Impact of Different Repair Assumptions on Repairable System Risk Assessment

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Abstract: In this study risk assessment is carried out to estimate the probability and magnitude of risk due to the unexpected system failure by considering different repair assumptions for repairable system. In order to measure the risk for different repair assumptions, the probability of failures and consequences are required. The probability of failure estimated using parametric Recurrent Data Analysis (RDA) approach while the consequences of failure analyzed based on reported data. Gas Turbine (GT) system was taken as a case study to verify the model. The results indicated that perfect repair assumption leads to minimum risk compared to imperfect and minimal repair assumptions. Based on results it was concluded that the maintenance team needs to follow perfect repair to mitigate the risk each time a failure happens.

Keywords: Parametric Recurrent Data Analysis (RDA), repairable system, repair assumptions, risk assessment

INTRODUCTION

A system failing to perform its functions can be restored to its original working condition by doing maintenance action is known as a repairable system (Lindqvist, 1999, 2006). For repairable systems, generally there are two main repair assumptions, either “as good as new” or “as bad as old”, but in reality the equipment lies somewhere in between these two conditions, which is called as imperfect repair or “better than old, but worse than new” (Doyen, 2005). The first two extreme assumptions for the repair were discussed by many researchers, are found not much practicable. These assumptions are less accurate compared to the imperfect maintenance assumption, because the failure nature of the repairable system depends much on the repair history of the system (Muhammad et al., 2009). The models mostly used to predict such assumptions are renewal processes including Homogeneous Poisson Process and Non Homogeneous Poisson Process. Such models were enough for simple system, but for complex repairable system there is a need of a more effective model (Lindqvist, 2008). Kijima and Sumita suggested a new approach called General Renewal Process (GRP) which is capable of covering all the three possible repair assumptions of repairable system (Muhammad et al., 2009). Kijima introduced two models, GRP Type-I and GRP Type-II. These models are considered special cases of Kijima virtual age model. According to GRP Type-I, repair removes only the portion of the age since last failure. However GRP Type II assumes that repair could remove the whole accumulated age for all previous failures.

Based on the repair assumptions discussed, a repairable system can be repaired once failure happens, but still it leads to risk due to unexpected failures. Hence, it is vital to assess risk of such unexpected failures to improve the system reliability. Many research works were carried on, related to risk assessment for repairable systems. Khan and Haddara (2003) and Wang et al. (2012) did quantitative risk assessment to minimize failure probability and failure consequences. Krishnasamy et al. (2005) assessed risk for power generating plant to mitigate maintenance cost including the cost of failure. Hu et al. (2009) conducted the risk assessment for petrochemical plant to schedule imperfect preventive maintenance. Khan and Haddara (2004) developed scheduled maintenance intervals based on the assessed risk value. Tan et al. (2011) performed risk assessment to categorize various equipments and selected the best maintenance strategies in Fujian oil refinery. Carazas and Souza (2010) applied the risk assessment concept to draw a decision making procedure for selecting maintenance policy for power plant equipment.

Risk assessment is an effective technique to minimize the failures and their consequences as stated in the literature. However, the effect of repair assumption which has the huge impact on the risk assessment of the repairable system was not integrated yet. Thus, in this study risk is evaluated based on different types of repair assumptions of repairable system. In order to measure the risk for different repair assumptions, the probability of failure estimated using parametric RDA approach. This approach is capable of predicting failures of repairable systems for all three types of repair assumptions. Meanwhile, the consequences of failure calculated based on reported data. Moreover, in this study only economic
consequences of failure are considered for risk assessment.

METHODOLOGY

In this study, three steps procedure was conducted to assess the risk of the system. In the first step, the probability of failure was analyzed using parametric RDA approach, secondly the consequences of failure were determined and thirdly; the risk quantification was carried out.

