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A New Multimodal Variational Formulation Analysis of Cylindrical Waveguide Uniaxial Discontinuities

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Abstract: The New Multimodal Varational Formulation (NMVF) was since successfully used to characterize the uniaxial discontinuities in the homogeneous rectangular waveguides, in the ridged rectangular waveguides and in the rectangular E-plane dielectric filled waveguides. In this study, we apply this method to characterize the uniaxial discontinuities between the homogeneous cylindrical waveguides. The behavior of two filter structures is highlighted. A good agreement is observed between the achieved results and those given in the literature.

Keywords: Accessible modes, impedance matrix, microwave devices, uniaxial discontinuities

INTRODUCTION

The study of discontinuities in circular waveguides have a particular interest regard to many publications dedicated on this subject which one can abundantly find in the literature (Balaji, 2011; Couffignal *et al*., 1994; Thabet and Riabi, 2007).

Many microwave devices such as multi-mode filters, impedance transformers, adapters,..., use circular transitions cells. To design these devices, it is necessary to have precise and fast method of electromagnetic modeling.

The electromagnetic modeling, which importance grows each day, profited from the marketing of more powerful computational tools which make it possible during these last decades to model more complex devices such as antennas feeding used in terrestrial and spatial modern telecommunications systems.

The significant development of the telecommunications field caused a keen demand of electromagnetic analysis and design tools and techniques. A diversity of numerical methods were thus been used to model microwave and millimeter-wave circuits such as: generalized S-matrix method (Arndt *et al*., 1986), finite difference method (Lotz *et al*., 1998), finite element method (Ilié Milan *et al*., 2004), boundary elements method (Wu and Litva, 1990), moment method (Zheng and Shen, 2005) and Variational Multimodal Method (VMM) (Tao and Baudrand, 1991; Nanan *et al*., 1991; Vuong, 1999).

The variational multimodal method is an integral one electromagnetic modeling method. It makes it possible to analyze the guiding structures starting from their impedance matrix and consequently from the individual S-matrix of discontinuities which compose them. Based on the concept of accessible and non-accessible modes, it makes it possible to reduce considerably the size of the matrix to be treated compared to the traditional methods such as the generalized S-matrix method. In its new formulation NMVF (Lilonga-Boyenga *et al*., 2007, 2008), the whole of discontinuities is characterizing by a single impedance matrix, leading to considerably reduced computing time and improved precision of the results.

The NMVF was used successfully to characterize with precision the uniaxial discontinuities and to design rectangular waveguides filters. In this study, we propose to apply it in the characterization of uniaxial discontinuities in homogeneous circular waveguides.

THEORY

Let us consider a structure composed by N cascaded uniaxial abrupt discontinuities as shown in Fig. 1. The current density at the z_i position is writing by:

$$\vec{J}_d(z_i) = (\vec{H}_t^{(i)}(z_i) - \vec{H}_t^{(i+1)}(z_i)) \times \vec{n} \tag{1}$$

$\overrightarrow{H_t}^{(i)}$ and $\overrightarrow{H_t}^{(i+1)}$ are the transverse magnetic fields on both sides of the interface of waveguides i and i+1 and, $\vec{n}$, the unit vector of the *z* direction of propagation.

Taking into account the conservation of the current on each interface, one can easily express the densities of surface current according to the tangential electric fields $\overrightarrow{E_t}(Z_i)$ at the different interfaces (Lilonga-Boyenga *et al*., 2007):

$$\vec{J}_d = \hat{Y}\vec{E}_t \tag{2}$$

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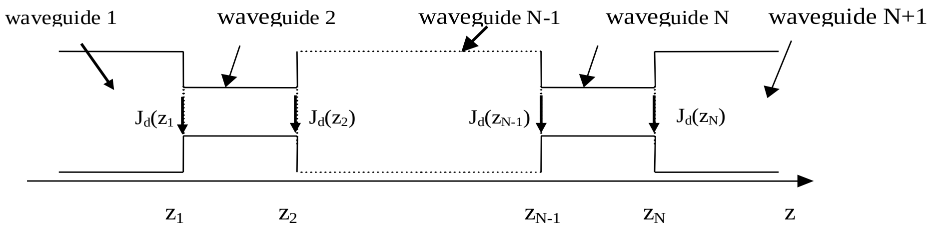

