It has good applicability to many work condition. Equation of state space could also be derived for analysis and application. So there model is now commonly used (Lin *et al.*, 2005).

**Improved PNGV model:** At present, some research about Li-ion battery proposed that an RC circuit ( $R_2, C_2$ ) should add to PNGV model made improved PNGV model, as shown in Fig. 3, to describe the charge and discharge characteristics (Dai *et al.*, 2010).

### PARAMETER IDENTIFICATION

**The basic theory of parameter identification with least square method:** For single-input single-output linear discrete system, the effective method of parameter identification is least squares method. The differential equation model can be described as follows:

$$A(q^{-1})y(k) = B(q^{-1})u(k) + e(k) \quad (1)$$

where,  $A(q^{-1}) = 1 + \sum_{i=0}^n a_i q^{-i}$ ,  $B(q^{-1}) = \sum_{i=0}^m b_i q^{-i}$ ,  $q^{-1}$  is delay operator and  $q^{-1}y(k) = y(k-i)$ ,  $e(k)$  is the square of residuals of model.

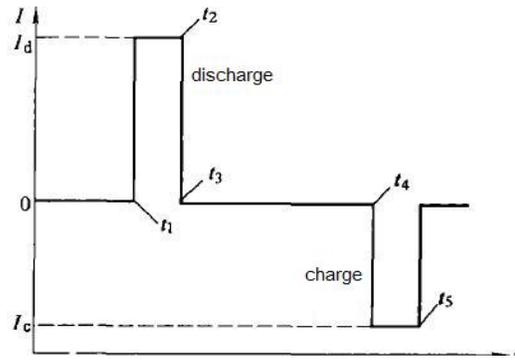


Fig. 4: HPPC test profile

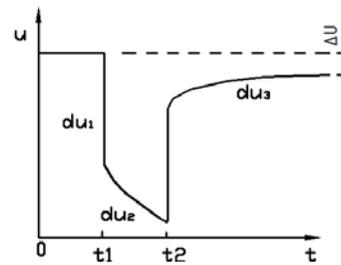


Fig. 5: Voltage response to HPPC test

Eq. (1) can be written in form of linear regression:

$$y(k) = h^T(k)\theta + e(k) \quad (2)$$

where,  $h^T(k) = [-y(k-1), \dots, -y(k-n), u(k), u(k-1), \dots, u(k-n)]$  and  $\theta^T(k) = [\alpha_1, \alpha_2, \dots, \alpha_n, b_0, b_1, \dots, b_n]$

Eq. (2) can be written in matrix form:

$$Y = H\theta + e \quad (3)$$

where,  $H = [h^T(n)h^T(n+1) \dots h^T(n+N)]^T$  and  $Y^T = [y(n)y(n+1) \dots y(n+N)]$ ,  $e^T = [e(n)e(n+1) \dots e(n+N)]$ .

The so-called least squares method is to use  $J$ , the square of residuals  $e(k)$ , as criterion, that is:

$$\frac{\partial J}{\partial \theta} = -2H^T(Y - H\theta) = 0 \quad (4)$$

An canonical equation can be get from Eq. (4), which is a  $2n$  dimensional linear equations. The estimated value of  $\theta$  is:

$$\hat{\theta} = (H^T H)^{-1} H^T Y \quad (5)$$

where,  $\hat{\theta}$  is the identified model parameter.

**Model parameter identification test:** Reference to the HPPC test of «Freedom CAR battery test manual», pulse discharge current is taken as excitation, as shown in Fig. 4. Model parameters can be identified base on

Table 1: Identified parameters table of three model

	C <sub>0</sub> (F)	C <sub>1</sub> (F)	C <sub>2</sub> (F)	R <sub>0</sub> (Ω)	R <sub>1</sub> (Ω)	R <sub>2</sub> (Ω)
PNGV model	582.4	117.3		0.1889	0.1309	
Improved PNGV model	582.4	232.1	25.04	0.1889	0.0129	0.081

voltage response data. To simplify the study, this test only carried on discharge test.