Failure probability: In this step, the failure probability of the system is defined using parametric RDA approach. The parametric RDA approach is based on GRP model, which provides a way to define the recurrence rate of repairable system failure overtime, by considering the repair effect on succeeding failure. Further RDA approach uses Power Law Model to estimate the probability of failure. The parameters of this model are calculated based on Maximum Likelihood Estimation (MLE) method. This model can be viewed as an extension of the Weibull distribution. As Weibull distribution governs the first system failure, but Power Law Model governs each succeeding system failure. Power Law intensity function can be written as (1) (Crow, 1990):

\[ \lambda(t) = \lambda \beta t^{\beta-1} \]

where,
\[ \lambda = \text{The scale parameter} \]
\[ \beta = \text{The shape parameter} \]
\[ t = \text{The system age.} \]

The value of each parameter is greater than zero. Hence, mean value of power law function is expressed as follow (2):

\[ E(N(t)) = \beta, \; t>0 \]

\( \lambda \) and \( \beta \) parameters can be estimated using Maximum Likelihood (ML) method, by (3) and (4) (Mettas and Zhao, 2005):

\[ \beta = \frac{n}{\sum_{i=1}^{n-1} \ln \left( \frac{t_i}{t_{i+1}} \right)} \]  
\[ \lambda = \frac{n}{t_n^\beta} \]

where, \( n \) shows \( n^{th} \) number of failure and \( t_i \) shows successive times to failure with \( 0<t_1<t_2<...<t_n \). The value of \( \lambda \) and \( \beta \) parameters may remain same or may change by different repair assumptions which are perfect repair, imperfect repair and minimal repair. After each type of repair the age of the system varies.

The age of the system can be defined based on the Kajima GRP Type-I and GRP-II models (Mettas and Zhao, 2005). Let assume a repairable system, where \( t_1, t_2, t_3, ..., t_n \) are the successive times for system failures. Let \( x_1, x_2, x_3, ..., x_n \) denote the time between failures for system. Also assume that some maintenance actions are taken after each failure to improve system performance. Let \( q \) be the maintenance effectiveness factor If value of \( q = 1 \) it's minimal repair, \( q = 0 \) perfect repair and if \( 0<q<1, \) it will be assumed imperfect repair.

Now GRP Type-I model assume that the \( i^{th} \) repair can remove accumulated age since \( i^{th} \) failure only. It can reduce the additional age \( x_i \) to \( q x_i \). Mathematically it can be represented by (5):

\[ v_i = v_{i-1} + qx_i = qt_i \]

where, \( v_i \) expresses the virtual age of repairable system after the \( i^{th} \) repair.

GRP Type-II model, assumes that up to the \( i^{th} \) failure virtual age has been accumulated to \( v_{i-1} + x_i \). In this type, \( i^{th} \) repair will remove the cumulative damage to system due to current and all previous failures, by reducing the virtual age to \( q(v_{i-1} + x_i) \). It can be estimated using (6):

\[ v_i = q(v_{i-1} + x_i) = q^2x_i + q^3x_2 + ... + x_i \]

Failure consequence analysis: Consequence analysis is the process for the quantification of the effect of the occurrence of each failure. Failure consequences in case of power plant include repair cost, loss of opportunity and maximum demand charge due to plant failure. Whenever system cannot fulfill required electricity capacity due to failure, it needs to use alternative electric supply source, which will impose maximum demand charge each time. The plant also has to pay for the amount of electricity consumed during that system down time. Then, failure consequences can be given as (7) (Nasir et al., 2012):

\[ \text{Consequences of failure} = \text{repair cost} + \text{loss opportunity cost} + \text{amount of electricity by alternate source cost} + \text{maximum demand charge cost} \]

\[ \sum (c_r + c_l + c_a + c_m) \]

- Repair cost estimation: Repair maintenance cost typically based on the cost of spare parts, labor cost etc. Cost of repair will be calculated using (8):

\[ \text{Cr} = NR_c \]

Now \( Cr \) is the total repair cost in MYR, \( N \) is the expected number of the failures and \( R_c \) is the cost of repair per failure.

- Loss opportunity cost estimation: This cost can be estimated using (9) (Márquez, 2007):
In (9) $C_E$ is the cost of electricity in MYR, $K$ is the amount of electricity plant supposed to produce in kW, $DT$ is the downtime in hours.

### Cost incurred due to alternative supply:
This cost can be expressed as in (10), it shows the costs incurred due to using alternative electricity supply source (Ray, 2007):

$$
C_{\text{incurred}} = C_E K \times \sum_{i=1}^{i=n} DT_i
$$

Cost incurred due to alternative supply $= C_E L \times \sum_{i=1}^{i=n} DT_i$  \( (10) \)

where $L$ is the amount of the electricity supplied from other source.

