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Parametric System Identification of Thermoelectric Cooler for Single Photon Avalanche Diode Application

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Abstract: The purpose of this study is to model the Thermoelectric Coolers (TEC) by means of computational intelligence system identification. Thermoelectric coolers are widely used in cooling, maintaining and stabilizing the temperature of the Single Photon Avalanche Diode (SPAD). SPAD is a temperature sensitive optoelectronic device, where even a slight variation in temperature can cause unstable performance in quantum efficiency, responsibility and dark counts. However, it is not a simple task to derive a mathematical model for TEC since it varies with the operating condition. In this study, Particle Swarm Optimization (PSO) was used to identify the mathematical model of the multistage TEC (1639733 from Element 14), which encapsulates dynamics of the SPAD, heat sink and components of the cooling heat exchanger. The model was validated by correlation tests, percentage accuracy and also by comparing its time and frequency responses against that of the TEC. It was found that the obtained model has a good representation of the actual system.

Keywords: Mathematical model, Particle Swarm Optimization (PSO), Single Photon Avalanche Diode (SPAD), system identification, Thermoelectric Cooler (TEC)

INTRODUCTION

The Single Photon Avalanche Diode (SPAD) is an optoelectronic device that is very sensitive to variation of operating temperatures; even a slight temperature variation results to poor performance of quantum efficiency (Huang and Duang, 2000), responsibility (Aly and El-Lail, 2006) and dark current (Singh, 1996). The Thermoelectric Cooler (TEC) is a main component used in cooling, maintaining and stabilizing the temperature of the single photon avalanche diode (Huang and Duang, 2000; Aly and El-Lail, 2006). Thus, a system that consist of SPAD, TEC and a custom mounting design of heat sink and the cooling heat exchanger components is specifically designed to fulfill the purpose, shown in Fig. 1.

SPAD supports and enables a host of numerous applications; Quantum Key Distribution (QKD), small-signal fluorescence, Light Detection and Ranging (LIDAR), photon counting and laser range finder (Singh, 1996). Specifically, the SPAD C30902 from Perkin Elmer has many other favorable factors such as high quantum efficiency, high responsibility, fast time response, wide spectral response range, wide operating temperature range and low noise.

There have been numerous works in modeling TECs using small-signal linearization (Huang and Duang, 2000), finite element analysis (Wey *et al*., 2006) and SPICE compatible equivalent circuit method (Lineykin and Ben-Yaakov, 2005). However, a drawback in those works is that the model structure must be known *a-priori*. There is no work on system identification of thermoelectric cooler found in the literature review. Therefore, in this study, the

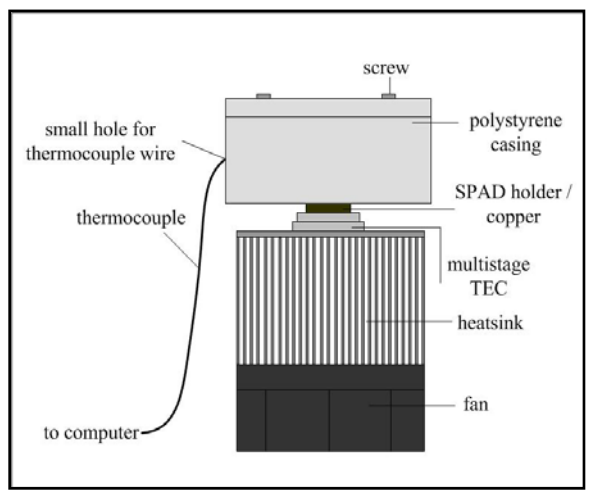

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Fig. 1: Front view cross section of SPAD, TEC and the custom mounting design