Fig. 1: Longitudinal view of N-uniaxial discontinuities structure

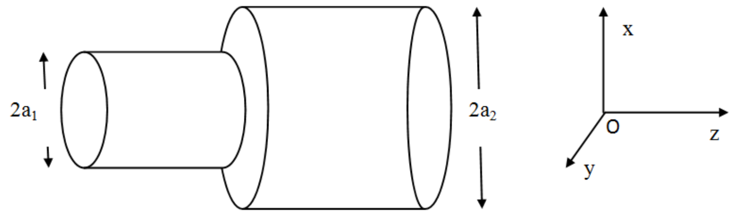

Fig. 2: Discontinuity between two homogeneous centered circular waveguides of radii a_1 and a_2

where,

$$\overrightarrow{J_d} = [\overrightarrow{J_d}(z_1)\,\overrightarrow{J_d}(z_2)\,\overrightarrow{J_d}(z_3)\ \dots\ \overrightarrow{J_d}\,(z_N)]^T$$

and

$$\overrightarrow{E_t} = [\overrightarrow{E_t}(z_1)\,\overrightarrow{E_t}(z_2)\,\overrightarrow{E_t}(z_3)\ \dots\ \overrightarrow{E_t}\,(z_N)]^T$$

T indicates the symbol of the transposed vector $\overrightarrow{E_t}(z_i)$, the tangential electric field at the interface i and $\hat{Y}$, the admittance operator of the overall structure.

The boundary conditions on the interfaces of discontinuities result in:

$$\begin{aligned}&\vec{J}\text{d} = 0 \text{ on dielectric interfaces}\\ &\vec{E}\text{t} = 0 \text{ on metallic interfaces}\end{aligned} \tag{3}$$

The variational form associated to the system (3) is:

$$f(\vec{E}_t) = \left\langle \vec{E}_t \middle| \hat{Y}\,\vec{E}_t \right\rangle \tag{4}$$

The solution of the system (3) is giving by the minimization of this above variational form. If the fields are developed on N_0 suitable test functions, after some algebraic handling, the minimization of (4) leads to the impedance matrix Z of the structure (Lilonga-Boyenga *et al*., 2008):

$$Z = -jN^{-1}\left|N\right|^{+\frac{1}{2}} U^{T*} Q^{-1} U \left|N\right|^{-\frac{1}{2}} \tag{5}$$

U and Q are scalar products matrix between modes of the waveguides sections and N the field normalization constant matrix. Matrix U and Q are evaluated from de coupling matrix given in Appendix. The symbol * indicates conjugate complex.

The scattering matrix S corresponding to this impedance matrix is then given by:

$$S = (Z + I)^{-1}(Z - I) \tag{6}$$

where, I is the unity matrix and N, the normalization constant matrix between modes.

RESULTS AND DISCUSSION

To valid the NVMF approach, two structures were analyzed: one step-discontinuity between two homogeneous circular waveguides of radii a_1 and a_2 and a double discontinuity formed by a thick iris of radius a_2 and length L inserted in a cylindrical waveguide of radius a_1. It should be noted that the characteristics of electromagnetic fields of TE and TM modes being propagated in the waveguides are analytical (Robert and Arlont, 1988).

One step-discontinuity between two homogeneous circular waveguides: The structure analyzed is sketched in Fig. 2. We first represent in Fig. 3, at the frequency of 12 GHz, the topology of the matrix of coupling between the modes of these two cylindrical centered waveguides on the assumption that the input waveguide of radius a_1 is supplied by the fundamental TE_{11} mode and where only the TE_{1n} and TM_{1n} modes are excited. Two cases are investigated:

1. a_1 = 10 mm and a_2 = 12.5 mm
2. a_1 = 11.165 mm and a_2 = 13.4 mm

As envisaged, this figure shows that, the coupling is stronger between the TE-TE modes on the one hand and the TM-TM modes on the other hand.