Through the pulse discharge, pulse amplitude is set as  $I_d = 20A$ . The pulse duration is  $t_2 - t_1 = 20s$ . The duration of the pulse end is  $t_4 - t_3 = 60s$ , Curve in Fig. 5 shows the voltage response to HPPC test (Jin *et al.*, 2012).

**Model parameter identification:** Model parameters can be identified according to the voltage response of pulse current results, combined with equivalent circuit model features. Internal resistance  $R_0$  can be identified using voltage drop segment  $du_1$  when pulse current is loaded, there is  $R_0 = du_1 / I$ .  $C_0$  reflect the change of open circuit voltage when pulse current is load from start to finish. So  $C_0 = \Delta Q / \Delta U = \int_0^t I(t) dt / \Delta U$ , where  $\Delta U$  is the open circuit voltage difference when pulse current is load from start to finish. Then  $C_0$  is solved out.

The zero-input response segment,  $du_3$ , reflects the process that RC circuit discharge to the loop when discharge current is removal. The voltage response value of a single RC-link is solved as  $du_3 = U_{c1} \exp(-t/\tau_1)$ . By least squares method, the voltage  $u_{c1}$  and time constant  $\tau_1$  of RC-link can be identified. The voltage response value of the second order of RC-link is  $du_3 = U_{c1} \exp(-t/\tau_1) + du_3 = U_{c2} \exp(-t/\tau_2)$ . Similarly, the parameters,  $u_{c1}$ ,  $u_{c2}$ ,  $\tau_1$ ,  $\tau_2$ , can also be identified by least square method.

Zero state response segment  $du_2$  reflects the battery polarization when current loaded. This phenomenon can be described as a capacitance charging process in equivalent circuit model. The voltage response value of a single RC-link is  $du_2$  and  $du_2 = IR_1(1 - \exp(-t/\tau_1))$ . Using PNGV model, the battery discharge losses should be get rid of from  $du_2$ . Then, resistance  $R_1$  and capacitance  $C_1$  could be identified using the least square method. The voltage value of the second order RC-link,  $du_2$ , is obtained, as:

$$du_2 = IR_1(1 - \exp(-t/\tau_1)) + IR_2(1 - \exp(-t/\tau_2))$$

Similarly, for improved PNGV model, resistance  $R_1$ ,  $R_2$  could be identified after excluding the discharging losses from  $du_2$ . Then, capacitance  $C_1$ ,  $C_2$  can be solved according to the time constant  $\tau_1$ ,  $\tau_2$  solved before. The identified parameters are shown in Table 1 (Jin *et al.*, 2012).

### MODEL VALIDATION

**The three types of model:** Thevenin model, PNGV model, improved PNGV model, which output  $u_t$  is found using Kirchoff's voltage law:

$$u_t = u_0 - IR_0 - u_{c1} \tag{6}$$

$$u_t = u_0 - IR_0 - u_{c1} - \frac{1}{C_0} \int_0^t Idt \tag{7}$$

$$u_t = u_0 - IR_0 - u_{c1} - u_{c2} - \frac{1}{C_0} \int_0^t Idt \tag{8}$$

The charging current and voltage of capacitance in RC-link are:

$$i_{c1}(t) = C_1 \frac{du_{c1}}{dt} \tag{9}$$

$$u_{c1}(t) = (i(t) - i_{c1}(t))R_1 \tag{10}$$

Through Laplace transform, the transfer function of RC circuit is:

$$\frac{U_{c1}(s)}{I(s)} = \frac{R_1}{1 + R_1 C_1 s} \tag{11}$$

Thus, the system's transfer function can be obtained through Laplace transform of Eq. (6) ~ (8). Three types of equivalent circuit simulation model are built on MATLAB/Simulink. Meanwhile, the discharge current is input to the simulation model according to test requirements, as shown in Fig. 6.

Simulation output is compared with the experimental values, shown in Fig. 7.

