Peak to Average Power Ratio Reduction using a Hybrid of Bacterial Foraging and Modified Cuckoo Search Algorithm in MIMO-OFDM System

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Abstract: The Partial Transmit Sequence which reduces the PAPR (Peak-to-Average Power Ratio) in Multiple Input Multiple Output Orthogonal Frequency Division Multiplexing (MIMO-OFDM) system using a novel optimization algorithm is proposed in this study. This novel optimization algorithm is based on a hybrid of Bacterial Foraging Optimization (BFO) and Modified Cuckoo Search algorithm (MCS) and is thus called HBFO-MCS. In HBFO-MCS, reproduction of individuals in a new generation is created, not only by swim and tumble operation as in BFO, but also by MCS. The natural reproduction step of BFO is swapped by the concept of searching best solutions as in MCS which then increases the possibility of generating the elite individuals for next generation. These enhanced reproduction step constitute the ready-to-perform population for the new generation once the initial population is performed by swim and tumble operation. Afterwards, discover probability is applied to abandon the worst solution due to the nature of MCS. HBFO-MCS is applied to optimize the best combination from a set of allowed phase factors in Partial Transmit Sequence (PTS) technique. The performance of HBFO-MCS is compared with BFO, Cuckoo Search (CS) and Modified cuckoo search MCS in the PAPR reduction in MIMO-OFDM system, accordingly proving its proficiency.

Keywords: Bacterial foraging algorithm, hybrid algorithm, MIMO-OFDM, modified cuckoo search algorithm, PAPR reduction, partial transmit sequence

INTRODUCTION

The advent evolutionary computation has inspired new resources for optimization problem solving, such as the optimal algorithms for phase factor optimization in Partial transmit sequence of MIMO-OFDM systems. In contrast to traditional computation systems which may be good at accurate and exact computation, but have brittle operations, evolutionary computation provides a more robust and efficient approach for solving complex real-world problems (Bäck and Schwefel, 1993; Fogel, 1995; Yao, 1999). Many evolutionary algorithms, such as Genetic Algorithm (GA) (Lain et al., 2011), Artificial Bee Colony algorithm (ABC) (Wang et al., 2010), Particle Swarm Optimization (PSO) (Wen et al., 2008; Mouhib et al., 2011), Bacterial Foraging Optimization (BFO) (Jing et al., 2008) and Modified Cuckoo Search (MCS) algorithm (Yang and Deb, 2010), have been proposed. Since they are heuristic and stochastic, they are less likely to get stuck in local minimum and they are based on populations made up of individuals with a specified behavior similar to biological phenomenon. These common characteristics led to the development of evolutionary computation as an increasing important field. Among existing evolutionary algorithms, the most well known branch is BFO. The Bacterial Foraging Optimization (BFO) algorithm is a biologically inspired computation technique which is based on mimicking the foraging behavior of E. coli bacteria. The information processing strategy of the algorithm is to allow cells to stochastically and collectively swarm toward optima. This is achieved through a series of three processes on a population of simulated cells:

- 'Chemotaxis' where the cost of cells is derated by the proximity to other cells and cells move along the manipulated cost surface one at a time (the majority of the work of the algorithm).
- 'Reproduction' where only those cells that performed well over their lifetime may contribute to the next generation.
- 'Elimination-dispersal' where cells are discarded and new random samples are inserted with a low probability.

It is the most robust and efficient algorithm in comparison with other presently available algorithms for global optimization of multi-objective, multi-
parameter design problems. Recently, a new evolutionary computation technique, the Cuckoo Search optimization (CS), is proposed (Yang and Deb, 2010). Like BFO, CS is initialized with a population of random solutions. Its development was inspired by the obligate brood parasitism of some cuckoo species by laying their eggs in the nests of other host birds (of other species). CS is based on three idealized rules:

- Each cuckoo lays one egg at a time and dumps its egg in a randomly chosen nest.
- The best nests with high quality of eggs will carry over to the next generation.
- The number of available hosts nests is fixed and the egg laid by a cuckoo is discovered by the host bird with a probability.

