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Significance Test of Reliability Evaluation with Three-parameter Weibull Distribution Based on Grey Relational Analysis

Xintao Xia, Yantao Shang, Yinping Jin and Long Chen
Mechatronical Engineering College, Henan University of Science and Technology, Luoyang 471003, China

Abstract: With the aid of the grey system theory, the grey relational analysis of the reliability with the three-parameter Weibull distribution is made for the Weibull parameter evaluation and its significance test. Via the theoretical value set and the experimental value set of the reliability relied on the lifetime data of a product, the model of the constrained optimization of the Weibull parameter evaluation based on the maximum grey relational grade. The grey significance of the reliability function with the three-parameter Weibull distribution is tested by means of the proposed criterion of the grey significance analysis of the reliability evaluation at the given grey confidence level. The cases of the helicopter component, the specimen and the ceramic material show that the grey relational analysis of the reliability is effective in the Weibull parameter evaluation and its significance test.

Keywords: Grey relational analysis, parameter evaluation, reliability, product, significance test, three-parameter weibull distribution

INTRODUCTION

For many products, such as spacecrafts, airplanes, bullet trains, nuclear reactors and submarines, the reliability is regarded as an important indicator for their safe and stable operation. In reliability analyses, the three-parameter Weibull distribution as a classical probability distribution is widely used in many scientific studies and engineering practices.

For example, Duffy *et al*. (1993a) described nonlinear regression estimators for the three-parameter Weibull distribution and made a reliability analysis of a turbopump blade; Ichikawa (1992) reviewed application of the three-parameter Weibull distribution in relation to reliability evaluation of ceramic components; Tsionas (2000) developed Bayesian analysis in the context of samples from three-parameter Weibull distributions and shown how to tackle the problems of prediction and estimation of reliability curves; Sürücü and Sazak (2009) considered a three-parameter Weibull distribution to model inter-failure times and used a robust estimation technique to estimate the unknown parameters; Bai and Mu (2011) used the three-parameters Weibull distribution to process the failure data of belt transportation system and presented the method of dynamic reliability evaluating of belt transportation system in mines by the practical example of the data statistical analysis; Ren *et al*. (2012) established the three-parameter Weibull distribution model of time between faults to improve the model accuracy of reliability evaluation of the machining center.

It can be seen that the primary concern in reliability analysis is to evaluate the Weibull parameters and so far many good results are achieved. But, from a standpoint of the statistical theory (Al-Rawi and Silva, 2012; Vetrova and Bardsley, 2012; Xia *et al*., 2012), the result of the parameter evaluation should follow a confidence level, that is, needs to conduct the significant test based on the fact that many complex and variable factors appear in life tests and the result may therefore be distorted, resulting in hidden dangers. For this reason, this study proposes a method for significance test of the reliability evaluation with the three-parameter Weibull distribution based on the grey relational analysis.

THEORETICAL VALUE SET AND EXPERIMENTAL VALUE SET OF RELIABILITY

The three-parameter Weibull distribution is defined as:

$$f(t;\eta,\beta,\tau)=\frac{\beta}{\eta}\left(\frac{t-\tau}{\eta}\right)^{\beta-1}\exp\left(-\left(\frac{t-\tau}{\eta}\right)^{\beta}\right) \quad (1)$$
$$t\geq\tau>0;\beta>0;\eta>0$$

where,

$f(t;\eta,\beta,\tau)$: The three-parameter Weibull distribution
t : A stochastic variable of the lifetime
(η,β,τ) : The mark of the Weibull parameters
η : The shape parameter

Corresponding Author: Xintao Xia, Mechatronical Engineering College, Henan University of Science and Technology, Luoyang 471003, China

β : The scale parameter
τ : The location parameter

The reliability function with the three-parameter Weibull distribution can be expressed as:

$$R(t;\eta,\beta,\tau)=\exp(-(\frac{t-\tau}{\eta})^{\beta}) \quad (2)$$

where,
$R(t;\eta,\beta$ and $\tau)$: The reliability function with the three-parameter Weibull distribution

For many products in engineering practices, the Weibull parameter (η, β, and τ) are unknown and need to be found out with the help of test evaluation. Thus, a lifetime experiment must be conducted. Assume that the lifetime data of a product is obtained as:

$$T=\{t_i\};t_1\le t_2\le\ldots\le t_i\le\ldots\le t_n;i=1,2,\ldots,n \quad (3)$$

where,
T = The lifetime data set
t_i = The ith lifetime datum in T
i = The sequence number of t_i
n = The number of the data in T

The lifetime data in T are substituted into Eq. (2) and the theoretical value of the reliability can be calculated by:

$$\mathbf{R}_0=\{R(t_i;\eta,\beta,\tau)\} \quad (4)$$

where,
$R(t_i;\eta,\beta,\tau)$: The theoretical value of the reliability
R_0 : The theoretical value set of the reliability

