

Experimental Evaluation of Gravity Wave Effects in a Stably Stratified Open Channel Flow

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Abstract: This study presents results of an experimental nature pertaining to a two-dimensional train of free internal gravity waves generated in a stably stratified pool of water bounded by the four sides of an open tank and free surface. The drag resistance due to the waves was measured by means of a spring-mass system connected between the moving mountain model and the winding mechanism of the apparatus. The mountain model was of particular importance in this study in that it was shaped in such a way that Fourier's transform could be employed in obtaining solution to the equations of motion. The analysis of the drag coefficient of the body was based strictly on the geometry of the mountain model. Fourier's analysis was employed to obtain the displacement of the fluid particle for each velocity variation.

Key words: Density stratification, drag resistance, Fourier's transform, gravity wave, mountain model, stratified atmosphere

INTRODUCTION

The first contributions to the history of waves date back to the late nineteenth century when researchers began to examine the waves set up in the lee of an obstacle at the bed of an open channel (Oyetunde, 2007; Ostrowski and szmytkiewicz, 2006). However, the first major contribution to the subject of wave forces included valuable results of theoretical and field studies which had very few comparisons with the field data. Mathematical relations for computing the forces exerted by breaking waves on breakwaters were also presented in the work. They were all derived from experiences in seas (Phillips, 1977; Long, 1953) and made a great contribution to the design of breakwaters, but they were not correlated with experimental data.

The general study of the flow of a fluid is very complex (Jamaluddin *et al.*, 2005; Oyetunde, 2007; Groves, 2004; Sen-Gupta, 1973). This is generally due to the irregularity of the waves and other complicated motions in the fluid flow. A usual approach to the problem has been to consider a simplification of the description of the fluid flow by linearizing the equation of the motion and the boundary conditions and by neglecting the viscosity of the fluid (Polton *et al.*, 2004). The time dependency of the solution is generally dealt with by a Fourier series substitution in the time domain (Lukomsky and Gandzha, 2003; Oyetunde and Okeke, 2004). In this way some simple classes of two-dimensional and three-

dimensional problems can be solved analytically. Nevertheless complete solution of the problem can seldom be obtained without recourse to experiment.

The purpose of this study is to consider the influence of major environmental forces on gravity waves in a stratified atmosphere using essentially an experimental technique. The study will establish, among others, detailed characterization of drag resistance effects of gravity waves for extrapolation to a stably stratified atmosphere especially as it affects free and forced rotation of astronomical satellites through it. Experimental evaluation and criteria for optimal control of the process will consolidate establishment of new results and contribution to knowledge.

MATERIALS AND METHODS

The apparatus and instrumentation: This consisted of a simple four pound force spring-mass system, a mountain model and an open tank measuring 3 m long, 0.1 m wide and 0.36 m deep. The mountain model (Obinabo (1978) has a ridge profile of the form:

$$Z = \frac{ab^2}{b^2 + x^2} \quad (1)$$

where, x and z are the spatial extent, and a and b the dimensions of the mountain model. This form (Fig. 1)

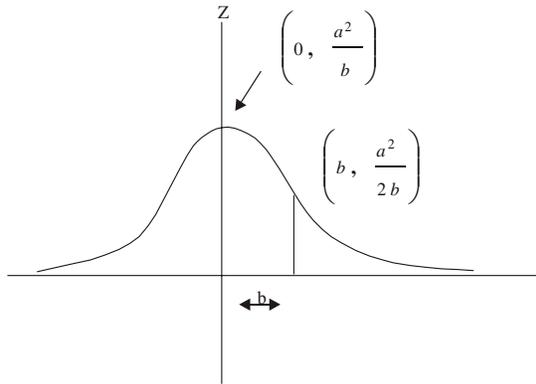


Fig. 1: Geometry of the mountain model $Z = \frac{ab^2}{b^2 + x^2}$

conforms to a simple Fourier's transform, and hence lends itself easily for use in the generalized Fourier's analysis in the solution of the wave equation.

The tank: Running down the entire length of the tank were two brass rails, one of circular cross-section, and the other rectangular. They were situated on either side of the top edges of the tank. A platform which was made to carry the mountain model was designed (Obinabo, 1978) such that it fitted satisfactorily on to the rails, and slid smoothly along them when pulled along by a winding mechanism. A steel wire was part of the winding mechanism and had one of its ends attached to one end of the platform and its length was stretched out and passed round the two simple pulley systems at the two ends of the tank. Between the other end of the steel wire and the platform was attached a spring-mass system by means of which it was possible to estimate the resistance encountered by dragging the mountain models through the stratified fluid.

