An Improved Data Correlation Algorithm for Multi-passive-sensor Tracking System

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Abstract: For improving the performance of the optimal assignment problem of data correlation of multi-passive-sensor system, an improved optimal assignment algorithm based on multi-source information fusion is put forward. The new algorithm takes advantage of the optimal solution and a certain number of near-optimal solutions of the traditional optimal assignment problem to construct a set of effective multi-tuple of measurement and constructs correlation probability fusing multi-source information between above effective multi-tuple of measurement and target track by using combination rule of D-S evidence theory. The result of simulation experiments shows that, compared with the traditional optimal assignment algorithm, the new algorithm not only improves the accuracy of multi-target tracking in different degrees but also saves a lot of time. So it is an effective data correlation algorithm for multi-passive-sensor system.

Keywords: Combination rule of D-S evidence theory, data correlation, multi-tuple of measurement, the optimal assignment algorithm

INTRODUCTION

When the dimension of the optimal assignment problem is bigger than or equal to 3, the technical complexity of solving the optimal assignment model increases exponentially with the increasing of the dimension of the assignment model (Han et al., 2010; Pattipati et al., 1992). And it is a NP-hard problem. The anti-jamming performances of the optimal assignment models which are instructed on the basis of different optimization criteria are different. Among them, the model error of the traditional optimal assignment problem constructed by using negative log likelihood ratio about measurement division is small. And it is more suitable for solving data correlation problem in dense target and clutter scenario. In good and general detection scenario, the tracking performance of the traditional optimal assignment problem is worse than that of the optimal assignment algorithm based on dynamic information. Deb et al. (1997) has done a deep research on the Lagrange relaxation algorithm of solving the traditional optimal assignment model and the study results show that it is a polynomial time algorithm. But with the increasing of the dimensions of the optimal assignment model and data correlation matrix, the calculation burden is still heavy. Studies have shown that the heavy calculation burden mainly comes from the calculations about the data correlation cost for all possible multi-tuple of measurement and solving of the assignment problem. Chummun et al. (2001) proposes a clustering algorithm which can avoid a large number of calculations from a great number of false location points. But this algorithm is not suitable for solving data correlation problem in dense target and clutter scenario. Goiri (2010) presents an assignment model which can avoid a lot of calculation for false correlations by using different attribute information and it has a certain effect in time savings. How to effectively fuse multi-source information to improve the tracking accuracy of multi-target and decrease the calculation burden of the traditional optimal assignment model still needs for further exploration and study.

In practice, owing to the complexity of detection environment and the measurement error of sensor, there usually exist certain model errors in multi-dimensional assignment model constructed by the measurements of sensors under complex detection scenario. If a model error of a multi-dimensional assignment problem is big, the measurement data correlation accuracy will not high even if one gets the accurate optimal solution of the assignment model.

In order to decrease the model error of the multi-dimensional assignment problem, one needs to effectively eliminate the interference of false location points. Popp et al. (2001) determines effective multi-tuple of measurement by using several good solutions of the optimal assignment model and weights the probability that the multi-tuple of measurement corresponds to a real target according to the idea of JPDA algorithm. And at last, it implements the optimal 2-Dimensional (2-D) point-track assignment between effective multi-tuple of measurement and target track under one-to-one feasible rule. As the optimal assignment on the unbalanced assignment problem may
lead to the fact that two or more multi-tuple of measurement corresponding to one track are assigned to different target tracks, so when the dimension of the optimal assignment problem is not high, this algorithm will not achieve the goal of eliminating the interferences of false location points effectively and decreasing the model error of the optimal assignment problem.

In view of the above analyses, one considers using certain number of good solutions of the traditional optimal assignment problem to compose effective multi-tuple of measurement and target track by fusing multi-source information using D-S combination rule of evidence theory, so as to reach the goal of improving the point-track correlation accuracy.