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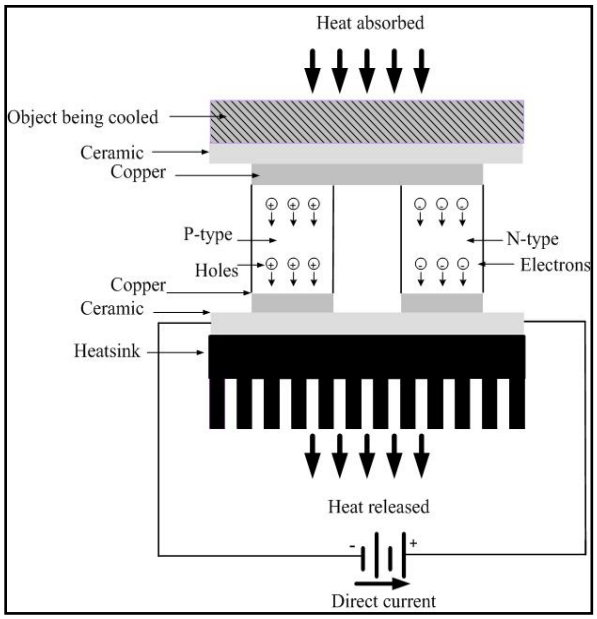

Fig. 2: Schematic diagram of a thermoelectric cooler

mathematical model of a TEC together with its mounting design with SPAD is obtained using the PSO optimization algorithm in MATLAB. Input-output data from the TEC was extracted, (in particular input voltage and output temperature were measured) and injected into the PSO algorithm. The benefit of this method is that *no-priori* knowledge of the system is needed.

The performance of the mathematical model was validated by the good quality correlation test within the normalized confidence band of $\pm 1.96/\sqrt{N}$ for 95% of the correlation result, good percentage accuracy of 98.03% by significantly small mean square error of 0.79003 in tracking the actual output of the system and successful stability test within the unit circle of the *z*-plane.

A highly nonlinear characteristic of TEC has made the normal equations derivation becoming more complex. Therefore, the objective of this study is to obtain a mathematical model that represents the behavior of the TEC by means of particle swarm system identification technique.

THERMOELECTRIC COOLER (TEC) OPERATING PRINCIPLE

Figure 2 shows the schematic diagram of a Thermoelectric Cooler (TEC). TEC is a heat pump that operates based on the mechanism of Peltier effect. Peltier effect is a situation when a junction consisting of two contradictory materials, when supplied with electrical current through it resulting in cooling effects (Marlow and Burke, 1995). TEC consist of p-type and n-type semiconductor. Typically, the p-type and n-type are made of bismuth-telluride material. A pair of p-type and n-type is referred as a "couple". A TEC may consist of hundreds of couples. These couples are arranged in arrays in order to make the most cooling effects. The arrays are connected electrically in series and thermally in parallel. For good housing rigidity, electrically insulated and thermally conducted, the TEC is fastened with plates of ceramic substrate on top and at the bottom of it (Marlow and Burke, 1995).

The p-type material has surplus of holes and therefore shortage of electrons. Meanwhile, n-type material has surplus of electrons and therefore shortage of holes. TEC is operated using direct current (dc). This permits electrons to flow through the interconnecting conductor from lower energy level in the p-type material to a higher energy level in the n-type material, consequently, absorbing the heat of the object being cooled on the cold side of the TEC. Meanwhile, the electrons from the n-type of the higher energy level go to the lower energy level in the p-type through the interconnecting conductor, consequently, releasing heat to the environment on the hot side of the TEC. TEC has many significant applications, in cooling many optoelectronics devices, for example avalanche photodiode, diode laser, solid-state detectors, photo detectors and many others (Nolas *et al*., 2001). Due to nonlinearities of TEC system, deriving mathematical model is not that simple. Thus, a computational intelligent system identification technique, which is Particle Swarm Optimization (PSO) has been adopted to obtain the TEC mathematical model.

PARTICLE SWARM OPTIMIZATION (PSO) METHOD

Particle Swarm Optimization (PSO) is an optimization tool that was inspired by birds flocking (Kennedy and Eberhart, 1995). The swarm intelligence technique is chosen because of its simple implementation, very few parameters needed and excellence in performance. PSO can be applied to neural network training, combinational optimization, pattern recognition, complication function and fuzzy system control.