Discovering operates on some set of worst nests and discovered solutions dumped from farther calculations. Given enough computation, the CS will always find the optimum (Whitley et al., 1994) but, as the search relies entirely on random walks, a fast convergence cannot be guaranteed. Three modifications to the CS method are made with the aim of increasing the convergence rate, thus making the Modified Cuckoo Search (MCS) method more practical for a wider range of applications but without losing the attractive features of the original method. Modified Cuckoo search algorithms provide more robust results than Particle swarm optimization and Artificial bee colony algorithm. An extensive detailed study of various structural optimization problems suggests that modified cuckoo search obtains better results than other algorithms.

Hybridization of evolutionary algorithms with local search has been investigated in many studies (Merz and Freisleben, 1997; Jaszkiewicz, 2001; Krasnogor and Smith, 2000). Such a hybrid is often referred to as a memetic algorithm (Knowles and Corne, 2000; Ishibuchi et al., 2003; Zhou and Peng, 2005). In this study, we propose a new algorithm that combines the evolution ideas of both BFO and MCS. In the reproduction operation of BFOs, individuals are reproduced or selected using Lévy flight of MCS directly to the next generation without any enhancement. To integrate this phenomenon into BFO, MCS is adopted to enhance the individual bacterium to achieve the position of best food on each generation. After this enhanced reproduction process, discovery probability is applied to eliminate the bacteria from their position. The new positions produced by the enhanced reproduction phenomenon are expected to perform better than some of those in original population and the poor-performed individuals will be weeded out from generation to generation. To demonstrate the searching ability of HBFOMCS, the optimization of phase factors of the partial transmit sequence in MIMO-OFDM system is considered.

MIMO-OFDM SYSTEM MODEL

In MIMO-OFDM system, a number of antennas are placed at the transmitting and receiving ends and their distances are separated far enough. The idea is to use realize spatial multiplexing and data pipes by developing space dimensions which are created by multi-transmitting and receiving antennas. The transmitted signal bandwidth is so narrow that its frequency response can be considered as being flat (Jones et al., 1994). Defining the channel matrix has N_t×N_r complex matrix, the elements of it are fading coefficients from the j-th transmit antenna to the i-th receive antenna. Assuming that a MIMO system with a transmit array of N_t antennas and a receive array of N_r antennas, the transmission can be expressed as:

\[ y = Hx + n \]  

where,
\[ y = N_r \times 1 \text{ receiving vector} \]
\[ x = N_t \times 1 \text{ transmitting vector} \]
\[ n = \text{Additive white Gaussian noise with autocorrelation matrix:} \]
\[ R_n = E \{nn^H\} = N_0 I_{NT} \]

where,
\[ I_{NT} = An \times N_t, \text{ identity matrix} \]
\[ N_0 = \text{Identical noise power of each receiving branch} \]

PAPR of the MIMO-OFDM signal: In an OFDM system, a high-rate data stream is split into N low-rate streams that are transmitted simultaneously by subcarriers, where N is the number of subcarriers. Each of the subcarriers is independently modulated using Phase-Shift Keying (PSK) or Quadrature Amplitude Modulation (QAM). The Inverse Fast Fourier Transform (IFFT) generates the ready-to-transmit OFDM signal. For an input OFDM block X = [X_0, X_1, ..., X_{N-1}]^T, each symbol in X modulates one subcarrier of \{f_0, ..., f_{N-1}\}. The N subcarriers are orthogonal, i.e., f_n = n/Δf, where Δf = 1/NT and T is the symbol period. The complex envelope of the transmitted OFDM signal in one symbol period is given by:

\[ x(t) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_n e^{j2πf nt}, 0 \leq t < NT \]
The PAPR of \( x(t) \) is defined as the ratio of the maximum instantaneous power to the average power, that is:

\[
\text{PAPR} = \frac{\max_{0 \leq t < N_N} |x(t)|^2}{E\{ |x(t)|^2 \}}
\]  

(3)

\[
E\{ |x(t)|^2 \} = \frac{1}{NT} \int_0^T |x(t)|^2 dt
\]  

(4)

However, most systems use discrete-time signals in which the OFDM signal is expressed as:

\[
x(k) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_n e^{j2\pi nk/LN}, k = 0,1, \ldots, LN - 1
\]  

(5)

where, \( L \) is the oversampled factor. It has been shown in (M’uller and Huber, 1997b) that the oversampled factor \( L = 4 \) is enough to provide a sufficiently accurate estimate of the PAPR of OFDM signals.

