

## Stochastic Discrete-Time Model and Simulation of Inventory Management System

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**Abstract:** Inventory management plays a very important role in logistics system. This study presents a discrete-time model for regularly order inventory management system considering the randomness of receiving goods of time and the volume of ordering determined by inventory difference. With the application of Simulink toolbox in MATLAB, an example of block diagram model in  $z$ -domain is built to simulate the dynamic change process of the inventory system, so that it can effectively assist and support decision-making.

**Keywords:** Discrete-time system model, inventory management, logistics, MATLAB, simulation

### INTRODUCTION

Inventory refers to the storage of goods for various objectives. Storage means logistics stagnation between intervals of goods leaving production process but having not yet entered the consumption process, which is an important part of logistics. It is the necessary condition of social expanded reproduction. And costs of inventory are considerable. Enterprises can reduce the amount of inventory to reduce storage costs. But, if the stock is too low often lead to out of stock, also will bring great loss to enterprises. Therefore, it is necessary to determine and maintain a reasonable inventory level. A commonly used inventory management method is periodic order. In the management system of fixed quantity ordering, order interval is varied and each time the order quantity is unchanged; in regular order management system, in contrast, each order quantity change whereas order interval is fixed (Wang, 1997). Many researches have studied the inventory management and control model (Yin *et al.*, 2007; Zhao, 2011; Wang, 2012; Taleizadeh *et al.*, 2013). To determine the inventory level, because the order and delivery cycle are influenced by many random factors, application of simulation method can obtain a better solution and can dynamically represent change of inventory level, which will provide decision-makers with more intuitive and clear decision information.

Some scholars studied the stochastic inventory system with MATLAB (Du and Jiao, 2007; Zhang and Xu, 2009; Shi and Xia, 2010). MATLAB is a tool software including mathematical calculation, graphics processing and program designing, which characteristics are concise grammar structure, efficient numerical computation, completed graphics function and easy to learn and use Zhang *et al.* (2006). In matrix algebra, numerical computing, digital signal processing, vibration theory, neural network control, dynamic simulation and other fields have a wide range of applications. Simulink is a powerful toolbox in

MATLAB that can be used for modeling, simulation and analysis of practical dynamic system. This software packet supports continuous, discrete and the mixture of both linear and nonlinear systems and multiple rate system with different sampling rate. It provides box modeling diagram window for the user, like using paper and pen to paint as easy, making modeling more intuitive, convenient and flexible. This study studies the modeling and simulation of periodical ordering inventory management system and its change of stocks. The application of Simulink to inventory system simulation can provide a convenient, practical and powerful tool for the study of inventory problem, so that make the analysis of logistics system more scientific, advanced and accurate.

### ANALYSIS OF PERIODICAL ORDERING INVENTORY SYSTEM

An inventory management system as shown in Fig. 1, it belongs to the periodical order management

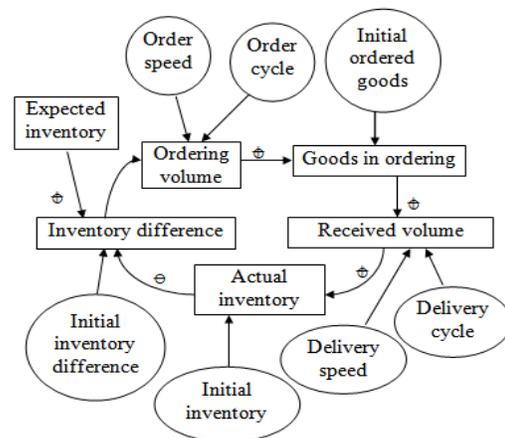


Fig. 1: Relationship of variables in periodical ordering inventory system

system. First we analyze the relationship among variables of the inventory system. It has two accumulation variables, inventory and order goods. It is a model studying dynamic process of the stock increased (Wang, 1997). It can be seen from the figure, comparing the expected inventory and actual inventory obtain the difference and then decision-maker determines the order velocity and the ordering cycle in order to determine the order quantity. Delay between order and delivery forms the goods in ordering, that means goods ordered but not received. Quantity of received goods is determined by the delivery schedule and delivery cycle according to the ability of the manufacturing plant. Inventory difference increases, the order quantity increases; receipt volume getting much makes the actual inventory increase; actual inventory increase makes the reduction of inventory difference. All the variables are influenced with each other.

**CONSTRUCTION OF SIMULATION MODEL**

According to above analysis, in order to study the process of the actual inventory amount close to the expected inventory, the mathematical model of the system is established as follows:

$$\left\{ \begin{array}{l} I[k] = I[k-1] + R2[k-1] \\ R2[k] = G[k] / W \\ E[k] = EV - I[k] \\ R1[k] = E[k] / Y \\ G[k] = G[k-1] + R1[k-1] - R2[k-1] \end{array} \right. \quad (1)$$

where,

$I$  = The actual inventory of beginning of every week

$EV$  = The expected inventory

$E$  = Inventory difference

$G$  = The goods in ordering

$R1$  and  $R2$  = Ordering speed and delivery speed respectively

$W$  = The delivery cycle

$Y$  = The ordering cycle

$k$  = The  $k$ th week

For example, let initial inventory  $I(0) = 100$ ;  $EV = 5000$ ,  $G(0) = 0$ ,  $W = 10$  weeks,  $Y = 5$  weeks.