In PSO, each bird is defined as "particle" flying through the dimensional space of $X = (x_1, x_2\ldots, x_k)$. They are randomly initialized with positions of $P = \{p_1, p_2\ldots, p_k\}$ and velocity of $V = (v_1, v_2\ldots, v_k)$. Each particle has their own fitness value determined by the objective function that represents a candidate of a possible solution to the optimization problem.

The particles approach to improve and move towards a better position is equivalent to how flock of birds improves their searching for food in a particular area (Kennedy and Eberhart, 1995).

The particles update to their new velocities by:

$$v_{k+1}^{i} = v_k^i + c_1 r_1 (p_k^i - x_k^i) + c_2 r_2 (p_k^g - x_k^g) \qquad (1)$$

where,
v^i_{k+1} = The particle updated velocity
v^i_k = The particle previous velocity
p^i_k = The particle best position reached so far
x^i_k = The previous particle position
p^g_k = The best swarm position reached so far
x^g_k = The previous swarm position

The equation is divided into three components, that shows that the particle's new velocity by adding the previous momentum velocity (first component) and the "thinking" mechanism of both particles best position reached so far, p^i_k (second component) and the best swarm position has reached so far, p^g_k (third component). The first component, v^i_k serves as a memory so that the particle will not alter direction drastically. The second component, so called the cognitive component, c_1r_1 (p^i_k - x^i_k) is to ensure that the particles attain to the previously found best position, p^i_k. The third component, the social component, c_2r_2 (p^g_k – x^g_k) serves to ensure the swarm attains the group norm or standard.
Then, the particles update to their new positions by:

$$x^i_{k+1} = x^i_k + v^i_{k+1} \quad (2)$$

where, c_1 and c_2 are the cognitive coefficient and social coefficient, while, r_1 and r_1 are the random numbers between 0 and 1. The values of c_1 and c_2 are carefully selected (Shi and Eberhart, 1998b).

Inertia weight: In order to balance the global search and local search throughout the searching process, an inertia weight, *w* was needed and introduced in the velocity Eq. (3) (Shi and Eberhart, 1998b):

$$v^i_{k+1} = w.v^i_k + c_1r_1(p^i_k - x^i_k) + c_2r_2(p^g_k - x^g_k) \quad (3)$$

Later, a new improvement of Time Varying Inertia Weight (TVIW) was introduced:

$$w = (w_1 - w_2)\left[\frac{\max iter - iter}{\max iter}\right] + w_2 \quad (4)$$

where, w_1 and w_2 are the initial and final inertia weight, respectively while iter and max iter is the current and final iteration, respectively (Shi and Eberhart, 1998a).

The optimum solution is guided by the two stochastic acceleration components c_1 and c_2. Thus, a Time Varying Acceleration Coefficient (TVAC) to dynamically reduce the cognitive coefficient, c_1 from 2.5 to 0 and dynamically increase the social coefficient, c_2 from 0 to 2.5 (Ratnaweera and Halgamuge, 2004). The reason is to enhance the global exploration at the beginning stage and to encourage the particles to converge towards the local exploration to achieve global optimum in the end of the findings:

$$c_1 = (c_{1f} - c_{1i})\frac{iter}{\max iter} + c_{1i} \quad (5)$$

$$c_2 = (c_{2f} - c_{2i})\frac{iter}{\max iter} + c_{2i} \quad (6)$$

Therefore, to achieve a global optimum searching result, a large inertia weight is necessary at the beginning of the searching process for particles global exploration, while a small inertia weight is essential towards the end of the searching process for particles local exploration.

Dynamic spread factor: Dynamic spread factor was introduced into the initial PSO algorithm in order to improve the result with guaranteed fast convergence (Shi and Eberhart, 1998a). The proposed algorithm result has better-quality in improving the basic PSO algorithm two main problems of premature convergence and preservation of diversity. The result shows that, in order for the particles to keep explore within the search space, it is important that they know their distributions and relative distances from one to another. The relative distances are measured in terms of precision and accuracy. Precision (spread) means the extreme distance between the best and the worst position of the particles with respect to the fitness function, stated in Eq. (7):

$$spread = x_{\max id} - x_{\min id} \quad (7)$$

While accuracy (deviation) means the average distance of particles from the global best position, depicted in Eq. (8):

$$deviation = \sum \frac{x_{id} - p_{gd}}{S} \quad (8)$$

where, *S* is the number of particles. Precision and accuracy are used to determine the dynamic spread factor, described in Eq. (9):

$$SF = 0.5\left(\frac{spread + deviation}{range_{\max} - range_{\min}}\right) \quad (9)$$

where, $range_{max}$ and $range_{min}$ are the maximum and minimum range of particle, respectively. So, when the particles drop in the neighborhood of global optimum, the dynamic spread factor will drop down extremely.