**DESCRIPTIONS OF THE TRADITIONAL OPTIMAL ASSIGNMENT ALGORITHM**

The likelihood function that multi-tuple of measurement \(Z_{i,t_2...t_s} = \{Z_{i,t_1}, Z_{i,t_2},..., Z_{i,t_s}\}\) associate with target \(t\) can be expressed as:

\[
A(Z_{i,t_2...t_s} \mid f) = \prod_{s=1}^{S} \left[ P_{dt} \cdot f(Z_{i,t_s}) \right]^{1-\delta_{0t_s}} \left[ 1 - P_{dt} \right]^{\delta_{0t_s}}
\]

where,
- \(f(Z_{i,t_s})\) = The probability density function that \(Z_{i,t_s}\) originates from target \(t\)
- \(P_{dt}\) = Detection probability of sensor \(s\)
- \(\delta_{0t_s}\) = Binary variables if \(t_s = 0\), i.e., target \(t\) is not detected by sensor \(s\), it is 1. Otherwise, it is 0

The likelihood function that each measurement is a false alarm or unrelated to target is:

\[
A(Z_{i,t_2...t_s} \mid \phi) = \prod_{s=1}^{S} \left( \frac{1}{\Psi_s} \right)^{n_s}
\]

where,
- \(\Psi_s\) : The size of the view of sensor \(s\)
- \(n_s\) : The number of target \(t\) detected by sensor \(s\)

Let:

\[
J = \min(- \ln \frac{A(Z_{i,t_2...t_s} \mid f)}{A(Z_{i,t_2...t_s} \mid \phi)})
\]

Then one can transform the correlation cost that multi-tuple of measurement \(Z_{i,t_2...t_s}\) associates with target \(t\) to the following formula:

\[
c_{i,t_2...t_s} = - \ln \frac{A(Z_{i,t_2...t_s} \mid f)}{A(Z_{i,t_2...t_s} \mid \phi)}
\]

\[
= \sum_{s=1}^{S} \left[ (1 - \delta_{0t_s}) \ln\left( \frac{\sqrt{2\pi} \cdot \sigma_{s}}{\Psi_s} \right) \right] + \frac{1}{2} \left( \frac{Z_{i,t_s} - \theta_{t_s}}{\sigma_{s}} \right)^2 \cdot \delta_{0t_s} \cdot \ln(1 - P_{dt_s})
\]

where, \(\sigma_s\) is the measurement error of sensor \(S\). Define binary association variables \(\rho_{i,t_2...t_s}\) as: if multi-tuple of measurement \(Z_{i,t_2...t_s}\) comes from a real target, \(\rho_{i,t_2...t_s} = 1\). Otherwise, \(\rho_{i,t_2...t_s} = 0\).

Then the measurement data correlation problem of multi-sensor can be described as the following optimal assignment problem:

\[
\min \sum_{i=1}^{n_1} \sum_{i=0}^{n_2} \cdots \sum_{i=0}^{n_S} c_{i,t_2...t_s} \rho_{i,t_2...t_s}
\]

Subject to:

\[
\sum_{i=1}^{n_1} \sum_{i=0}^{n_2} \cdots \sum_{i=0}^{n_S} \rho_{i,t_2...t_s} = 1; \quad \forall i_1 = 1,2\ldots n_1
\]

\[
\sum_{i=0}^{n_2} \sum_{i=0}^{n_3} \cdots \sum_{i=0}^{n_S} \rho_{i,t_2...t_s} = 1; \quad \forall i_2 = 1,2\ldots n_2
\]

\[
\vdots
\]

\[
\sum_{i=0}^{n_1} \sum_{i=0}^{n_2} \cdots \sum_{i=0}^{n_{S-1}} \rho_{i,t_2...t_s} = 1; \quad \forall i_S = 1,2\ldots n_S
\]

**IMPROVED MEASUREMENT DATA CORRELATION ALGORITHM**

In order to improve the tracking performance of the traditional optimal assignment algorithm and decrease the time spent of the algorithm, one considers fusing the assignment process of the multi-dimensional assignment algorithm of measurement data correlation with the point-track 2-D assignment process between the optimal multi-tuple of measurement and target. That is one considers using the optimal solution and a certain number of near-optimal solutions of the problem (5) - (6) to determine effective multi-tuple of measurement which are permitted to participate in the 2-D point-track assignment process and introducing the radiation source carrier frequency (RF), Pulse Repetition Interval (PRI), Pulse Width (PW) information to 2-D point-track correlation process by using combination rule of D-S evidence theory. Then a point-track correlation probability fusing state and above multiple feature.
information can be got. And the state of multi-target will be updated according to the result of a 2-D assignment model whose effective matrix consists of above fusion point-track correlation probabilities.