This is a discrete-time system model. Determine the  $z$ -transform of Eq. (1),  $f[k] \xrightarrow{z} F(z)$ ,

$$F(z) = \sum_{k=-\infty}^{+\infty} f[k]z^{-k}$$

And according to the time shifting property of  $z$ -transform, i.e.,  $f[k-n] \xrightarrow{z} z^{-n}F(z)$  (Oppenheim *et al.*, 1997), then:

$$\left\{ \begin{array}{l} I(z) = z^{-1}I(z) + z^{-1}R2(z) \\ R2(z) = G(z) / W \\ E(z) = EV - I(z) \\ R1(z) = E(z) / Y \\ G(z) = z^{-1}G(z) + z^{-1}R1(z) - z^{-1}R2(z) \end{array} \right. \quad (2)$$

In Simulink, it contains Continuous, Discrete, Sinks (output), Sources (input), Linear, Nonlinear, Math, Connectors and Extra links submodel Library and each submodel library contains the corresponding function module. Users can also custom and create their own modules. Using the display module can see the running results at the same time of simulation.

Hence according to Eq. (2), a system block diagram is built with delay links and a series of arithmetic links to simulate this system. Taking into account the delivery cycle being affected by various random factors make the delivery time is not fixed, so using a normal distribution random number with predetermined mean to simulate. In the Simulink Library Browser window select the constant module and random number module from the Sources model library, the unit delay module from the discrete library, the sum and product module from the Math library, the oscilloscope block from the Sink library, as well as the mux block from the Signals and Systems library. Connect these blocks thus forming the model diagram of inventory management system simulation in model window that is shown in Fig. 2. Parameters of the order cycle and the expected inventory module are set to 5 and 5000. The mean of delivery cycle module is set to 10, the variance is set to 1 and while the sample time is set to 1.

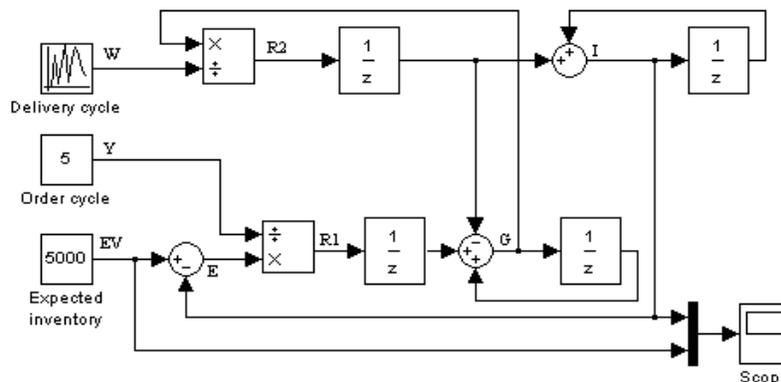


Fig. 2: Block diagram of simulation model of inventory system

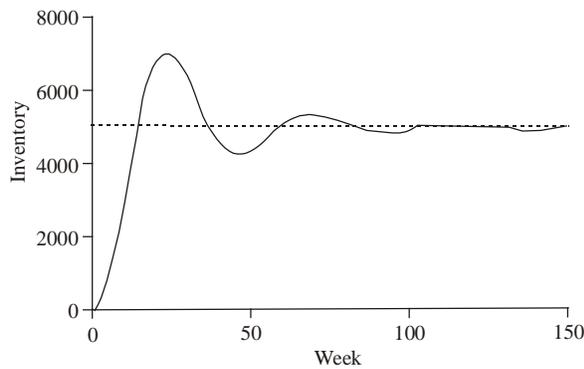


Fig. 3: Simulation result of inventory management system

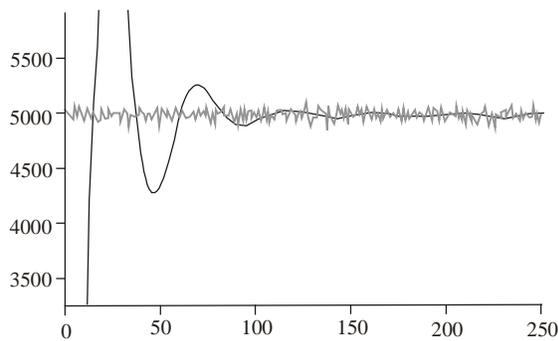


Fig. 4: Simulation result of changing the expected inventory to random number

### ANALYSIS OF SIMULATION RESULT

As an example, set the parameters of simulation time is 150, which means to simulate 150 weeks. After the simulation, open Scope can see this inventory management system simulation results, as shown in Fig. 3.

From the curve can be seen, the system oscillate but in the end will tend to a stable value, namely the actual inventory fluctuate around the expected volume of inventory and gradually approaches the desired inventory. If choose different order cycle, then reach the expected inventory time will be different. Small overshoot means less excess inventory thus can reduce the cost of inventory. Proper selection of parameters, the transition process time to stable state can be shortened. Decision-makers can choose different order time and simulate several times, then compare the overshoot and the transition process time to determine a reasonable order cycle, so that the cost of inventory can meet the requirements.

If change the expected inventory to be random demand, normal distribution with parameters of mean = 5000 and variance = 1600 and simulation time set to 250, the simulation result is shown in Fig. 4. The expected inventory turns to fluctuated curve and the

time of actual inventory approaching the stable state is a little longer, it also has slight fluctuation near expected value all the steady time comparing to the fixed expected inventory.

### CONCLUSION

Inventory management plays an important role in logistics management. The determination of inventory level, ordering time and speed and order quantity, affects the normal production of one enterprise. Reasonable inventory can shorten the circulation of goods time, accelerate the development of production, reduce costs, reduce logistics cost and improve the comprehensive economic benefits. Using Simulink toolbox of MATLAB is convenient and efficient for the modeling and simulation, which can provide accurate and intuitive information for decision-makers, to speed up the decision-making process and improve the veracity of decision. The result of the simple application to inventory system is consistent with analytic method that proved the effective of the simulation model, so the modeling toolbox is also suitable for other complex systems.

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