Under the condition of unknown Weibull parameters, the Nelson method (Nelson, 1990) and the Johnson method (Johnson, 1970) can be employed to make a nonparametric estimation for the empirical values of the reliability, as follows:

$$\mathbf{R}_1=\{r(t_i)\} \quad (5)$$

where,
$r(t_i)$: Called the ith empirical value of the reliability
R_1 : The empirical value set of the reliability

According to the Johnson method (Johnson, 1970), the empirical value of the reliability is expressed as an expectation, as follows:

$$r(t_i)=1-\frac{i}{n+1};i=1,2,\ldots,n \quad (6)$$

According to the Nelson method (Nelson, 1990), the empirical value of the reliability is expressed as a median, as follows:

$$r(t_i)=1-\frac{i-0.3}{n+0.4} \quad (7)$$

In reliability evaluation, the empirical value r (t_i) in R_1 can be considered as the experimental value of the reliability due to its corresponding to and depending on the lifetime datum t_i in T and R_1 can accordingly be called the experimental value set of the reliability.

With this, two sets, R_0 and R_1, that are, respectively, the theoretical value set and the experimental value set, are obtained and they can be used for an estimation of the three Weibull parameters by means of the grey relational grade.

Weibull parameter evaluation: The systems can be divided into three types: the black system, the white system and the grey system. The black system is not perceived one, the white system is perceived one and the grey system is a partly perceived and unascertained system between the black and white systems.

The grey relational grade as a very important concept in the grey system theory is widely used in many fields of science and technology (Xia *et al*., 2012, 2010; Wen *et al*., 2011; Sridhar *et al*., 2012; Palanikumar *et al*., 2012). For example, it can be employed to evaluate the difference of the attributes of systems according to the geometry shape of data series. The bigger the value the grey relational grade takes, the smaller the difference of the attributes of systems; or else, the bigger the difference. The grey relational grade is given by:

$$r=r(\eta,\beta,\tau)=\frac{1}{n}\sum_{i=1}^{n}\frac{\min_i|r(t_i)-R(t_i;\eta,\beta,\tau)|+\xi_0\max_i|r(t_i)-R(t_i;\eta,\beta,\tau)|}{|r(t_i)-R(t_i;\eta,\beta,\tau)|+\xi_0\max_i|r(t_i)-R(t_i;\eta,\beta,\tau)|} \quad (8)$$

where,
$r\in(0,1)$: The grey relational grade of the experimental value set R_1 to the theoretical value set R_0
$\xi_0\in(0,1)$: The distinguishing coefficient given in advance

The most common distinguishing coefficient ξ_0 is 0.5 in the Weibull parameter evaluation based on the maximum grey relational grade.

The optimal estimation of the Weibull parameter (η, β, τ) should makes the grey relational grade r maximum, as follows:

$$r \to r_{\max} = \max_{(\eta,\beta,\tau)} r(\eta,\beta,\tau) \tag{9}$$

The constraint conditions in Eq. (9) are given by:

$$0 < \tau \le t_1 \tag{10}$$

$$\eta > 0 \tag{11}$$

$$\beta > 0 \tag{12}$$

Equation (9) to (12) is called the model of the constrained optimization of Weibull parameter evaluation based on the maximum grey relational grade. With the help of the model, the optimal estimation for the Weibull parameter can be obtained as:

$$(\eta,\beta,\tau) = (\eta^*,\beta^*,\tau^*) \tag{13}$$

where,
(η^*, β^*, τ^*) : The optimal estimation for (η, β, τ)

The model means that the experimental value set R_1 is approaching to the theoretical value set R_0 along with the optimal estimation (η^*, β^*, τ^*) of the Weibull parameter (η, β, τ), when the grey relational grade r is approaching to the maximum grey relational grade $r_{\max}$.

GREY SIGNIFICANCE TEST OF RELIABILITY EVALUATION

Considered the first element R (t_1; η^*, β^*, τ^*) in R_0 as the reference value, two grey relational grades are defined as:

$$\gamma_0(\xi) = \frac{1}{n}\sum_{i=1}^{n} \frac{\min_i |R(t_i;\eta^*,\beta^*,\tau^*) - R(t_1;\eta^*,\beta^*,\tau^*)| + \xi \max_i |R(t_i;\eta^*,\beta^*,\tau^*) - R(t_1;\eta^*,\beta^*,\tau^*)|}{|R(t_i;\eta^*,\beta^*,\tau^*) - R(t_1;\eta^*,\beta^*,\tau^*)| + \xi \max_i |R(t_i;\eta^*,\beta^*,\tau^*) - R(t_1;\eta^*,\beta^*,\tau^*)|} \tag{14}$$