Consequently, will cause the inertia weight to drop down, refer to Eq. (10):

$$w = \exp\left(\frac{-iter}{SF \times \max iter}\right) \tag{10}$$

Thus, results in faster convergence and higher precision. The cognitive component, c_1 is described in Eq. (11):

$$c_1 = 2\left(1 - \frac{iter}{\max iter}\right) \tag{11}$$

so that it will linearly decreases with time from 2 to 0 to have better global exploration at the beginning of the search and converge to local exploration towards the end of the search, while social component, c_2 is maintained at 2 so that the particles are pulled towards the global optimum (Latiff and Tokhi, 2009).

PARAMETRIC SYSTEM IDENTIFICATION USING PSO

Parametric system identification is defined by the model structure and parameters from a given data input and output to describe the real physical system (Ljung, 1999). Also, it is a method to forecast the output of the system once the parameters of the mathematical model is identified. In other words, parametric system identification is a practice of control and prediction of a real world physical system. The determination of mathematical model using Particle Swarm Optimization (PSO) is realized by minimizing the mean square error prediction output to the actual output of the system. There are four steps in determining parameter system identification:

- Collection of information about the system
- Determination of a model structure to represent the system
- Selection of the parameters mathematical model
- Validation of the mathematical model (Toha and Tokhi, 2010)

Collection of input and output experimental data of the system: The collection of input and output experimental data of the system is taken up to 1400 points to ensure that the information are rich in spectral density corresponding to the system bandwidth by exciting all the dynamic modes of interest (Ljung, 1999). This parameter system identification is designated in the discrete time domain due to the reason that we sampled the experimental data. It is important to make the collected data almost continuous by a