### Maximum demand charge cost:
This cost can be expressed as in (11); it shows maximum demand charge cost:

$$
\text{Max: demand charge cost per failure} = E \times (\text{max: demand in kW})
$$

where, $E$ = fixed cost in MYR/kWh of maximum demand

### Risk assessment:
It is systematic analysis to quantify probability and magnitude of losses due to system failure. Mathematically, it can be represented as (12) (Modarres, 2006):

$$
R = \sum_i f_i \times c_i
$$

where,

- $R$ = The expected risk value
- $f_i$ = Expected frequency of failures
- $c_i$ = The consequences of failure

### RESULTS AND DISCUSSION

#### Case study:
Gas turbine equipment is taken as a case study. This gas turbine is operating at campus Gas District Cooling (GDC) plant which has the capacity of 4.2 MW. One year gas turbine performance data was used to estimate the failure probability. The data was collected during the peak hours between 8 am to 5 pm for week days. The limit for minimum production capacity is based on the work done by Majid et al. (2011) on similar configured system. Whenever system output was below 1500 kW limit, it was considered as failure of the system. Time to failure (TTF) and cumulative time to failure of the gas turbine are shown in Table 1.

#### Table 1: TTF for gas turbine

<table>
<thead>
<tr>
<th>Failures No</th>
<th>TTF (Hrs)</th>
<th>Cumulative TTF (Hrs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>297</td>
<td>297</td>
</tr>
<tr>
<td>2</td>
<td>630</td>
<td>927</td>
</tr>
<tr>
<td>3</td>
<td>171</td>
<td>1098</td>
</tr>
<tr>
<td>4</td>
<td>603</td>
<td>1701</td>
</tr>
<tr>
<td>5</td>
<td>1287</td>
<td>2988</td>
</tr>
<tr>
<td>6</td>
<td>81</td>
<td>3069</td>
</tr>
<tr>
<td>7</td>
<td>207</td>
<td>3276</td>
</tr>
</tbody>
</table>

#### Table 2: Model selection based on MLE value

<table>
<thead>
<tr>
<th>Parameters and LK value</th>
<th>Kijima Type I</th>
<th>Kijima Type II</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>1.461617</td>
<td>1.6536</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.00008</td>
<td>0.00002</td>
</tr>
<tr>
<td>$q$</td>
<td>0.048474</td>
<td>0.233078</td>
</tr>
<tr>
<td>LK value</td>
<td>-43.387868</td>
<td>-43.139447</td>
</tr>
</tbody>
</table>

#### Table 3: Kijima type II parameters at different q values

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$q$</th>
<th>$\beta$</th>
<th>$\lambda$</th>
<th>$q$</th>
<th>$\beta$</th>
<th>$\lambda$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>1.3394</td>
<td>1.6536</td>
<td>1.0774</td>
<td>1</td>
<td>1.0774</td>
<td>0.00002</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.0002</td>
<td>0.00002</td>
<td>0.00098</td>
<td>0.0002</td>
<td>0.00002</td>
<td>0.00098</td>
</tr>
</tbody>
</table>

#### Table 4: Risk estimation for perfect repair

<table>
<thead>
<tr>
<th>Year</th>
<th>Expected Cumulative no: of failures</th>
<th>Consequences per failure (MYR)</th>
<th>Expected risk value (MYR)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5.882</td>
<td>165600</td>
<td>974059.2</td>
</tr>
<tr>
<td>2</td>
<td>12</td>
<td>165600</td>
<td>1987200</td>
</tr>
<tr>
<td>3</td>
<td>18.14</td>
<td>165600</td>
<td>3003984</td>
</tr>
<tr>
<td>4</td>
<td>24.524</td>
<td>165600</td>
<td>4061174.4</td>
</tr>
<tr>
<td>5</td>
<td>30.57</td>
<td>165600</td>
<td>5062392</td>
</tr>
</tbody>
</table>

#### Table 5: Risk estimation for imperfect repair

<table>
<thead>
<tr>
<th>Year</th>
<th>Expected Cumulative no: of failures</th>
<th>Consequences per failure (MYR)</th>
<th>Expected risk value (MYR)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5.774</td>
<td>165600</td>
<td>956174.4</td>
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<tr>
<td>2</td>
<td>12.07</td>
<td>165600</td>
<td>1998792</td>
</tr>
<tr>
<td>3</td>
<td>18.456</td>
<td>165600</td>
<td>3056313.6</td>
</tr>
<tr>
<td>4</td>
<td>24.956</td>
<td>165600</td>
<td>4132713.6</td>
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<tr>
<td>5</td>
<td>31.25</td>
<td>165600</td>
<td>5175000</td>
</tr>
</tbody>
</table>