and

$$\gamma_1(\xi) = \frac{1}{n}\sum_{i=1}^{n} \frac{\min_i |r(t_i) - R(t_1;\eta^*,\beta^*,\tau^*)| + \xi \max_i |r(t_i) - R(t_1;\eta^*,\beta^*,\tau^*)|}{|r(t_i) - R(t_1;\eta^*,\beta^*,\tau^*)| + \xi \max_i |r(t_i) - R(t_1;\eta^*,\beta^*,\tau^*)|} \tag{15}$$

where,
$\gamma_0(\xi)$: The grey relational grade of R (t_i; η^*, β^*, τ^*) to R (t_1; η^*, β^*, τ^*)
$\gamma_1(\xi)$: The grey relational grade of r (t_i) to R (t_1; η^*, β^*, τ^*)
$\xi \in (0, 1)$: The distinguishing coefficient to be solved in the grey significance analysis of the reliability evaluation
R (t_1; η^*, β^*, τ^*) : The reference value

The grey difference is defined as:

$$d(\xi) = |\gamma_1(\xi) - \gamma_0(\xi)| \tag{16}$$

The grey difference can be employed to describe the difference degree between R_0 and R_1. Clearly, the grey difference varies with the distinguishing coefficient. The maximum grey difference can accordingly be given by:

$$d^* = \max_{\xi=\xi^*} d(\xi) \tag{17}$$

where
ξ^* : The optimum distinguishing coefficient

The equivalence coefficient of R_0 and R_1 is defined as:

$$f = \begin{cases} 1-\dfrac{d^*}{\eta}; & d^* \in (0,\eta] \\ 0 \quad ; & d^* \in [\eta,1] \end{cases} \tag{18}$$

where,
$f \in [0,1]$: The equivalence coefficient of R_0 and R_1
$\eta \in (0,1]$: The weight

Let the grey significance level $\alpha \in [0,1]$ and then grey confidence level can be given by:

$$P_G = (1-\alpha)\times 100\% \tag{19}$$

The most common grey significance levels are 0.05 in the grey significance analysis of the reliability evaluation based on the maximum grey relational grade.

The relation of the grey confidence level with the weight is:

$$\eta = \frac{1}{\lambda}\left(1-\frac{P_G}{100}\right) \tag{20}$$

where,
$\lambda \in [0,1]$: The level that also is called a cut of the relation between R_0 and R_1

The grey confidence level P_G is able to describe the reliability of the relation between R_0 and R_1 and the criterion of the grey significance analysis of the reliability evaluation is given below.

Criterion: if

$$f \geq \lambda \tag{21}$$

Then the relation between R_0 and R_1 is close, meaning that the result of the reliability evaluation is of significance at the P_G grey confidence level; otherwise the relation between R_0 and R_1 is not close, meaning that the result of the reliability evaluation is of no significance at the P_G grey confidence level.

The significance of the reliability function with the three-parameter Weibull distribution can be tested with the help of the given grey confidence level and the criterion of the grey significance analysis of the reliability evaluation. If the result of the reliability evaluation is of significance at the P_G grey confidence level, the reliability function R (t; η*, β*, τ*) with the three-parameter Weibull distribution is of value in the engineering practice studied; or else, the reliability function R (t; η*, β*, τ*) with the three-parameter Weibull distribution is of no value in the engineering practice studied.

In the light of the least information principle in the grey system theory, the optimum level is λ = 0.5.

CASE STUDIES

Case 1: This is a case of the helicopter component. In failure analysis, the lifetime data set of a helicopter component is obtained, as follows (n = 13) (Luxhoy and Shyur, 1995):

T = (156.5, 213.4, 265.7, 265.7, 337.7, 337.7, 406.3, 573.5, 573.5, 644.6, 744.8, 774.8, 1023.6)

For the Weibull parameter evaluation, the results are (η*, β*, τ*) = (451.9850, 1.6561, 97.2279) and r_{max} = 0.7682. The diagram of R_0 and R_1 (the median of the reliability) are presented in Fig. 1. Overall, the trend of the experimental value set R_1 is in very good accordance with that of the theoretical value set R_0, but its uncertainty and fluctuation relative to R_0 is big.

For the grey significance test of the reliability evaluation, let α = 0.05 and λ = 0.5, then the results are P_G = 95% and f = 0.234<0.5. From the criterion, the reliability function R (t; η*, β*, τ*) with the three-parameter Weibull distribution is of no value in the engineering practice studied at the 95% grey confidence level (this likely roots in the big uncertainty and fluctuation of R_1 relative to R_0 as shown in Fig. 1).