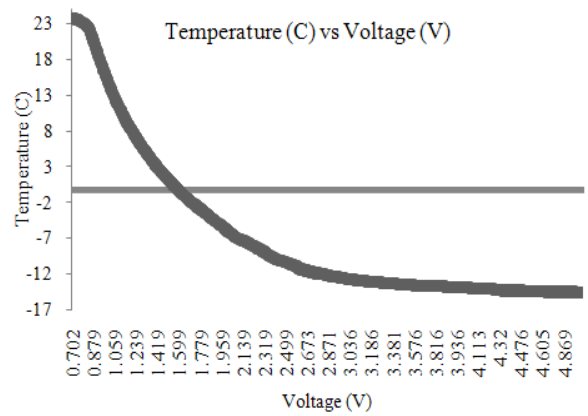

Fig. 3: The collected experimental data input and output of the system

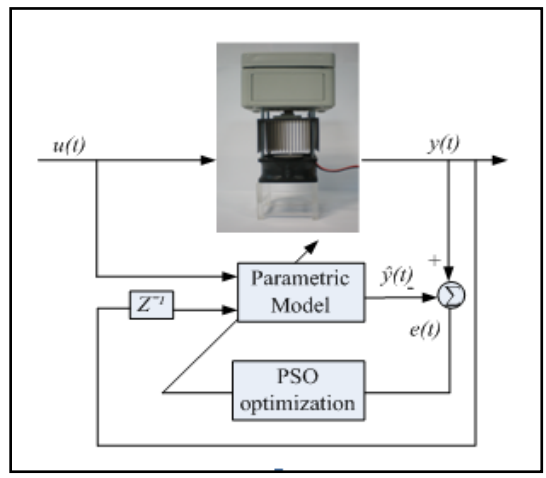

Fig. 4: PSO based parametric system identification of TEC for SPAD application

significant small sampling time, to represent the actual system. Hence, 1400 input and output experimental data points were taken with the sampling time of 1 second. The input, u (t), from 0 up to 5V with the small increment of 0.003V is supplied to the TEC. The corresponding output, y (t), temperature of the SPAD at the cold side of the TEC were taken. Figure 3 shows the result of the collected experimental data consisting of the output temperature corresponding to the input voltage of the system. The simulation of PSO was run in the m-file of MATLAB on a PC (Intel (R) Core (TM) 2 Duo CPU, 2.93 GHz, 1.96 GB of RAM) by extracting the obtained input and output data, producing a mathematical model to represent the dynamic model of the TEC mounting design including the SPAD, heat sink and the cooling heat exchanger components.

Determination of the model structure: It is important to determine the order of the mathematical model to represent the actual system. Improper lower order

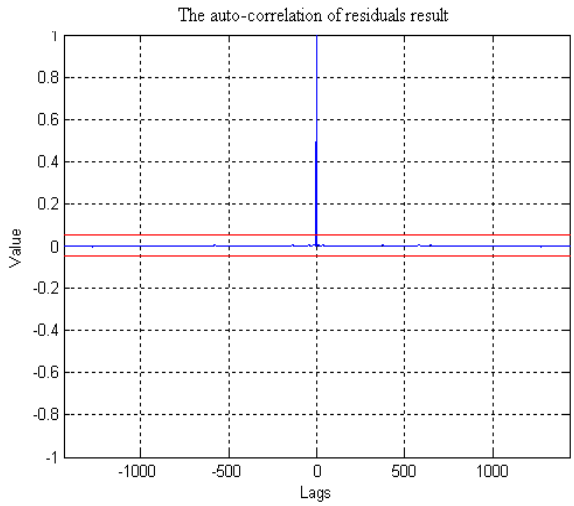

Fig. 5: The auto-correlation of residuals result

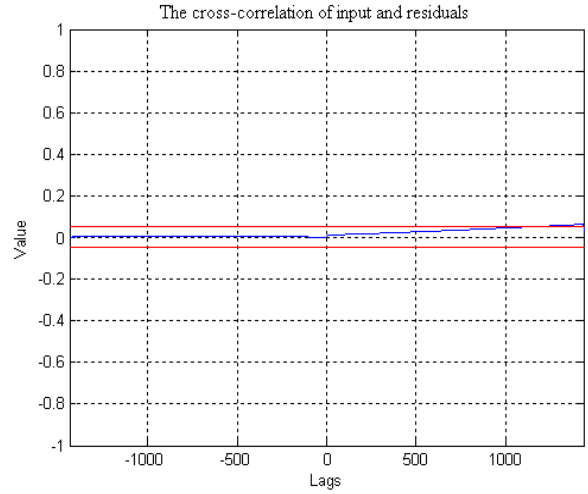

Fig. 6: The cross-correlation of input and residuals

model may imply that the mathematical model is inadequate, while improper higher order model may escalate the mathematical model uncertainties.

From Fig. 4, Eq. (12) is the model structure of auto regression with exogenous input (ARX) was chosen to represent the model structure of the system:

$$\hat{y}(t) = -\sum_{i=1}^{N} a_i \times y(t-i) + \sum_{j=0}^{M} b_j \times u(t-j) \qquad (12)$$

where,

$\hat{y}$ (t) = The predicted output
a_i = The denominator
b_j = The numerator
N = The number of the denominator
M = The number of the numerator
u (t) = The input
y (t) = The actual output

Second order model was chosen because it gives better representation of the system dynamics and produced less mean square error compared to the other order models. Hence, an easier way to represent Eq. (12) is by using the backshift operator z^{-1}:

$$\hat{y}(t) = \left(\frac{b_0 z^{-1} + b_1 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}} \right) \qquad (13)$$

Equation (14) is the Mean Square Error (MSE), which is the objective function for the PSO optimization process. The result of the MSE should converge to zero:

$$e(t) = MSE = \frac{1}{N} \sum_{t=1}^{N} \left| y(t) - \hat{y}(t) \right|^2 \qquad (14)$$