Complementary Cumulative Density Function (CCDF) is a commonly used performance criterion to show the PAPR reduction and it is described as:

\[
\text{CCDF} = \Pr \{\text{PAPR}(x) > \text{PAPR}_0\}
\]  

(6)

where, \( \text{PAPR}_0 \) is a certain level of PAPR.

CCDF denotes the probability that the PAPR of the data symbol exceeds the given threshold and is given by:

\[
\Pr \{\text{PAPR}(x) > \text{PAPR}_0\} = 1 - (1-e^{-\text{PAPR}_0})^N
\]  

(7)

From Eq. (9) CCDF of MIMO-OFDM is much less than in Eq. (7).

**Partial transmit sequence for PAPR reduction:** The block diagram of the PTS method is shown in Fig. 1. In the MIMO-OFDM PTS, the input data vector \( X \) is encoded with space time encoder into two vectors \( X \) and \( Y \). Each of the data vector \( X \) and \( Y \) are considered as an individual OFDM transmission and the PTS method is applied individually for each of them. Three partitioning methods have been proposed in the literature and we choose the random partitioning method, which provides the best PAPR reduction performance. The PTS operation is explained for the space time encoded data \( X \) and is similar to the other data \( Y \). The input data block \( X \) is partitioned into \( M \) disjoint sub-blocks (Jing et al., 2009; Khan et al., 2010; Cimini and Sollenberger, 2000), \( X_m, m = 1, 2 \ldots, M \) such that:

\[
X = \sum_{m=0}^{M-1} X_m
\]  

(10)

Sub-blocks are combined to minimize the PAPR in the time domain. \( L \) times oversampled time domain signal of \( X_m \) is denoted as \( x_m, m = 1, 2 \ldots, M \), which are obtained by taking an IFFT of length \( NL \) on \( X_m \) concatenated with \((L-1) N\) zeros. Each \( x_m \) is multiplied by a phase weighting factor \( b_m = e^{j\phi_m} \), where \( \phi_m \in (0, 2\pi) \) for \( m = 1, 2 \ldots, M \). The goal of the PTS approach (M’uller and Huber, 1997a; Han and Lee, 2004) is to find an optimal phase weighted combination to minimize the PAPR value. The transmitted signal in the time domain after combination can be expressed as:
\[ x'(b) = \sum_{i=1}^{M} b_i x_i \quad (11) \]

where \( x'(b) = [x_1'(b), x_2'(b), \ldots, x_{W-1}'(b)] \)

In general, the selection of the phase factor is limited to a set with finite number of elements to reduce the search complexity. The set of allowed phase factors is:

\[ P = \{e^{j2\pi l/W}, l = 1, 1, \ldots, W-1\} \quad (12) \]

where, \( W \) is the number of allowed phase factors. We can fix a phase factor without any performance loss. There are only \( M-1 \) free variables to be optimized and hence \( W^{M-1} \) different phase vectors are searched to find the global optimal phase factor. The search complexity increases exponentially with \( M \), the number of sub-blocks. The aim in the PTS is to find the optimal phase factor. The phase factors are as follows:

- \( b_M = \{\pm 1\} \) if \( W = 2 \)
- \( = \{\pm 1, \pm j\} \) if \( W = 4 \)
- \( = \{\pm 1, \pm j, +0.707\pm 0.707j, -0.707\pm 0.707j\} \) if \( W = 8 \)

Therefore, the Side Information (SI) consists of \( b \) and the length of the SI is \( R = (M-1) \log_2(W) \) bits (Jiang et al., 2007; Zhang et al., 2009).

**MINIMIZE PAPR USING A HYBRID OF BFO AND MCS (HBFOMCS) ALGORITHM**

In order to get the OFDM signals with the minimum PAPR, a suboptimal combination method based on the hybrid of Bacterial Foraging Optimization (BFO) and Modified Cuckoo Search (MCS) algorithm is proposed to solve the optimization problem of PTS. The hybrid algorithm with lower complexity can get better PAPR performance. The minimum PAPR for PTS method is relative to the problem: Minimize:

\[ f(b) = \max_{E[|x(b)|^2]} \{\sum_{i=1}^{M} |x_i'(b)|^2\} \quad (13) \]

subject to \( b \in \{e^{j\phi_m}\}_W \quad (14) \)

where \( \phi_m \in \frac{2\pi k}{W}, k = 0, 1, \ldots, W-1 \)

The proposed HBFOMCS combines BFO with MCS to form a hybrid HBFOMCS. This hybrid of BFO with MCS algorithms can always produce a better algorithm than either the BFO or the MCS algorithms alone (Jing et al., 2008; Yang and Deb, 2010). In this section, basic concepts of BFO and MCS are introduced, followed by a detailed introduction of HBFOMCS in the next section.