Case 2: This is a case of some specimen. Under the same level of stress the fatigue life data set of the specimen is obtained as (n = 20) (Deng *et al*., 2004):

T = (350, 380, 400, 430, 450, 470, 480, 500, 520, 540, 550, 570, 600, 610, 630, 650, 670, 730, 770, 840)

For the Weibull parameter evaluation, the results are (η^*, β^*, τ^*) = (338.2694, 2.2505, 259.5402) and r_{max} = 0.9486. The diagram of R_0 and R_1 (the expectation of the reliability) are presented in Fig. 2. Clearly, the experimental value set R_1 is in very good accordance with the theoretical value set R_0 and its uncertainty and fluctuation relative to R_0 is very small.

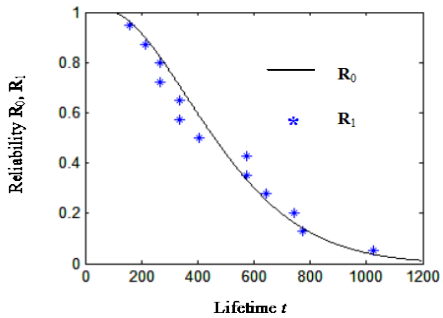

Fig. 1: Theoretical value set R_0 and experimental value set R_1 of reliability in case 1

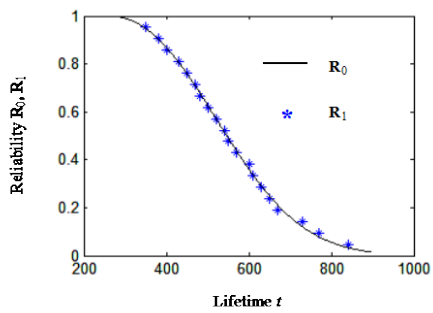

Fig. 2: Theoretical value set R_0 and experimental value set R_1 of reliability in case 2

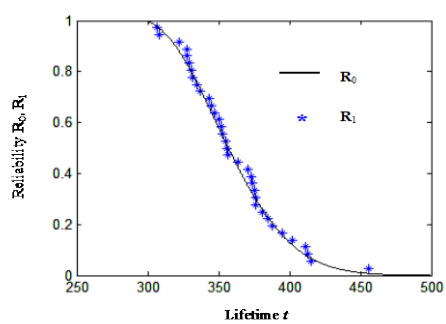

Fig. 3: Theoretical value set R_0 and experimental value set R_1 of reliability in case 1

For the grey significance testing of the reliability evaluation, let α =0.05 and λ = 0.5, then the results are P_G = 95% and f = 0.519>0.5. From the criterion, the reliability function R (t; η^*, β^*, τ^*) with the three-parameter Weibull distribution is of value in the engineering practice studied at the 95% grey confidence level (this likely roots in the very small uncertainty and fluctuation of R_1 relative to R_0 as shown in Fig. 2).

Case 3: This is a case of the ceramic material. In life tests, Duffy *et al*. (1993b) obtained the lifetime data set of aluminum oxide ceramics, as follows (n = 35):

T = (307, 308, 322, 328, 328, 329, 331, 332, 335, 337, 343, 345, 347, 350, 352, 353, 355, 356, 357, 364, 371, 373, 374, 375, 376, 376, 381, 385, 388, 395, 402, 411, 413, 415, 456)

For the Weibull parameter evaluation, the results are (η^*, β^*, τ^*) = (73.7708, 2.0351, 295.3628) and r_{max} = 0.7296. The diagram of R_0 and R_1 (the expectation of the reliability) are presented in Fig. 3. It is not hard to see that the experimental value set R_1 is in very good accordance with the theoretical value set R_0 and its uncertainty and fluctuation relative to R_0 is very small.

For the grey significance test of the reliability evaluation, let α =0.05 and λ = 0.5, then the results are P_G = 95% and f = 0.714>0.5. From the criterion, the reliability function R (t; η^*, β^*, τ^*) with the three-parameter Weibull distribution is of value in the engineering practice studied at the 95% grey confidence level (this likely roots in the very small uncertainty and fluctuation of R_1 relative to R_0 as shown in Fig. 3).

Three case studies above show that grey relational analysis of the reliability with the three-parameter Weibull distribution is effective in the Weibull parameter evaluation and its significance test.

CONCLUSION

The model of the Weibull parameter evaluation based on the maximum grey relational grade means that the experimental value set of the reliability is approaching to the theoretical value set of the reliability along with the optimal estimation of the Weibull parameters, when the grey relational grade is approaching to the maximum grey relational grade.

The significance of the reliability function with the three-parameter Weibull distribution can be tested with the help of the given grey confidence level and the criterion of the grey significance analysis of the reliability evaluation.

The cases of the helicopter component, the specimen and the ceramic material show that the grey relational analysis of the reliability with the three-parameter Weibull distribution is effective in the Weibull parameter evaluation and its significance test.

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