

## Comparison of Return Stroke Current Profiles for Transmission-Line-Type and Traveling-Current-Source-Type Models

<sup>1</sup>J.O. Adepitan and <sup>2</sup>E.O. Oladiran

<sup>1</sup>Department of Physics and Telecommunication, Tai Solarin University of Education, Ijagun, Ijebu, Nigeria

<sup>2</sup>Department of Physics, University of Ibadan, Ibadan

**Abstract:** The study is aimed at determining the dependence of the current along a channel on the model used, assuming the same base current. We compared three transmission-line-type models, namely: Transmission Line (TL), Modified Transmission Line with Linear decay, Modified Transmission Line with Exponential decay and two traveling-current-source-type models: Bruce-Golde (BG) and Traveling Current Source (TCS) models. The current profiles along the channel at different heights predicted by these models are presented and discussed. Comparison is based on the assumption that all the models have the same base current. It was found that at low heights and within a time window frame of 15  $\mu$ s, the currents of the transmission-line-type models predict a zero value at one time or the other with a maximum turning point following some 1 $\mu$ s after. A linear relationship is predicted between the current peak and the channel height. A discontinuity of current peak was observed at high heights. No zero value of current was recorded in case of TCS both at low and high channel heights.

**Keywords:** Channel base current, channel height, lightning channel, lightning models, return stroke current

### INTRODUCTION

Lightning return stroke models are categorized into four classes:

- **The gas dynamic or “physical” models:** These are primarily concerned with the radial evolution of a short segment of the lightning channel and its associated shock wave.
- **Electromagnetic models:** These are usually based on the so-called lossy thinwire antenna approximation of the lightning channel. These models involve a numerical solution of Maxwell’s equations to find the current distribution along the channel from which remote electric and magnetic field can be computed
- **The distributed circuit models, also called RLC transmission line models:** They can be viewed as an approximation to the electromagnetic models and they represent the lightning discharge as a transient process on a transmission line characterized by resistance, inductance and capacitance, all per unit length. These models are used to determine the channel current versus time and height and can therefore also be used for the computation of remote electric and magnetic fields.
- **The last class is the engineering models:** In which a spatial and temporal distribution of the channel

current (or the channel line charge density) is specified based on such observed lightning return stroke characteristics as current at the channel base, the speed of the upward propagating wave front and the channel luminosity profile.

Rakov and Uman (1998) classified a number of frequently used “engineering” return stroke models into two categories, transmission-line-type models and traveling-current-source-type models, with the implied location of the current source and the direction of the current wave as the distinguishing factors. The current source in the transmission-line-type models is often visualized to be at the lightning channel base where it injects an upward-traveling current wave that propagates behind and at the same speed as the upward-propagating return stroke front. The current source in the traveling-current-source-type models is often visualized as located at the front of the upward-moving return stroke from which point the current injected into the channel propagates downward to ground at the speed of light. Traveling-current-source-type models can also be viewed as involving current sources distributed along the lightning channel that are progressively activated by the upward-moving return stroke front, releasing the charge deposited by the preceding leader (Rachidi *et al.*, 2002).

In the Transmission Line (TL) model the return stroke process is modeled as a current wave injected at the

Table 1: Transmission-line type models for  $t \geq (z'/v)$

Models	$P(z')$	$v$	Current equation
Transmission Line(TL) Uman and McLain (1969)	1	$vf$	$I(z', t) = I(0, t - z'/v)$ for $t \geq z'/vf$ $I(z', t) = 0$ for $t < z'/vf$
Modified Transmission Line with Linear current decay (MTLL) Rakov and Dulzon (1991)	$1 - z'/H$	$vf$	$I(z', t) = [1 - z'/H]$ $I(0, 1 - z'/v)$ for $t \geq z'/vf$ $I(z', t) = 0$ for $t < z'/vf$
Modified Transmission Line with Exponential (MTLE) current decay Nucci <i>et al.</i> (1988)	$\exp(-z'/\lambda)$	$vf$	$I(z', t) = (0, 1 - z'/v)$ $\exp(-z'/\lambda)$ for $t \geq z'/vf$ $I(z', t) = 0$ for $t < z'/vf$

$H$  = total channel height = constant,  $v = v_r =$  constant,  $\lambda =$  constant = current decay constant

Table 2: Traveling current source-type models for  $t \geq z'/vf$

Models	$P(z')$	$v$	Current equation
Bruce and Golde (BG) Bruce and Golde (1941)	1	$\infty$	$I(z', t) = I(0, t) f$ or $t \geq z'/vf$
Traveling Current Source (TCS) Heidler(1985)	1	$-c$	$I(z', t) = I(0, 1 + z'/v)$ for $t \geq z'/vf$ $I(z', t) = 0$ for $t < z'/vf$

$P(z')$  is model-dependent attenuation function

base of the lightning channel and propagating upward along the channel with neither attenuation nor dispersion and at an assumed constant speed. In the Traveling Current Source (TCS) model the return stroke process is modeled as a current source traveling upward at an assumed constant speed and injecting a current wave into the channel, which then propagates downward at the speed of light and is absorbed at ground without reflection.