**Basic concepts of BFO:** In recent years *E. coli* introduced the Bacterial Foraging Optimization algorithm (BFO) for numerical optimization problems (Yang and Deb, 2010). In BFO, a bacterium keeps foraging on food population by two basic steps, tumble and swim. The main goal of a bacterium is to optimize the best food position within the pre-defined iterations. Chemo-taxis is the initial step for a bacterium in which \( N_c \) numbers of swim steps are to be followed. When a bacterium completes each swim it calculates the fitness of current food position and compares it with the previous position. Once the fitness of the current food decreases, then the previous position of the bacterium gets changed in the moving direction which is known as tumble. Tumble is a unit walk in any random direction on food sources. In case a bacterium optimizes best fitness in the next swim steps, it keeps moving in the same direction up to \( N_s \) swim steps. Alternation between the swim and tumble steps involves in one chemo-taxis iteration.

After completion of \( N_c \) chemo-taxis steps, a bacterium finds \( M \) best food positions, therefore \( N \) number of bacteria optimize \( S = N_c^*N \) number of food positions. Reproduction and elimination dispersal are the augment steps in BFO. In the reproduction step, bacteria in the \( S \) food positions are arranged in descending order and are splitted into two i.e., \( S_r = S/2 \). The bottom half of \( S \) bacteria are removed as their fitness are low. The bottom half are replaced by the healthiest bacteria of the top position and are placed in the same food position for the next iteration. After the completion of \( N_r \) reproduction steps the elimination-dispersal step is performed based on the elimination probability \( E_{pr} \). In elimination-dispersal few bacteria are removed from the food position randomly based on \( E_{pr} \). The removed bacteria are then replaced by new bacteria selected randomly from the available bacterial population. The entire process is repeated again and it continues up to the defined value of elimination dispersal. The optimum combination is obtained once the elimination dispersal is over.

\( N^*N_c^*N_s \) is the measure of computation complexity or the number of searches for the proposed BFO-PTS algorithm, where \( N \) is the number of bacteria, \( N_c \) is the number of chemo-taxis and \( N_s \) is the number of swim steps initialized for optimization.

**Basic concepts of MCS:**

**Cuckoo Search (CS):** Is an optimization algorithm developed by Yang and Deb (2009). It was inspired by the obligate brood parasitism of some cuckoo species by laying their eggs in the nests of other host birds (of other species). Some host birds can engage direct conflict with the intruding cuckoos. For example, if a host bird discovers the eggs are not their own, it will either throw these alien eggs away or simply abandon...
its nest and build a new nest elsewhere. Cuckoo Search (CS) uses the following representations.

Each egg in a nest represents a solution and a cuckoo egg represents a new solution. The aim is to use the new and potentially better solutions (cuckoos) to replace a not-so-good solution in the nests. In the simplest form, each nest has one egg. The algorithm can be extended to more complicated cases in which each nest has multiple eggs representing a set of solutions. CS is based on three idealized rules:

- Each cuckoo lays one egg at a time and dumps its egg in a randomly chosen nest.
- The best nests with high quality of eggs will carry over to the next generation.
- The number of available host’s nests is fixed and the egg laid by a cuckoo is discovered by the host bird with a probability $p_d$. Discovering operate on some set of worst nests and discovered solutions dumped from farther calculations. Given enough computation, the CS will always find the optimum (Jing et al., 2008; Whitley et al., 1994; Yang and Deb, 2009) but, as the search relies entirely on random walks, a fast convergence cannot be guaranteed.