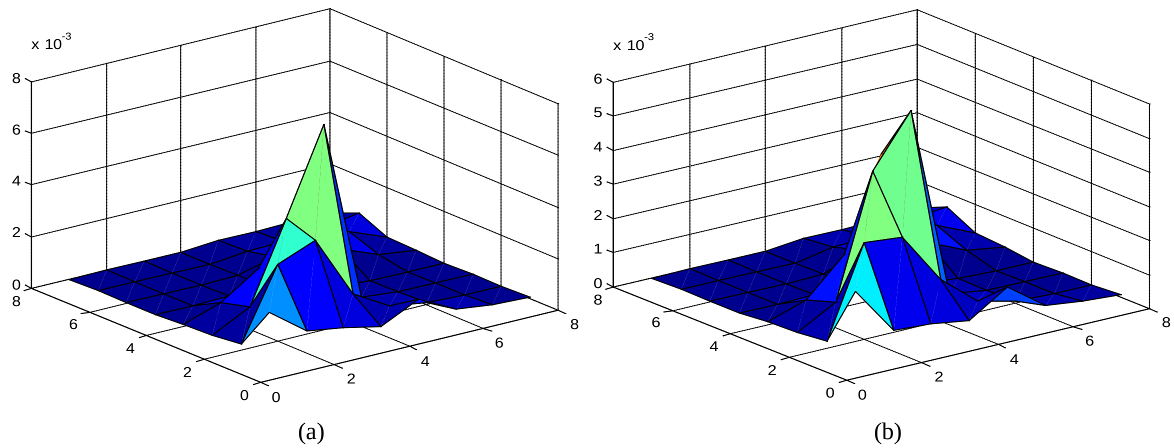

Fig. 3: Topology of matrix of coupling between two homogeneous centered cylindrical waveguide modes, (a) a_1 = 10 mm and a_2 = 12.5 mm, (b) a_1 = 11.165 mm and a_2 = 13.4 mm

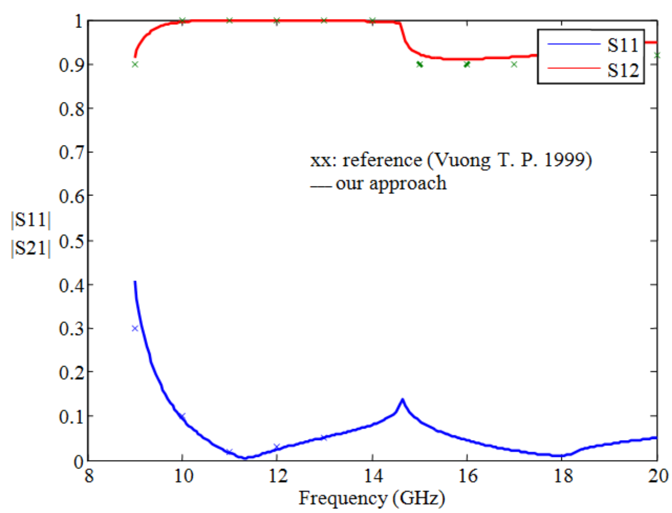

Fig. 4: Frequency response of one step-discontinuity (a_1 = 10 mm and a_2 = 12.5 mm)

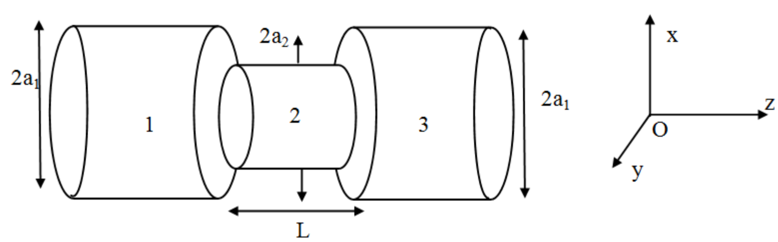

Fig. 5: Double discontinuity circular waveguide (1) - iris (2) - circular waveguide (3)