The arrangement of the winding mechanism was such that the drive could be transmitted from the hydraulic gear box and electric motor down to the winding via a 'vee' belt. The motor, however, experienced a major disadvantage in that its speed could not be reversed without stopping it completely. Prior to the commencement of this experiment, no instrumentation was set up for monitoring the speed of the mountain models. Rough estimates were obtained by considering two points on the tank and using a stop watch to record the time the models travelled from one point to the other. Since this speed is considered a very important parameter in this work it was necessary, as a first approach to the experimental work, to calibrate the speeds of the motor. This was possible by connecting to the speed varying wheel of the motor a knob with distinctive circular graduations on it. Now using a stop watch and

considering the distance moved by the model each time it was possible to produce a graph of the speeds of the models against the dial graduations on the knob, and in this way values of speed in metres per second were assigned to the graduation on the knob.

Video apparatus: this consisted basically of a Sony video camera coupled to a Sony video recorder. The tank was illuminated by the light source of the video tape apparatus.

Density stratification: Density stratification was achieved by introducing lighter layers of salt solutions first from the underneath of the tank so that the denser layers when introduced later raised the lighter layers without interfering with the stratification already achieved (Fig. 2). The success of this method was enhanced by using a large funnel to which a considerably long rubber tube was connected. The free end of the rubber tube was lowered so that it rested on the bottom of the tank while the funnel was supported, on the tank, by the two edges. Nine mega litres of tap water was first introduced into the tank via the funnel-and-tube arrangement. The next layer to be introduced was the same quantity of water in which 200 mL of salt was dissolved. Throughout the stratification process the volume of water was kept constant while the quantity of salt dissolved was varied to give the required density gradient.

Experimental procedure: With the tank filled up with stratified tap water to a height of 0.25 m. The 'alpine' mountain model was secured in the gripping slot of the platform and dragged, along the length of the tank, in the fluid. The values of the forces registered on the spring-mass system were recorded against the corresponding speeds. The water in the tank was siphoned out and the tank was set up for the next experiment. This involved stratification of the fluid and it entailed introducing water with different concentration of salt into the tank in such a way that a satisfactory variation in steepness of the density gradient of the fluid was achieved. In order that the waves set up could be observed, tracers were inserted into the fluid. These tracers were droplets of mixtures of carbon tetrachloride and turpentine of known densities and coloured with paint. By varying the mixing densities of these tracers, it was possible to define distinct layers of stratification by the presence of straight lines formed by the arrangement of the tracers which settled in positions according to their densities. When the fluid had settled down, the mountain model was pulled along the tank through the fluid and the values of the resistance to motion were obtained for each speed and recorded as usual. Internal waves were set up during this motion and its motion was recorded with a video-tape apparatus.