Modified Cuckoo Search (MCS): Three modifications to the cuckoo search are made with the aim of increasing the convergence rate, thus making the method more practical for a wider range of applications but without losing the attractive features of the original method (Yang and Deb, 2009). The three modifications are:

- In the CS, $\alpha$ is constant whereas in MCS, the value of $\alpha$ decrease as the number of generations $G_n$ increases. This is done to encourage more localized searching as the individuals, or the eggs, get closer to the solution.
- In the CS, there is no information exchange between individual nests and, fundamentally, the searches are performed independently. Therefore, adding up information exchange between the eggs tries to formulate convergence to minimum.
- In the CS, there is no restriction to the nests which have worst solutions. So, adding restriction in MCS for the nest which participated in a generation with worst solutions or eggs. In MCS, reduction of worst nest is applied to begin the next generation and search complexity gets decreased when compared with the CS.

$(N\star G_n)/2$ is the measure of computation complexity or the number of searches for the proposed MCS-PTS algorithm, where $N$ is the initial number of nests considered for optimization and $G_n = N-1$ is the number of generations considered for computation.

Hybrid of BFO and MCS (HBFOMCS): The Hybrid optimization algorithm, HBFOMCS consists of three major operators:

- Population or initialization
- Swim and tumble
- Reproduction with Lévy flight

This hybrid optimization algorithm utilizes the capable properties of MCS in BFO algorithm to achieve the better search ability with less computational complexity. The reproduction and elimination dispersal operation of BFO is replaced with the searching ability of MCS using Lévy flight. The Hybrid optimization algorithm, HBFOMCS consists of three major operators.

Initialization: In HBFOMCS, MCS work with the same population. Initially, $N$ individuals constituting the population should be randomly generated as in MCS. These individuals may be regarded as nests in terms of MCS, or as positions of bacteria in terms of BFO. In addition, the optimization parameters, such as maximum tumble, maximum swim, discover probability and maximum Lévy step size should be initialized. After initialization, new nests on the next generation are created by swim, tumble and reproduction operations.

Swim and tumble: In each generation, after the fitness values of all the nests in the same population are calculated, then the fitness values of $N$ nests are calculated to bring out the individuals (position or nest) with best solutions (food or egg). In case, if maximum swim does not generate best individuals then tumble procedure is performed to change the direction of bacteria or cuckoo. These individuals are regarded as elites. Instead of performing unlimited tumble steps as in BFO, HBFOMCS confines the maximum number of tumble steps as initialized. This enhancement procedure tries to imitate the maturing phenomenon in nature, where individuals will become more appropriate to the environment after attaining data from the society. Furthermore, by using this enhanced procedure in tumble step, the never-ending situation can be avoided while an ultimate individual has been generated in swim step.

Reproduction with Lévy flight: To produce well performing individuals, in the reproduction operation nests are reproduced by enhanced reproduction. To reproduce the nest, the Lévy flight of MCS is used and the discover probability is applied to select the
individuals. Thus, the top nests are reproduced after their fitness values are compared with the new individuals produced by Lévy flight.

The adopted reproduction may be regarded as a kind of elite reproduction and is used to increase the searching ability. As in nature, individuals selected have guaranteed ability to search new individuals will achieve better fitness than those by usual reproduction as in BFO. After top nests are reproduced, bottom nests or individuals are replaced by Lévy flight without comparing the fitness function. From the perspective of BFO, where bottom individuals are replaced by applying elimination dispersal probability (discover probability).

\((N*G_n)\) is the measure of computation complexity or the number of searches for the proposed HBFOMCS-PTS algorithm, where \(N\) is the initial number of nests considered for optimization and \(G_n = N-1\) is the number of generations considered for computation.

The mathematical model of hybrid algorithm is explained as follows:

**HBFOMCS algorithm:**

\[
\begin{align*}
A & \leftarrow \text{Max Lévy Step Size} \\
\text{MaxTumble} & \leftarrow \text{Allowed tumble steps} \\
\text{MaxSwim} & \leftarrow \text{Allowed swim steps} \\
P_d & \leftarrow \text{Discover Probability} \\
P_s = 1 - P_d & \leftarrow \text{Select Probability} \\
S & \leftarrow \text{Split Position} \\
\text{Initialize a population of } n \text{ nests:} \\
& \leftarrow f(x_i)
\end{align*}
\]

**SIMULATION RESULTS**

Hybrid BFOMCS algorithm is applied to search the better combination of phase factor for PTS. In the study, we select the phase factor \(b = \{-1, 1\}^M\) or \(b = \{-1, 1, j, -j\}^M\) or \(b = \{±1, ±j, +0.707±0.707j, -0.707±0.707j\}^M\). In the proposed HBFOMCS-PTS technique we optimize the best phase factor from \(W^{M-1}\) combinations where \(M\) is number of sub-blocks and \(W\) is the allowed phase factor. To evaluate and compare the performance of the HBFOMCS-PTS algorithm for MIMO-OFDM PAPR reduction, numerous simulations have been conducted. In order to get CCDF, 1000 random symbols are generated. The transmitted signal is oversampled by a factor of \(L = 4\) for accurate PAPR.