SIMULATION RESULTS

The mathematical model obtained from the PSO simulation in the *z*-plane is:

$$H(z) = \frac{-1.805\,z + 0.8054}{z^2 - 0.7745\,z + 0.7718} \qquad (15)$$

In the *s*-plane, the mathematical model is:

$$H(s) = \frac{-113.3\,s - 4925}{s^2 + 16.19s + 4916} \qquad (16)$$

There are three ways to validate the mathematical model system:

- Correlation test
- Percentage accuracy
- Mean Square Error (MSE) in tracking the actual output of the system

Correlation test: These following correlation test conditions to validate the mathematical model have been full filled. The graphs are shown in Fig. 5 to 9 (Billings and Voon, 1986):

$$\phi_{ee}(t) = E\left[\varepsilon(t-\tau)e(t)\right] = \delta(\tau) \qquad (17)$$

$$\phi_{u\,e}(\tau) = E\left[u(t-\tau)e(t)\right] = 0_{\forall\tau} \qquad (18)$$

$$\phi_{u^2e}(\tau) = E\left[(u^2(t-\tau) - \bar{u}^2(t)e(t)\right] = 0 \;\; \forall\tau \qquad (19)$$

$$\phi_{u^2e^2}(\tau) = E\left[(u^2(t-\tau) - \bar{u}^2(t)e^2(t)\right] = 0 \;\; \forall\tau \qquad (20)$$

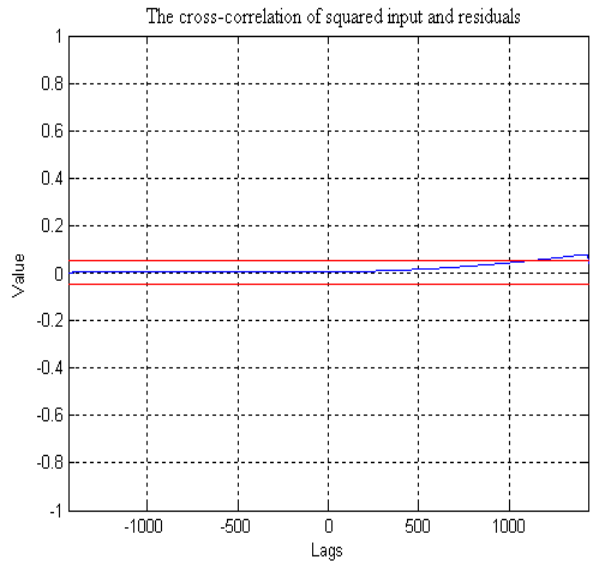

Fig. 7: The cross-correlation of squared input and residuals

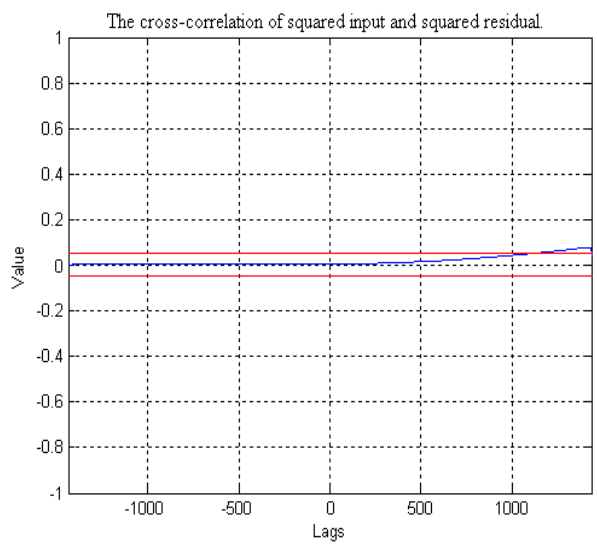

Fig. 8: The cross-correlation of squared input and squared residual

$$\phi_{e(eu)}(\tau) = E\left[e(t)e(t-1-\tau)u(t-1-\tau)\right] = 0 \quad \tau \geq 0 \tag{21}$$