From the pulse current response result of different model, all the three kinds of model can accurately describe the voltage response during the process of zero state response. Thevenin model has larger model errors during the process of zero-input response. The main reason is that the model doesn't describe the discharging loss. This make large different between open-circuit voltage and the experimental values. The voltage response of improved PNGV model is more accurate than PNGV model at the beginning of zero-input state. But if the still time is long enough, the two model will toward the same value and will satisfy the experimental value finally. The result shows that Thevenin model is not suitable for the Li-ion battery of electric vehicles. The voltage response value of PNGV model and improved PNGV model can both fit the experimental value well. But in comparison, the improved PNGV model has higher accuracy

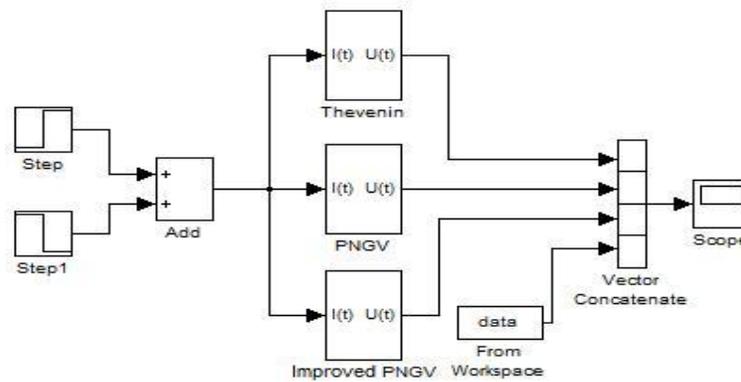


Fig. 6: System simulation model

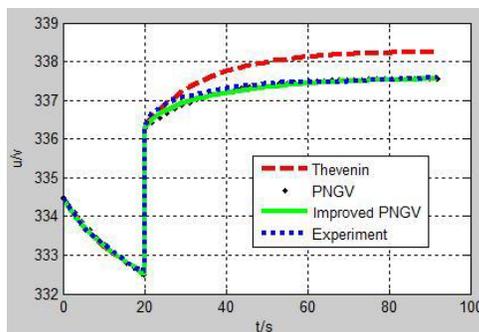


Fig. 7: Simulation values compared with experimental values

because it uses two RC circuits to describe the polarization.

### CONCLUSION

Three kinds of equivalent circuit model are presented based on Li-ion battery. The result of voltage response can be obtained through HPPC test. The battery model is built on MATLAB/Simulink. The simulation result is compared with experimental result. The final comparison indicated that Li-ion battery with improved PNGV model has higher accuracy.

### ACKNOWLEDGMENT

The authors gratefully acknowledge the support of the Beijing University of Aeronautics and Astronautics

and the Guilin College of Aerospace Technology. The study is supported by the project of featured program construction and curriculum integration of Guangxi high school under Grant No.GXTSZY125.

### REFERENCES

- Dai, H., X. Wei and Z. Sun, 2010. An inner resistance adaptive model based on equivalent circuit of lithium-ion batteries. *J. Tongji Univ. Nat. Sci.*, 38(1): 98-102.
- Jia, Y., D. Xie, Y. Gu, Q. Ai, Z. Jin and J. Gu, 2011. Classification and characteristics of equivalent circuit models for ev's battery. *Power Energy.*, 32(6): 516-521.
- Jin, F., H. Yongling and B. Ran, 2012. The method research of parameter identification of Li-ion battery base on least square method. *Adv. Mater. Res.*, 490-495: 3854-3858.
- Lin, C., B. Qiu and Q. Chen, 2005. Comparison of current input equivalent circuit models of electrical vehicle battery. *Chinese J. Mech. Eng.*, 41(12): 76-81.
- Yang, Y., T. Tao-Feng, Q. Da-Tong and H. Ming-Hui, 2012. PNGV equivalent circuit model and SOC estimation algorithm of lithium batteries for electric vehicle. *J. Syst. Simul.*, 24(4): 938-942.