In our simulation, 16-QAM modulation with \(N = 256\) sub-carriers is used and the phase factor \(W = 2\) is chosen. When larger phase factor, for example, \(W = 4, 8, 16\) and \(32\) are chosen, the similar simulation results can be obtained, while the performance will be better.

In Fig. 2 and 3, some results of the CCDF of the PAPR are simulated for the MIMO-OFDM system with 256 subcarriers, in which \(M = 4\) and 16 sub-block employing random partition and the phase weight factor \(W = 32\) and 2, uniformly distributed random variables are used for PTS. As we can see that the CCDF of the PAPR is gradually promoted upon increasing the numbers of generations due to the limited phase weighting factor. As the numbers of generation are increased, the CCDF of the PAPR has been improved. For a generation \(G_n = 30\) and 20, we can see that the HBFOMCS based PTS technique is capable of attaining a near OPTS technique performance, when \(Pr(PAPR > PAPR_0) = 10^{-2}\).
Fig. 2: CCDF of HBFOMCS technique for different Gen ($G_n$) when $M = 4$ W = 16

Fig. 3: CCDF of HBFOMCS technique for different Gen ($G_n$) when $M = 4$ and W = 32

Fig. 4: CCDF of the PAPR with the PTS technique searched by HBFOMCS technique when $N = 256$, $M = 2, 4, 8$ and 16

the degree of improvement is limited when $G_n$ is above 30. On the other hand, the computational complexity is increased with $G_n$. Only a slight improvement is attained for increasing $G_n = 20$ to 30. The computational complexity of $G_n = 20$ is double of that of $G_n = 10$.

Fig. 5: Comparisons of HBFOMCS-PTS technique under different phase weight factors and $M = 4$ sub-block

Fig. 6: Comparisons of HBFOMCS-PTS technique under different phase weight factor and $M = 8$ sub-block

Fig. 7: Comparisons of HBFOMCS-PTS technique for W = 2 and $M = 16$ sub-block
Hence, based on the trade-off between the PAPR reduction and computational complexity, \( G_n = 20 \) is a suitable choice for our proposed HBFOMCS based PTS technique.

Figure 4 shows the simulated results of the HBFOMCS assisted PTS technique, in comparison against normal MIMO-OFDM for various sub-blocks \( M \). M is chosen as 8 and 16. In particular, the PAPR of an MIMO-OFDM signal exceeds 9.8 dB for \( 10^2 \) of the possible transmitted MIMO-OFDM blocks. However, by introducing PTS approach with \( M = 16 \) clusters partition with phase factor \( W = 2 \), at \( 10^2 \) PAPR reduces to 5.6 dB. In short, new approach can achieve a reduction of PAPR by approximately 4.2 dB at the \( 10^2 \) PAPR compared with the original MIMO OFDM. Thus, the performance of the techniques is better for larger \( M \) since larger numbers of vectors are searched for larger \( M \) in every update of the phase weighting factors.

Moreover, it can be observed that probability of very high peak power has been increased significantly if PTS techniques are not used. As the number of sub-blocks and the set of phase weighting factor are increased, the performance of the PAPR reduction becomes better. However, the processing time gets longer because of much iteration. From Fig. 4, as expected, the improvement increases as number of clusters increases. Thus, using the HBFOMCS technique, we can obtain better results than presented previously.