Secondly, for the case (1), using a computer code developed in MATLAB software, we evaluated the S-parameters of this discontinuity. Figure 4 illustrates frequency response of this discontinuity. We observe a good transmission on the frequency band used. However, one observes a resonance at the frequency of 14.6 GHz due to the interaction of the incident wave with the higher TM_{11} mode of the output waveguide. These results are in agreement of those obtained by Vuong (1999), whom used the classical variational method coupled to the electric equivalent circuit's method.

Double discontinuity: The variational method used to characterize a simple transition between two circular waveguides, can be extend to the characterization of a

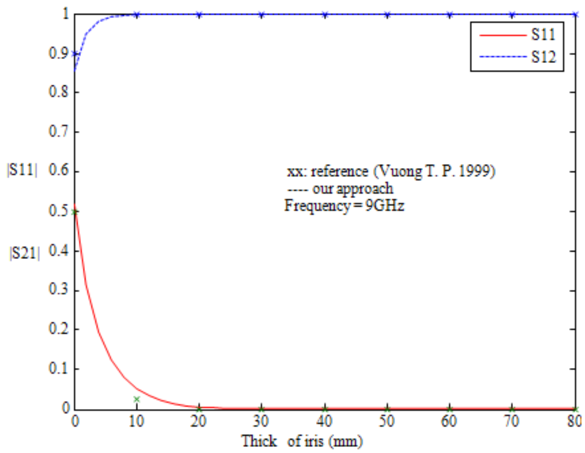

Fig. 6: Response of double discontinuity versus iris thickness

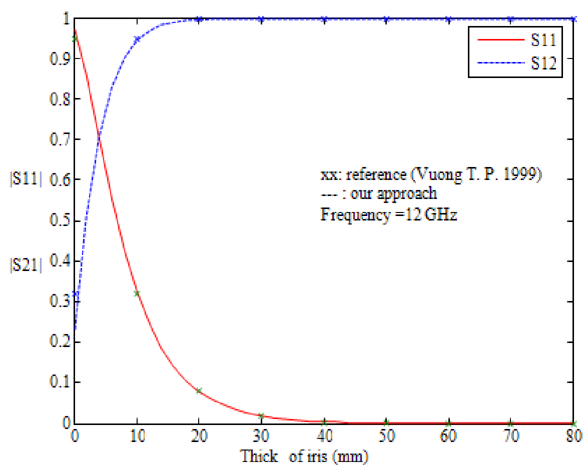

Fig. 7: Response of double discontinuity versus iris thickness

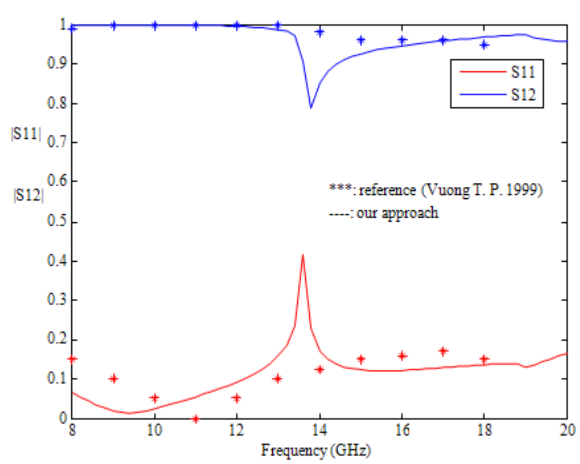

Fig. 8: Frequency response of the double discontinuity for a_1 = 13.4 mm and a_2 = 11.165 mm

double or more transition. In the classical variational method, if the scattering parameters S_1 and S_2 of the two individual discontinuities, separated by a waveguide section of length L, are determined, the cattering matrix S of the double discontinuity is obtained by the chaining method which requires a study of convergence, increasing thus considerably the computing time (Lilonga-Boyenga *et al*., 2007).