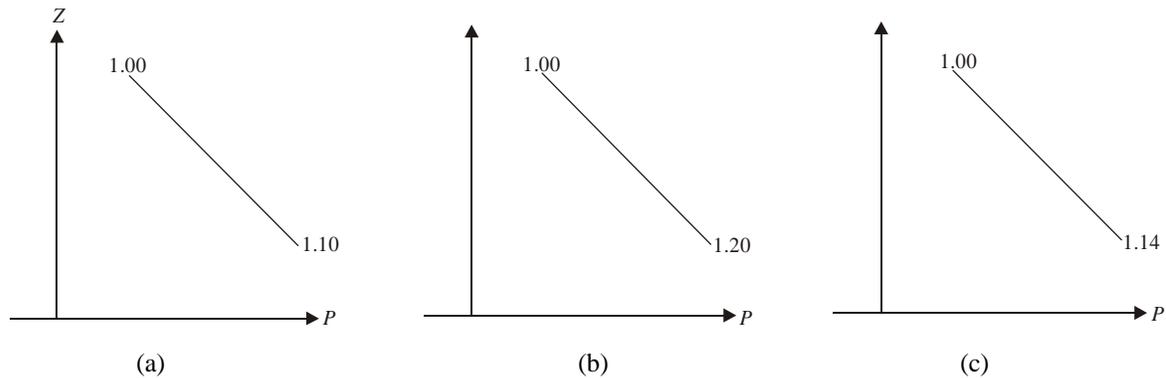


Fig. 2: Profile of the density stratification

Table 1: Experimental results

1 st Results			2 nd Results			3 rd Results		
Speed/mg	Scale reading (bf)	Drag N	Speed/mg	Scale reading (bf)	Drag N	Speed/mg	Scale reading (bf)	Drag N
0.03	0.750	3.34	0.03	0.375	1.67	0.03	0.313	1.39
0.06	0.875	3.89	0.06	0.500	2.22	0.06	0.563	2.50
0.10	1.250	5.56	0.10	0.688	3.06	0.10	0.625	2.78
0.14	1.625	7.23	0.14	0.813	3.61	0.14	0.750	3.34
0.18	1.875	8.34	0.18	1.000	4.45	0.18	1.063	4.73
0.21	2.000	8.90	0.21	1.188	5.28	0.21	1.250	5.56

Table 2: Parameters obtained by measurement of the waves produced

Speed(m/s)	Wavelength(m)	Amplitude(m)
0.03	16.4×10^{-2}	1.5×10^{-2}
0.06	27×10^{-2}	2.0×10^{-2}
0.10	67.6×10^{-2}	4.2×10^{-2}
0.14	82×10^{-2}	4.1×10^{-2}
0.18	waves produced here	
0.21	were impossible to record	

$$a = 2.8125 \times 10^{-2} \text{ m}$$

$$b = 5.625 \times 10^{-2} \text{ m}$$

$$x = 11.25 \times 10^{-2} \text{ m}$$

$$z = 12 \times 10^{-2} \text{ m}$$

CONCLUSION

The theory upon which this work was done is primarily analogous to an atmospheric problem. The laboratory model was an over-simplification of the practical problem in that in the atmosphere there exist many complicated boundary conditions. The experimental work was carried out by in a stably stratified fluid confined in an open channel. The surface boundary of the fluid was also considered as a part of the boundary of confinement of the fluid. In this way all possible boundary complications were avoided.

The results of the study showed a linear relationship between the drag and the velocity of travel of the mountain model in the fluid. Wave amplitude was also shown to bear similar relationship with the drag. The measured values of the drag were quite small but were within the expected range. The Morrison's equation for the drag consists of the velocity and inertia drag components; the inertia drag component is a function of the acceleration of the fluid. Since there was no acceleration of the fluid, the inertia term becomes eliminated from the Morrison's equation so that the total drag was given by the velocity drag component. The analysis of the drag coefficient of the body was based strictly on the geometry of the mountain model. Fourier's analysis was employed to obtain the displacement of the fluid particle for each velocity variation.

RESULTS AND DISCUSSION

The drag due to the waves was measured by means of a spring-mass system connected between the moving mountain model and the winding system of the apparatus. A mountain model designed by Obinabo (1978) was used to generate the gravity waves in the stratified fluid. The approach was of particular importance in that it was shaped to represent mountains and hills in our geographical environment that generate gravity waves in the atmosphere (Flanagan, 1969). The shape of the model (Obinabo, 1978) was to facilitate use of the Fourier's transform in the solution of the equations of motion describing the waves. This analysis (Table 1) gave the theoretical values of the amplitude of the waves, and a linear relationship was obtained between the drag resistance and the velocity. wave amplitude also bears similar relationship with the drag. The measured values of the drag (Table 2) were quite small but they were within the expected magnitude range for the laboratory model considered.

Other readings:

$$A_s = 19.065 \times 10^{-3} \text{ m}^2$$

$$h = 0.25 \text{ m}$$

The analysis of the result by Fourier's method gave rise to a method of obtaining theoretical values of the wave amplitudes which can be compared with the experimental results. The results show that the formation of the "rotors" is a function of the wave amplitude, and they tend to present complications in the flow of the fluid which may affect the values of the drag. streamline plots for values of the amplitude ranging from small to large were shown demonstrate the development of rotor regions in the fluid flow, the effects of which have considerable influence on the values of the drag. Use of the force equations and assumptions in their derivation as well as those assumptions made in the formulation of the linearized wave equation reveal that there is considerable room for improvements.

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