#### Table 6: Risk estimation for minimal repair

<table>
<thead>
<tr>
<th>Year</th>
<th>Expected Cumulative no: of failures</th>
<th>Consequences per failure (MYR)</th>
<th>Expected risk value (MYR)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6</td>
<td>165600</td>
<td>993600</td>
</tr>
<tr>
<td>2</td>
<td>12.661</td>
<td>165600</td>
<td>2096661.6</td>
</tr>
<tr>
<td>3</td>
<td>19.597</td>
<td>165600</td>
<td>3245263.2</td>
</tr>
<tr>
<td>4</td>
<td>26.718</td>
<td>165600</td>
<td>4424500.8</td>
</tr>
<tr>
<td>5</td>
<td>33.98</td>
<td>165600</td>
<td>5627088</td>
</tr>
</tbody>
</table>

#### Selection of the model:
From Kajima virtual age models GRP Type-I and GRP-II selection were done based on MLE technique. Greater the MLE value of the model, best will be the statistical fit for the given data. Based on this assumption results of the estimated parameters are depicted in Table 2. The likelihood (LK) value of GRP Type-II is greater than GRP Type-I. This shows the best statistical fit is GRP Type-II for the given time to failure data of gas turbine.

#### Estimation of parameters for GRP Type-II model at different q value:
After selecting GRP Type-II, the parameter estimation was done by setting $q = 0, 0 < q < 1$ and 1. As discussed in methodology, when the value of $q$ is 0, the system follows perfect repair whereas if $q$ is 1, the system repair is minimal. If $q$ is between 0 and 1,
Based on these assumptions, the $\lambda$ and $\beta$ values were estimated and are indicated in Table 3. The results of the Table 3 showed $\beta$ value is more than 1, which means the failure rate is increasing.

**Estimation of expected number of failures:** Knowing the expected failure frequency is essential to evaluate the risk of failure. The cumulative expected number of failures for different repair assumptions is indicated in Fig. 1-3. Fig. 1 show the expected cumulative number of failures when the system follows perfect repair. At the end of year one, there is a possibility to have 5.882 failures. Fig. 2 shows the failure numbers for imperfect repair and it was observed that at the end of year one the number of failures is 5.774, but the failure frequency increases rapidly as compared to perfect repair. Due to increasing failure frequency, expected cumulative number of failures for next five years is more for imperfect repair as compare to perfect repair as shown in Table 5. Fig. 3 shows the cumulative number of failures for minimal repair and it was observed that the expected number of failures at the end of year one is 6. The gas turbine has more frequency of failure if it is supposed to be repaired by minimal repair assumption for each time.

**Risk quantification:** The downtime was extracted from the system failure data available and the labor cost rates and other production related costs were assumed based on reported data. The consequences per failure estimated using (7) are 1,65,600MYR each time for the gas turbine.

Risk quantification for five years was done for different repair assumption using (12) and the results are depicted in Table 4-6. The total risk value for perfect repair is about 5062392MYR, while the risk for imperfect repair was about 5175000MYR. The gas turbine incurs high risk value when it adapts minimal repair which is about 5627088MYR. The results revealed that perfect repair could minimize the risk of failure by 11.1 and 2.17% compared to minimal repair and imperfect repair respectively. Thus, perfect repair action would mitigate the risk through increasing availability and reliability of the system. If the system gets high availability and reliability, there will be less frequency of failure and maintenance cost.

**CONCLUSION**

In this study, risk assessment was carried out on repairable system considering different repair assumptions. For the analysis, failure probability and consequences of the failure were required. For failure probability, parametric RDA method was used, which is more advance and effective method in predicting the failure frequency of repairable systems for all three types of repair assumptions, which are perfect repair, minimal repair and imperfect repair. The consequences of the failure calculated based on the reported data.

Gas turbine equipment was taken as case study. The results revealed that perfect repair could minimize the risk of failure for GT by 11.1 and 2.17% compared to minimal repair and imperfect repair respectively. Thus, perfect repair action would minimize the risk by
increasing availability and reliability of the GT. If the GT gets high availability and reliability, there will be less frequency of failure and maintenance cost.

Even if the perfect repair minimizes the risk, the implementation of this repair assumption strategy apparently looks more costly. Thus, this study can be extended to cost benefit analysis, to get more realistic results.

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