**Description of the improved measurement data correlation algorithm:** As the correlation expressions between different types of information and target track are different, therefore, one needs to determine the point-track correlation function expression before fusing multi-source information. The point-track correlation function expressions based on different types of information are described as follows.

**Correlation function expression based on state estimate:** For each multi-tuple of measurement got by components of several satisfactory solutions solving problem (5)-(6), one uses the following equations:

\[
\begin{align*}
\tan z_1(i_1) &= \frac{(y_i - y_1)}{(x_i - x_1)} \\
\tan z_2(i_2) &= \frac{(y_i - y_2)}{(x_i - x_2)} \\
&\vdots \\
\tan z_S(i_S) &= \frac{(y_i - y_S)}{(x_i - x_S)}
\end{align*}
\]

(7)

To estimate the position of target \( t \) corresponding to the above multi-tuple of measurement. Here, \( z_j(i_j) \) is the bearing measurement of the \( j \)th sensor and \( (x_j, y_j) \) is the position coordinates of the \( j \)th sensor.

Take the above estimate point as a fusion measurement corresponding to the multi-tuple of measurement and use it to construct the point-track correlation function based on state estimate as follows:

\[
A_t(1) = A_t(k) = \exp\left(-\frac{1}{2}v^T_k(k) \cdot [S^T_k(k)]^{-1} \cdot v_k(k+1) / \sqrt{2\pi S^T_k(k)} \right)
\]

(8)

where, \( i = 1,2,\cdots, m; t = 1,2,\cdots, T \). \( m \) is the confirmed number of valid measurements at current moment, here, it is the number of effective multi-tuple of measurement; \( T \) is the number of target; \( v_k \) is the residual vector that track \( t \) is from measurement \( i \); \( S_k \) is the corresponding residual covariance matrix.

**Correlation function expression based on radiation source carrier frequency:** If the type of target frequency is fixed and the corresponding central value is \( g_r \), then the frequency difference between the \( i \)th frequency measurement and the frequency of target is \( \Delta g_{it} = |g_{it} - g_r| \) and the correlation function between frequency measurement and the frequency of target can be defined as (Wang et al., 2006):

\[
A_t(2) = \begin{cases} 
0, & \Delta g_{it} \leq g_e \\
\frac{|\Delta g_{it} - g_e|}{g_e}, & g_e < \Delta g_{it} < 2g_e \\
1, & \Delta g_{it} \geq 2g_e 
\end{cases}
\]

(9)

**Correlation function expression based on pulse repetition interval:** Let us suppose that the pulse repetition interval is repetition frequency fixed and the central value is \( u_t \), then the difference of pulse repetition interval between the \( i \)th pulse repetition interval measurement and the pulse repetition interval of target is \( \Delta u_{it} = |u_{it} - u_t| \), i.e., the correlation function between pulse repetition interval measurement and the target can be described as:

\[
A_t(3) = \begin{cases} 
0, & \Delta u_{it} \leq u_e \\
\frac{\Delta u_{it} - u_e}{u_e}, & u_e < \Delta u_{it} < 2u_e \\
1, & \Delta u_{it} \geq 2u_e 
\end{cases}
\]

(10)

**Correlation function expression based on pulse width:** The point-track correlation function expression based on pulse width \( A_t(4) \) is similar to (10).