In Fig. 5 to 7, for a fixed number of clusters, the phase weighting factor can be chosen from a larger set of \( \{2, 4, 8, 16, 32\} \). It is shown that the added degree of freedom in choosing the combining phase weighting factors provides an additional reduction. When the number of phase weighting factor \( W = 16 \) and 32 and number of sub-blocks \( M = 4 \), PAPR can be reduced about 3.5 dB at \( 10^2 \) from 9.8 to 6.3 dB. When \( W = 2 \) and 4 and \( M = 8 \), at \( 10^2 \) PAPR can be reduced about 3.2 dB from 9.8 to 6.5 dB and about 4.4 dB from 9.8 to 5.6 dB. When \( W = 2 \) and \( M = 16 \), at \( 10^2 \) PAPR can be reduced about 4.2 dB from 9.8 to 5.6 dB. As the number of sub-blocks and the set of phase weighting factor are increased, the performance of the PAPR reduction becomes better. However, the processing time gets longer because of much iteration.

The iteration number of proposed technique is shown in Table 1. For \( M = 16 \) and \( W = 2 \) the OPTS technique requires 32,768 iterations per OFDM frame, while HBFOMCS-PTS technique requires 812 iterations only. The complexity of HBFOMCS-PTS is only 0.024\% (812/32768) of that of the optimal PTS technique. For \( M = 4 \) and \( W = 32 \) the OPTS technique requires 32,768 iterations per OFDM frame, while HBFOMCS-PTS technique requires 1122 iterations only. The complexity of HBFOMCS-PTS is only 0.034\% (1122/32768) of that of the optimal PTS technique. The iteration number of the proposed technique for other combination of phase weight factors and number of sub-blocks are also shown in Table 1.

Table 1: The computational complexity comparisons of HBFOMCS-PTS technique and OPTS under different phase factors (W) and sub-blocks (M)

<table>
<thead>
<tr>
<th>Combinations</th>
<th>OPTS</th>
<th>HBFOMCS-PTS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Search complexity (( W^{M+1} ))</td>
<td>PAPR (dB)</td>
</tr>
<tr>
<td>( W = 2 ) M = 8</td>
<td>128</td>
<td>6.5</td>
</tr>
<tr>
<td>( W = 2 ) M = 16</td>
<td>32,768</td>
<td>5.5</td>
</tr>
<tr>
<td>( W = 4 ) M = 8</td>
<td>16,384</td>
<td>5.6</td>
</tr>
<tr>
<td>( W = 16 ) M = 4</td>
<td>4096</td>
<td>6.2</td>
</tr>
<tr>
<td>( W = 32 ) M = 4</td>
<td>32768</td>
<td>6.3</td>
</tr>
</tbody>
</table>

Table 2: The computational complexity comparisons of HBFOMCS-PTS, MCS-PTS, BFO-PTS and OPTS under different phase factors (W) and sub-blocks (M)

<table>
<thead>
<tr>
<th>Methods</th>
<th>Computational complexity</th>
<th>( W = 2 ) M = 8</th>
<th>( W = 2 ) M = 16</th>
</tr>
</thead>
<tbody>
<tr>
<td>OPTS</td>
<td>( W^{M+1} = 128 )</td>
<td>6.5</td>
<td>-</td>
</tr>
<tr>
<td>( W^{M+1} = 32768 )</td>
<td>-</td>
<td>-</td>
<td>5.30</td>
</tr>
<tr>
<td>BFO-PTS</td>
<td>( N^<em>N_s</em>N_s = (5^*4^3) = 60 )</td>
<td>6.9</td>
<td>-</td>
</tr>
<tr>
<td>( N^<em>N_s</em>N_s = (5^*4^3) = 900 )</td>
<td>-</td>
<td>-</td>
<td>5.75</td>
</tr>
<tr>
<td>MCS-PTS</td>
<td>( N(N+1)/2 = (10^2) = 55 )</td>
<td>6.7</td>
<td>-</td>
</tr>
<tr>
<td>( N(N+1)/2 = (40^2) = 820 )</td>
<td>-</td>
<td>-</td>
<td>5.70</td>
</tr>
<tr>
<td>HBFOMCS-PTS</td>
<td>( N^*G_n = 8^*7 = 56 )</td>
<td>6.5</td>
<td>-</td>
</tr>
<tr>
<td>( N^*G_n = 29^*28 = 812 )</td>
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<td>-</td>
<td>5.50</td>
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For both the phase factor combinations shown in the table HBOMCS algorithm outperforms BFO and MCS in terms of PAPR reduction and computation complexity. For $M = 16$ and $W = 2$ the OPTS technique requires 32,768 iterations per OFDM frame to obtain a PAPR value of 5.3 dB, while HBOMCS-PTS technique requires 812 iterations only to obtain a PAPR value of 5.5 dB which is 0.2 dB greater than the optimal PAPR value and 0.2 dB less than MCS-PTS (820 searches) and 0.3 dB less than BFO-PTS (900 searches). For $M = 8$ and $W = 2$ the OPTS technique requires 128 iterations per OFDM frame to obtain a PAPR value of 6.5 dB, while HBOMCS-PTS technique requires 55 iterations only to obtain the same PAPR value of 6.5 dB which 0.2 dB less than MCS-PTS (55 searches) and 0.4 dB less than BFO-PTS (60 searches).