Equation (21) is the normalized correlations of two given sequences, ψ_1 (t) and ψ_2 (t):

$$\hat{\phi}_{\psi_1\psi_2}(\tau) = \frac{\sum_{t=1}^{N-\tau}\psi_1(t)\psi_2(t+\tau)}{\sqrt{\sum_{t-1}^{N}\psi_1^{\,2}(t)\sum_{t-1}^{N}\psi_2^{\,2}(t)}} \tag{22}$$

The result of the normalized correlation is within $\pm 1.96/\sqrt{N}$ for 95% of the confidence band, shown in

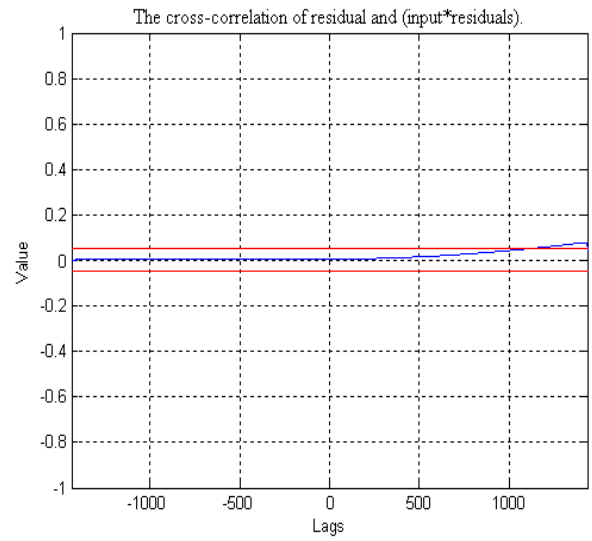

Fig. 9: The cross-correlation of residual and (input*residuals)

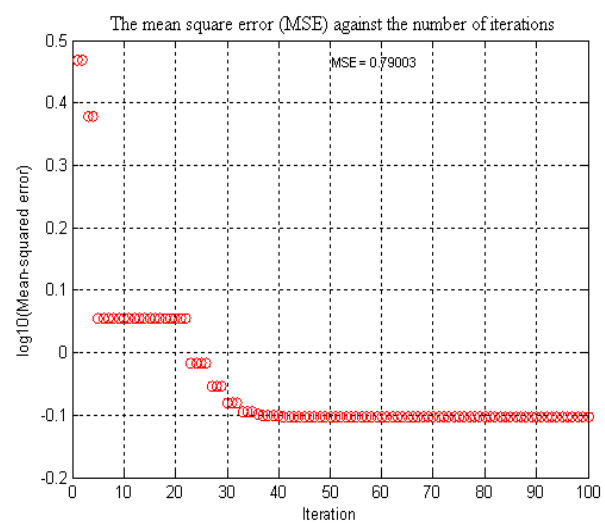

Fig. 10: The Mean Square Error (MSE) against the number of iterations

Fig. 5 to 9, where N is the number of data series (Thomson *et al*., 1996). The red lines are the confidence band while the blue lines are the correlation test results.

Percentage accuracy and Mean Square Error (MSE) in tracking the actual output of the system: The PSO predicted output managed to track the actual output of the system by 98.03% of percentage accuracy. Figure 10 shows the best minimum Mean Square Error (MSE) against the number of iterations, which is 0.79003. It is shown from Fig. 10 that the MSE significantly converged from the beginning towards the end. The elapsed time to complete the iterations is 151.973231 sec.