For the calculation of the lightning return-stroke electromagnetic field, a spatial-temporal distribution of the current along the channel  $i(z', t)$  must be assumed. To this purpose several models have been proposed. It is to be observed that only models in which the return-stroke current  $i(z', t)$  can be simply related to a specified channel base (ground-level) current  $i(0, t)$  are suitable, since only the channel-base current can be measured directly and only for it experimental results are available.

The most commonly adopted engineering return-stroke models used to calculate lightning-induced voltages are summarized in Table 1 and 2:

The Transmission Line (TL) model is the most widely used model of the lightning return stroke and is the simplest of the models in the transmission-line-type category. The TL model has been primarily employed to estimate return stroke peak currents and peak current derivatives from measurements of the peak electric field and peak electric field derivative, respectively, with an assumed return stroke speed. In the TL model the current specified at the base of the channel  $I(0, t)$  is assumed to propagate upward along the channel with a constant speed  $v$ , the speed of the return stroke front. The current at a given height  $z'$  is equal to the current at ground at time  $z'/v$  earlier.

The Traveling Current Source (TCS) model, proposed by Heidler (1985), is the simplest member of the

category of traveling-current-source-type models. In the TCS model, the current source is implied to be at the upward propagating (at constant speed  $v$ ) return stroke front and the current wave propagates downward with the speed of light  $c$  to the Earth where it vanishes (which implies that the channel is terminated in its characteristic impedance). The current at a given height  $z'$  is equal to the current at ground at time  $z'/c$  later.

We will identify and discuss significant features of the current profiles of engineering return stroke models.

## METHODOLOGY

For the current at the channel base  $i(0, t)$  of ground-initiated lightning return stroke, analytical expression (Heidler, 1985) is adopted:

$$i(0, t) = \frac{I_0}{\eta} \frac{\left(\frac{t}{\tau_1}\right)^n}{1 + \left(\frac{t}{\tau_1}\right)^n} \exp\left(-\frac{t}{\tau_2}\right) \quad (1)$$

where,

$$\eta = \exp\left[-\left(\frac{\tau_1}{\tau_2}\right)\left(\frac{n\tau_2}{\tau_1}\right)^{(1/n)}\right] \quad (2)$$

- $I_0 =$  Amplitude of the channel-base current
- $\tau_1 =$  Front time constant
- $\tau_2 =$  Decay time constant
- $\eta =$  Amplitude correction factor

$n =$  Exponent (2 ... 10)

The function allows for the adjustment of the current amplitude by varying  $I_0$

Sum of two functions given in Eq. (1) was chosen so as to obtain the overall waveshape of the current as observed in typical experimental results. The parameters listed in Table 1 were chosen. These values were adapted from Berger *et al.* (1975). The same undisturbed base current was employed in the comparison of the transmission-line type and traveling-current-source-type models.

Adapting MTLE, TL, MTLL, BG and TCS models, the current at various heights ( $z' = 200, 300$  m, 1, 2, 3 and 4 km) and a time window frame of between 0 and 15  $\mu$ s were calculated. Most literature relating to propagation of lightning over the ground adopted the value of  $n = 2$ . The same value is also adopted in this work. Wave speed of 0.5  $c$  is assumed. The cloud height,  $H = 5$  km for tropic was adopted (Aina, 1971).