To validate efficiency of the NFVM approach, in which the study of convergence is not necessary, we studied the case of a double discontinuity formed by a thick iris of radius a_2 and length L, inserted between two circular waveguides of the same radius a1 as shown in Fig. 5, at 9 and 12 GHz. The waveguides and the irisare concentric.

Using the same computer code, we calculated the S-parameters of this double discontinuity at the frequencies of 9 and 12 GHz. Figure 6 and 7 illustrates the response of the structure against the thickness of the iris. We observe a good transmission when the thickness of the iris lies between 10 and 80 mm at 9 GHz and between 20 and 80 mm at 12 GHz. However, the reflexion decreases gradually as the length of the iris increases. We also compared our results with those obtained by Vuong (1999). In both cases, one notes a very good agreement between our approach and that of this reference.

The same computer code was used to calculate the S-parameters of the double discontinuity formed by two cylindrical waveguides of the same radius a_1 = 13.4 mm, separated by a thick cylindrical iris of L = 1 mm length and of radius a_2 = 11.165 mm.

Frequency response of this double discontinuity is represented on Fig. 8. Between 8 and 13.66 GHz, the transmission is almost good. One observes a resonance at the frequency of 13.66 GHz resulting to the interaction with the TM_{11} mode of the iris. We also compared our results with those obtained by Vuong (1999). As Fig. 8 shows, a good agreement can be noted.

The weak difference is the precision brought by the NFVM in which all higher modes in the iris are taking into account in calculations contrary to the VMM traditional one.

CONCLUSION

In this study, we used the NVMF to characterize two cylindrical waveguide filter structures presenting uniaxial discontinuities. The results obtained were compared with results available in the literature. We noted a good agreement between our results and those of the references. This study made it possible to show that the problems of uniaxial discontinuities in the cylindrical waveguide can be tackled with a higher degree of accuracy and a greater flexibility with a profit of the significant computing time using the NFVM. This tool of analysis and design of the microwave devices presenting uniaxial discontinuities will have to be able to apply to more complex and more varied structures such as elliptic filters and open structures.

APPENDIX

The expressions of the normalized components of the fields of the various TE and TM modes being propagated in a homogeneous cylindrical waveguide of radius a can be written in polar coordinates in the following form (Robert and Arlont, 1988):

TE modes:

$$\begin{cases} E_{r1m} = -\sqrt{\frac{2}{\pi}}\frac{jJ_1\left(\frac{t'_{1m}}{a}r\right)}{rJ_1(t'_{1m})\sqrt{t'^2_{1m}-1}}\cos\theta \\ E_{\theta 1m} = \sqrt{\frac{2}{\pi}}\frac{J'_1\left(\frac{t'_{1m}}{a}r\right)t'_{1m}}{aJ_1(t'_{1m})\sqrt{t'^2_{1m}-1}}\sin\theta \\ E_{z1m} = 0 \end{cases} \quad (A1)$$

$$\begin{cases} H_{r1m} = -\sqrt{\frac{2}{\pi}}\frac{j\beta_{1m}t'_{1m}J'_1\left(\frac{t'_{1m}}{a}r\right)}{\omega\mu_0 aJ_1(t'_{1m})\sqrt{t'^2_{1m}-1}}\sin\theta \\ H_{\theta 1m} = -\sqrt{\frac{2}{\pi}}\frac{J_1\left(\frac{t'_{1m}}{a}r\right)}{\omega\mu_0 rJ_1(t'_{1m})\sqrt{t'^2_{1m}-1}}\cos\theta \\ H_{z1m} = \sqrt{\frac{2}{\pi}}\frac{J_1\left(\frac{t'_{1m}}{a}r\right)K^2_{c1m}}{\omega\mu_0 J_1(t'_{1m})\sqrt{t'^2_{1m}-1}}\sin\theta \end{cases} \quad (A2)$$