In order to fuse different types of measurement information by using combination rule of D-S evidence theory, one needs to convert the correlation function between different types of measurement and target track to basic probability assignment function. And then use the following combination rule of D-S evidence theory:

\[
m(C) = \sum_{\Delta A_t \cap B_k = \emptyset} \frac{m_1(A_t) m_2(B_k)}{1 - K} , \quad \forall C \subset U, C \neq \emptyset
\]

(11)

\[ K = \sum_{\Delta A_t \cap B_k = \emptyset} m_1(A_t) m_2(B_k) < 1 \]

(12)

to fuse several evidences based on different measurements to an integrated evidence from a single sensor. Continuing to use (11), one can further fuse above integrated evidences based on multi-source information fusion from different sensors to a joint basic probability assignment function \( m_{it} \) \((j = 1,2,\cdots, m; t = 1,2,\cdots, T)\). And then \( m_{it} \) can be converted into the correlation probability \( f_{it} \) between multi-tuple of measurement and target track. Then the point-track correlation probability matrix can be denoted as:
Thus, the point-track correlation problem between effective multi-tuple of measurement and target track can be described as the following 2-D assignment problem:

\[
\min \sum_{i=0}^{m} \sum_{t=0}^{T} f_{it} x_{it}
\]

Subject to:

\[
\begin{align*}
\sum_{i=0}^{m} x_{it} &= 1; \quad \forall t = 1, 2, \cdots, T \\
\sum_{t=0}^{T} x_{it} &= 1; \quad \forall i = 1, 2, \cdots, m
\end{align*}
\]  

(13)

Steps of the improved measurement data correlation algorithm:

Step 1: Solve the problem (5) - (6) and get the optimal solution and a number of near-optimal solutions.

Step 2: Construct a set of effective multi-tuple of measurement by components of above good solutions. For each multi-tuple of measurement, one fuses multi-source information from the same sensor by using combination rule of D-S evidence theory and gets the basic probability assignment function based on multi-source information fusion of single sensor. Continuing to fuse above basic probability assignment functions from different sensors using combination rule of D-S evidence theory, then one can get the joint basic probability assignment function \( m_{it} (i = 1, 2, \cdots, m; t = 1, 2, \cdots, T) \) which corresponds to the effective multi-tuple of measurement.

Step 3: Transform the joint basic probability assignment function \( m_{it} \) to the correlation probability \( f_{it} \) and then one can get the corresponding correlation probability matrix \( F_{it} = (f_{it}) \) between the effective multi-tuple of measurement and target track.

Step 4: Solve the 2-D assignment problem whose effective matrix is \( F_{it} = (f_{it}) \) and get the optimal point-track matching result. Then Kalman filter will be used to update the state of multi-target.

SIMULATIONS

Simulation scenarios: Let us suppose that three bearing-only passive sensors are used to locate targets.
stability. By comparing Fig. 1 to 4, one can see that as the interval between targets decreases, multi-target tracking accuracy of SD algorithm and FMFSD algorithm subsequently becomes poor. But FMFSD algorithm is still better than SD algorithm. And it can be seen from Fig. 3 to 6, that as the interval between targets further decreases, the convergence of SD algorithm becomes worse. But FMFSD algorithm fusing multi-source information still can track multi-target stably.

The above simulation results show that, in different simulation environment, the tracking performance of FMFSD algorithm fusing multi-source information is always superior to SD algorithm. That is to say, the tracking performance and tracking stability of the traditional optimal assignment algorithm can be improved in different degrees through fusing multi-source information effectively.

As can be seen from Table 1, FMFSD algorithm takes a relatively short time spent. That is because FMFSD algorithm improves correlation degree between the effective multi-tuple of measurement and target track by fusing multi-source information, which decreases the time spent of solving the point-track assignment problem. It also can be seen from Table 1 that, the advantage on time spent of FMFSD algorithm is more and more obvious as the number of targets increases. It can be seen that compared with SD assignment algorithm, FMFSD algorithm has good real-time performance and it is more suitable for engineering application.
CONCLUSION

The traditional optimal assignment algorithm is studied in this study. And on this basis, an improved optimal assignment algorithm fusing multi-source information has been put forward. The new optimal assignment algorithm improves the point-track correlation expression by introducing multi-source information to effective multi-tuple of measurement. Simulation results show that, compared with the traditional optimal assignment algorithm, the new algorithm not only improves the multi-target tracking accuracy in varying degrees and has good track stability, but also decreases the time spent of SD algorithm greatly. How to further decrease the time spent of the improved optimal assignment algorithm is worthy of future study.

REFERENCES