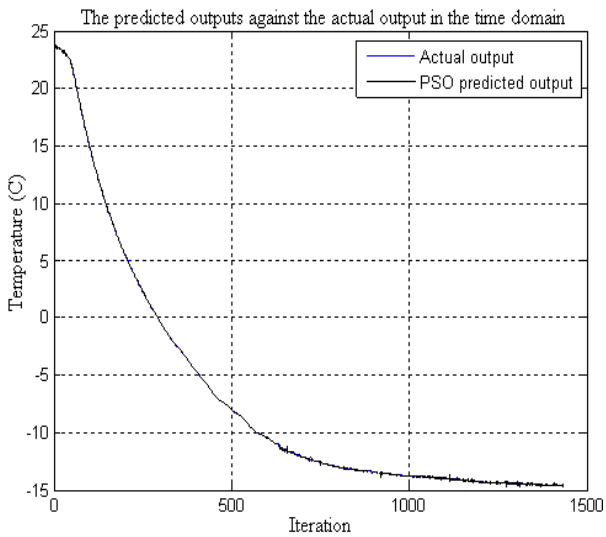

Fig. 11: The predicted outputs against the actual output in the time domain

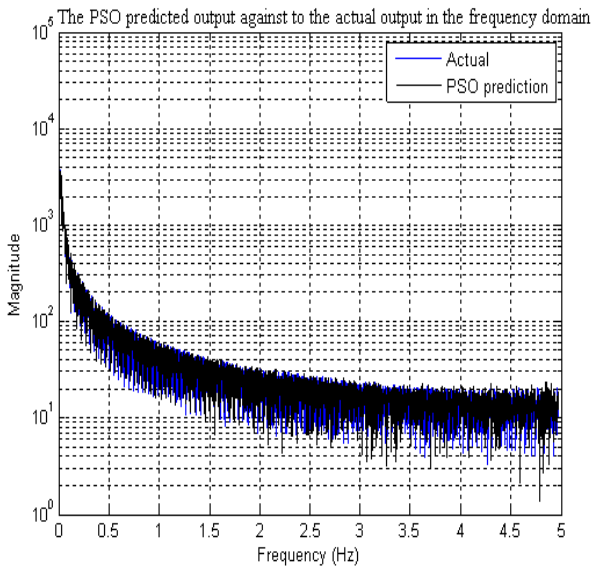

Fig. 12: The PSO predicted output against to the actual output in the frequency domain

The predicted output trailing the actual output is presented in Fig. 11 and 12 for the time and frequency domain. The blue lines are the actual output while the black lines are the PSO predicted output.

Stability test: The stability of the system is determined directly from its transfer function where all zeros and poles must be in the unit circle of the *z*-plane. From Fig. 13, all the polynomial roots of the determined transfer function are located inside the unit circle of the z-plane, hence, the estimated mathematical model is stable.

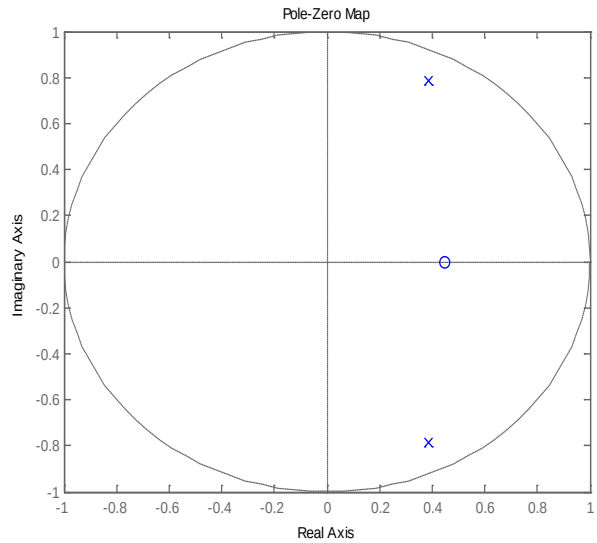

Fig. 13: The stability test of the transfer function in the *z*-plane

CONCLUSION

In this study, the parameter system identification using Particle Swarm Optimization (PSO) algorithm has been carried out to determine the mathematical model of multistage TEC for SPAD application. The PSO algorithm is introduced with the dynamic spread factor to produce better-quality in improving the premature convergence and preservation of diversity. The PSO algorithm extracts the collected input and output data of the TEC system which has been obtained experimentally. Auto Regression with exogenous input (ARX) is found to be the second order model structure of the system. The minimization of Mean Square Error (MSE) was used as the objective function of PSO results to significantly small mean square error. The mathematical model was validated with correlation test, percentage accuracy in tracking the actual output of the system which shows that the TEC has been modeled closely representing the actual system.

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