The performance of the proposed methods can be compared with the existing optimization algorithms for OFDM such as PSO and ABC in terms of computational complexity and PAPR reduction. Table 3 shows the comparison of computational complexity among different methods for phase factor $W = 2$, $M = 8$ sub blocks. PSO optimization algorithm (Wen et al., 2008) can obtain a PAPR reduction of 0.4 dB (8 dB) greater than the optimum value of 7.6 dB. Our proposed algorithms can obtain a better PAPR reduction i.e., BFO achieves the same PAPR reduction as PSO, 0.4 dB (6.9 dB) greater than the optimum value of 6.5 dB but MCS achieves a better PAPR reduction of 0.2 dB (6.7 dB) greater than the optimum value of 6.5 dB and HBOMCS achieves superior PAPR reduction of obtaining exactly the same optimal value of 6.5 dB.

The computational complexity reduction is much better for the proposed algorithms. PSO can achieve the above PAPR reduction in 88 searches which is 40 searches less than the optimum computation of 128 searches. BFO algorithm can achieve the same PAPR reduction in 60 searches which is 68 searches less than the optimum computation. Similarly MCS and HBOMCS can achieve better PAPR reduction in 55 searches which is 73 searches less than the optimum computation.

Table 4 shows the comparison of computational complexity among different methods for phase factor $W = 2$, $M = 16$ sub blocks. ABC optimization algorithm (Yajun et al., 2010) can obtain a PAPR reduction of 0.35 dB (6.8 dB) greater than the optimum value of 6.45 dB and PSO algorithm can obtain a PAPR reduction of 0.65 dB (7.1 dB) greater than the optimum value of 6.45 dB. In comparison BFO achieves 0.45 dB (5.75 dB) greater than the optimum value of 5.3 dB and MCS achieves a better PAPR reduction of 0.4 dB (5.7 dB) greater than the optimum value of 5.3 dB and HBOMCS achieves superior PAPR reduction of 0.2 dB (5.5 dB) greater than the optimum value of 5.3 dB.

The computational complexity comparisons of HBOMCS-PTS technique with MCS, BFO and OPTS under different phase weight factors and number of subblocks are shown in Table 2.
The computational complexity reduction is much better for the proposed algorithms. PSO and ABC algorithm can achieve the above PAPR reduction in 900 searches which is 31868 searches less than the optimum computation. BFO algorithm can achieve the same PAPR reduction as ABC and better than PSO in same number of searches. But MCS and HBFO MCS can achieve better PAPR reduction in 820 and 812 searches respectively which is less than the computation required by ABC and PSO. From the simulation results it is observed that the Hybrid of BFO and MCS (HBFO MCS) algorithm can give a better PAPR reduction with a minimum computational complexity.

CONCLUSION

In this study, we formulate the phase weighting factors searching of PTS as a particular combination optimization problem and we apply the hybrid BFOMCS technique to search the optimal combination of phase weighting factors for PTS to obtain almost the same PAPR reduction near to that of optimal PTS while keeping low complexity. Simulations results show that hybrid BFOMCS-based PTS method is an effective method to compromise a better trade-off between PAPR reduction and computation complexity. By appropriate selection of phase weighting factors according to the required performance and tolerable complexity, the proposed hybrid BFOMCS-PTS can give better PAPR reduction with less complexity. Additionally, the performance of the proposed method was slightly degraded compared to that of optimum method, PTS. However, the computational complexity of the proposed hybrid BFOMCS method was remarkably lower than that of optimum method and the individual BFO and MCS method.

REFERENCES