TM modes:

$$\begin{cases} E_{r1m} = -\sqrt{\frac{2}{\pi}}\frac{jJ'_1\left(\frac{t_{1m}}{a}r\right)}{at_{1m}J_2(t_{1m})}\cos\theta \\ E_{\theta 1m} = \sqrt{\frac{2}{\pi}}\frac{jJ_2\left(\frac{t_{1m}}{a}r\right)}{rt_{1m}J_2(t_{1m})}\sin\theta \\ E_{z1m} = \sqrt{\frac{2}{\pi}}\frac{k^2_{c1m}J_1\left(\frac{t_{1m}}{a}r\right)}{r\beta_{1m}t_{1m}J_2(t_{1m})}\cos\theta \end{cases} \quad (A3)$$

$$\begin{cases} H_{r1m} = -\sqrt{\frac{2}{\pi}}\frac{j\omega\varepsilon J_1\left(\frac{t_{1m}}{a}r\right)}{r\beta_{1m}J_2(t_{1m})}\sin\theta \\ H_{\theta 1m} = -\sqrt{\frac{2}{\pi}}\frac{j\omega\varepsilon J'_1\left(\frac{t_{1m}}{a}r\right)}{a\beta_{1m}J_2(t_{1m})}\cos\theta \\ H_{z1m} = 0 \end{cases} \quad (A4)$$

where,

J_1 = The one order Bessel function of the first kind

$J`_1$ = Its derivative and $t_{1m}$ and $t`_{1m}$ the m^{th} zero of J_1 and $J`_1$, respectively

γ_{1m} = $j\beta_{im}$: The propagation constant in the waveguide interior medium and ε, its permittivity

The integral defining the coupling between the modes m and n of the electromagnetic fields in waveguides 1 and 2 is given by the matrix element:

$$C_{mn} = \iint_S \vec{E}_m^{(1)} \times \vec{H}_n^{(2)^*}\vec{n}ds \quad (A5)$$

This expression can also be writing:

$$C_{mn} = \iint_S \vec{E}_m^{(1)}\vec{J}_n^{(2)^*}ds \quad (A6)$$

$J_n^{(2)}$, being the current density of the n^{th} mode at the interface in waveguide 2 and S the section of aperture. Therefore the coupling element matrix is obtained using transformation to line integral (Luciano and Giorgio, 1994):

TE-TE coupling:

$$C_{mq}^{te-te} = \frac{2a_1\beta^*_{1q}K^2_{C1q}t'_{1q}J'_1\left(\frac{t'_{1q}}{a_2}a_1\right)}{\left(K^2_{C1m}-K^2_{C1q}\right)a_2\omega\mu_0 J_1\left(t'_{1q}\right)\sqrt{\left(t'^2_{1m}-1\right)\left(t'^2_{1q}-1\right)}} \quad (A7)$$

TE-TM coupling:

$$C_{mq}^{te-tm} = \frac{2\omega\varepsilon J_1\left(\frac{t_{1q}}{a_2}a_1\right)}{\beta_{1q}t_{1q}J_2(t_{1q})\sqrt{t'^2_{1m}-1}} \quad (A8)$$

TM-TE coupling:

$$C_{mq}^{tm-te} = 0 \quad (A9)$$

TM-TM coupling:

$$C_{mq}^{tm-tm} = \frac{2\omega\varepsilon K^2_{C1q}J'_1(t_{1m})J_1\left(\frac{t_{1q}}{a_2}a_1\right)}{\left(K^2_{C1m}-K^2_{C1q}\right)\beta^*_{1q}t_{1q}J_2(t_{1m})J_2(t_{1q})} \quad (A10)$$

$K_{c1\alpha}$, is the cutoff wave number of the mode α defined as follow:

$$K^2_{c1\alpha} = \omega^2\mu_0\varepsilon - \beta^2_{1\alpha} \ (\alpha = m, q)$$

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