Table 3: Typical values of parameters applied to base current. (Berger *et al.*, 1975)

$I_1$ (kA)	$\tau_{r1}$ ( $\mu$ s)	$\tau_{r2}$ ( $\mu$ s)	$n_1$	$I_2$ (kA)	$\tau_{r1}$ ( $\mu$ s)	$\tau_{r2}$ ( $\mu$ s)	$n_2$
10.7	0.25	2.5	2	6.5	2.1	230	2

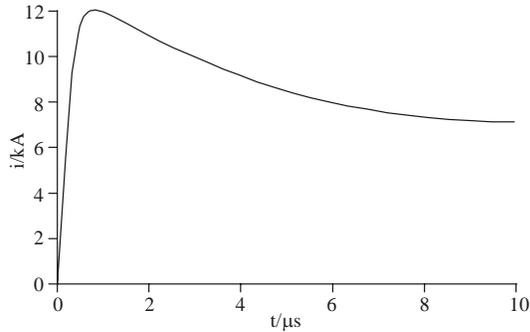


Fig. 1: Profile of undisturbed base-current,  $i(0, t)$ , same for all models, using the above typical parameters. The total channel height,  $H = 5$  km, Return stroke speed,  $v = 0.5c$  ( $m/\mu s$ )

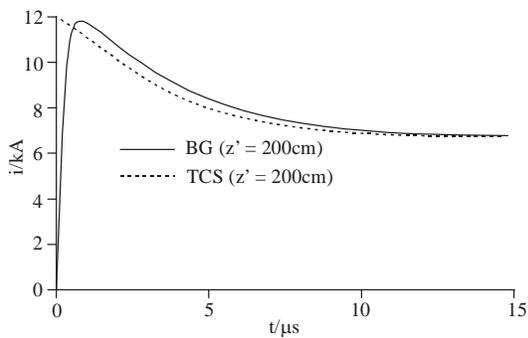


Fig. 2: Current as a function of time at height,  $z' = 200$  m (BG and TCS models)  $\mu s$

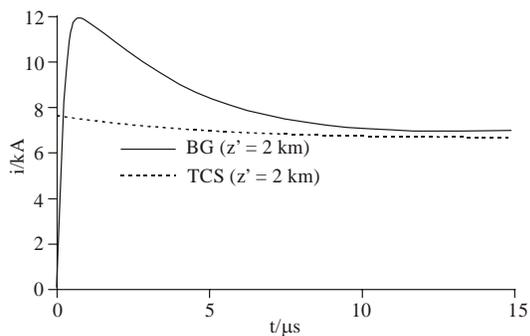


Fig. 3: Current as a function of time at height,  $z' = 2$  km, cloud height,  $H = 5$  km (BG and TCS models)

### RESULTS AND DISCUSSION

The profile of the common undisturbed base current is shown in Fig. 1 using the parameters in Table 3. Figure 2 and 3 show the profiles of TCS type models at channel

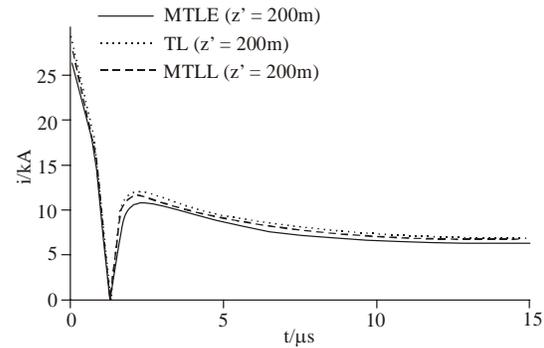


Fig. 4: Current as a function of time at height,  $z' = 200$  m, cloud height,  $H = 5$  km. (MTLE, TL and MTLL models)

Table 4: Peak values of the currents with different return stroke models at different heights ( $v = 0.5c$ )

Model	$Z' = 200$ m	$Z' = 300$ m	$Z' = 500$ m	$Z' = 1$ km
MTLE	10.9 kA	10.9 kA	9.7 kA	7.6 kA
MTLL	11.6 kA	10.9 kA	10.9 kA	9.9 kA
TL	12.1 kA	12.1 kA	12.1 kA	12.1 kA

heights 200 m and 2 km, respectively. For channel height  $z' = 200$  m, a time lag of  $0.65 \mu s$  occurred between the peaks of the current of BG model compare and that of TCS. It was observed that the current almost coincided in both case beyond the time for the peak values of the currents. Figure 3 revealed that at high channel height, say  $z' = 2$  km, the current of TCS model is almost constant with time., with no peak value.

**Case A-low heights:** Figure 4 presents return stroke current profile as a function of time,  $t$ , within a window frame of  $15 \mu s$  and channel height,  $z' = 200$  m for MTLE, TL and MTLL models. In case of MTLE model, at this height, the current dropped rapidly from  $27.1$  kA at time  $t = 0$  to a minimum turning point with current,  $i = 0$  within  $1.3 \mu s$ . The current picked up to a maximum turning point with peak current,  $I_p = 10.9$  kA within the next  $0.9 \mu s$ . Thereafter the current decreased gradually with time.

TL and MTLL models followed the same wave form as that of MTLE. It is observed that the minimum turning point of the three transmission-line-type models coincide at time  $t = 1.3 \mu s$ . Also the maximum turning point occurred at the same time,  $t = 2.2 \mu s$  with slight variation in the current peak as shown in Table 4.

Figure 5 presents return stroke current profile as a function of time,  $t$ , within a window frame of  $15 \mu s$  and channel height,  $z' = 500$  m for MTLE, TL and MTLL models. In case of MTLE model, at this height, the current dropped rapidly from  $67.3$  kA at time  $t = 0$  to a minimum turning point with current,  $i = 0$  within  $3.4 \mu s$ . The current picked up to a maximum turning point with peak current,  $I_p = 13.2$  kA within the next  $0.9 \mu s$ . Thereafter the current decreased gradually with time.

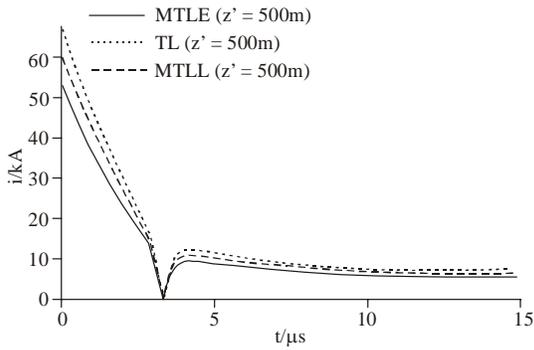


Fig. 5: Current as a function of time at height,  $z' = 500$  m, cloud height,  $H = 5$  km (MTLE, TL and MTLL models)

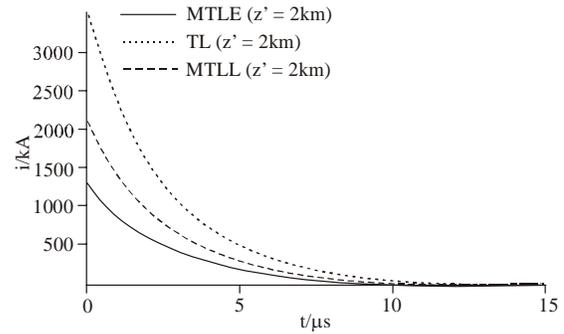


Fig. 8: Current as a function of time at height,  $z' = 2$  km, cloud height,  $H = 5$  km (MTLE, TL and TLL models)

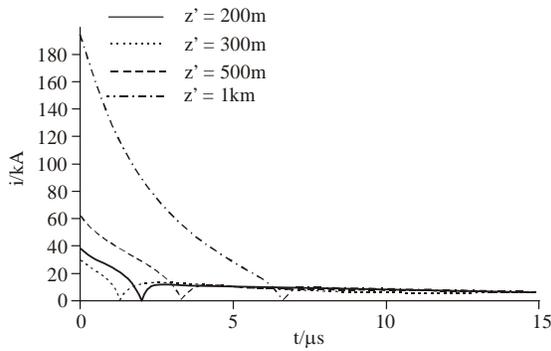


Fig. 6: Current as a function of time for MTLL model

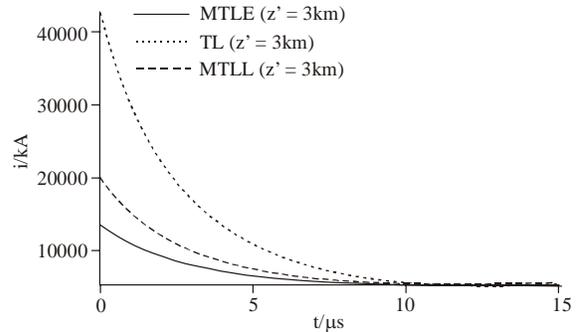


Fig. 9: Current as a function of time at height,  $z' = 3$  km, cloud height,  $H = 5$  km. (MTLE, TL and MTLL models)

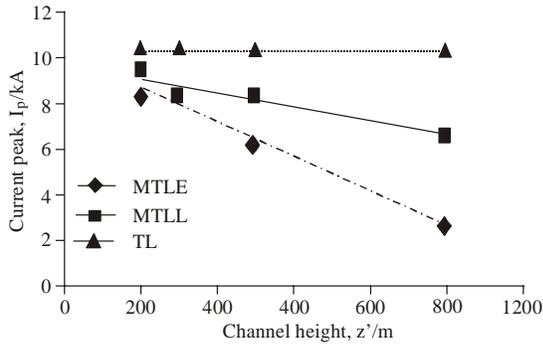


Fig. 7: Relationship between current peak,  $I_p$  and channel height,  $z'$

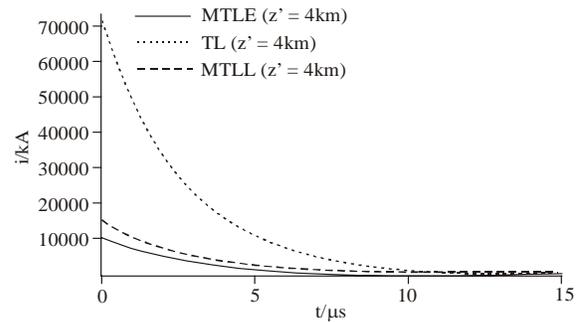


Fig. 10: Current as a function of time at height,  $z' = 4000$  m, cloud height,  $H = 5$  km. (MTLE, TL and MTLL models)

TL and MTLL models followed the same wave form as that of MTLE.

Figure 6 revealed that in case of MTLL, a time delay of 2  $\mu$ s was observed in the wave form at  $z' = 500$  m relative to that at  $z' = 200$  m. The delay time of waveform was 5.3  $\mu$ s in case of channel height,  $z' = 1$  km relative to that of  $z' = 200$  m. It is also observed that peak current attenuates with increase in channel height (Table 4 and Fig. 6). The same pattern of time delay in wave form was observed for both MTLE and TL. A linear relationship is established between the peak current,  $I_p$  and the channel height,  $z'$  (Fig. 7)

**Case B-high heights:** Figure 8 to10 represent return stroke current profiles as a function of time,  $t$ , within a window frame of 15  $\mu$ s and channel height,  $z' = 2, 3$  and 4 km for MTLE, TL and MTLL models, respectively. The wave forms are the same for transmission-line-type models. The minimum and maximum turning points observed in current profiles at low heights are discontinuous at heights  $z' = 2$  km and above. A rapid increase in the values of the current with height is observed.

## CONCLUSION

We compared three transmission-line-type models, namely: Transmission Line (TL), Modified Transmission Line with Linear decay, Modified Transmission Line with Exponential decay and two traveling-current-source-type models: Bruce-Golde (BG) and Traveling Current Source (TCS) models. The current profiles along the channel at different heights predicted by these models are presented and discussed. Comparison is based on the assumption that all the models have the same “undisturbed base current”. At low heights and within a time window frame of 15 $\mu$ s, the currents of the transmission-line-type models predict a zero value at one time or the other with a maximum turning point following some 1  $\mu$ s after. A linear relationship is predicted between the current peak and the channel height. There is discontinuity of current peak at high heights. No zero value of current was recorded in case of TCS both at low and high channel heights.

## REFERENCES

- Aina, J.I., 1971. Lightning discharge studies in a tropical area. *J. Geomagn. Geoelectr.* 23(3): 347-358.
- Berger, K., R.B. Anderson and H. Kroninger, 1975. Parameters of lightning flashes. *Electra*, No. 41.
- Bruce, C.E.R. and R.F. Gold, 1941. The lightning Discharge. *J. Int. Electr. Eng.*, 88: 487-520.
- Heidler, F., 1985. Traveling Current Source Model for LEMP Calculation. Paper Presented at the 6th International Symposium on Electromagnetic Compatibility, Swiss Fed. Inst. of Technol., Zurich, Switzerland.
- Nucci, C.A., C. Mazzetti, F. Rachidi and M. Ianoz, 1988. On lightning Return Stroke Models for LEMP Calculations, Paper Presented at the 19th International Conference on Lightning Protection, O<sup>o</sup> VE, Graz, Austria.
- Rakov, V.A. and A.A. Dulzon, 1991. A modified transmission line model for lightning return stroke field calculation. In *Proceeding of 9th International Symposium on EMC, Zurich, Switzerland*, 44H1, pp: 229-235.
- Rakov, V.A. and M.A. Uman, 1998. Review and evaluation of lightning return stroke models including some aspects of their application. *IEEE T. Electromagn. Compat.*, 40: 403-426.
- Rachidi, F., V.A. Rakov, C.A. Nucci and J.L. Bermudez, 2002. The effect of vertically extended strike object on the distribution of current along the lightning channel. *J. Geophys. Res.*, 107(23): 4699, Doi: 10.1029/2002JD002119.
- Uman, M.A. and D.K. McLain, 1969. Magnetic field of lightning return stroke. *J. Geophys. Res.*, 74: 6899-6910.