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Mathematical Modeling of Age Specific Marital Fertility Rates of Bangladesh

Rafiqul Islam
Department of Population Science and Human Resource Development,
University of Rajshahi, Rajshahi -6205 Bangladesh

Abstract: The purpose of this study is to build up mathematical models to age specific marital fertility rates (ASMFRs) and forward cumulative ASMFRs of Bangladesh. For this, the secondary data of ASMFRs have been taken from Bangladesh Demographic and Health Survey (BDHS). It is observed that ASMFRs follows quadratic polynomial model in 1996 and one degree polynomial model i. e. simple linear regression model in 1999-2000. On the other hand, forward cumulative ASMFRs follow cubic polynomial models in both cases. Model validation technique, cross-validity prediction power (CVPP), , has been applied to these mathematical models to check the validity of the models.

Key words: Age specific marital fertility rates (ASMFRs), polynomial model, variance explained (R^2), cross validity prediction power (CVPP), F-test

INTRODUCTION

Mathematical modeling not only in Demography but also in Population Studies in our state has hardly ever been used. In up-to-the-minute era, mathematical model is very sophisticated mechanism to express data mathematically. Mathematical model is finally very important for the estimation of population projections and estimations. In fact, mathematical model is essentially an endeavor to find out structural relationships and their dynamic behavior among the various elements in Demography. In Demography, mathematical models are mostly two types: non-deterministic or stochastic and deterministic. The variables in the stochastic models are in the form of probability distribution. On the other hand, deterministic models are used to explain the functional relationship between variables that take definite values. In this study, deterministic models have been introduced. Additionally deterministic models are also classified into two classes: stationary population models and time series population models. Stationary population models have only been discussed in the present study.

It was reviewed the relationship of total separation rates and separation rates due to death with their age variable and found a semi-log function of the type (Ali, 1994). Age distribution, age specific death rates (ASDRs) and the number of persons surviving at an exact age x (l_x) for male population of Bangladesh in 1991 follow modified negative exponential model, 4th degree polynomial model and 3rd degree polynomial model, respectively (Islam, Islam, Ali and Mostofa, 2003). In Islam and Ali (2004), it was found that age specific fertility rates (ASFRs) follows slightly modified biquadratic polynomial model where as forward and backward cumulative ASFRs follow quadratic and cubic polynomial model, respectively in the rural area of Bangladesh. In Islam (2004), it is seen that infant mortality rate follows semi-log linear model while crude death rate, life expectancy at birth for male and female follow simple linear regression models.

Traditionally, one can sketch some graphs of the demographic parameters. But, very few of us know what types of functional form are appropriate for the parameters in the context of Bangladesh. In this study, an attempt has been made here to find out what types of mathematical models are more appropriate to ASMFRs and forward cumulative ASMFRs of Bangladesh. Thus, the main objectives of this study are as follows:

- to build up mathematical models to ASMFRs and forward cumulative ASMFRs of Bangladesh, and
- to apply cross-validity prediction power (CVPP) and F-test to these models to verify how much these models are valid or not.

This paper is mainly organized into four sections. First section is introduction. Section two contains sources of data and methodology in which polynomial model, mathematical model fitting, model validation, shrinkage of the fitted model as well as F-test are briefly described here. Results and discussion are narrated in section four. Lastly, section five concludes the conclusion of this paper.

MATERIALS AND METHODS

Sources of data: A secondary data on ASMFRs in 1996 and 1999-2000 of Bangladesh have been taken from Bangladesh Demographic and Health Survey (BDHS) (Mitra and Associates, 1997 & 2001) which is shown in Table 1.

Methodology:

Polynomial Moidel: An expression of the form

$y = f(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + ... + a_nx^n$ $(a_o \neq 0)$ (Waerden, 1948)

where a_o is the constant term; a_i is the coefficient of x_o^i (i =1, 2, 3, ..., n) but $a_1, a_2, ..., a_n$ are also constants and n is the positive integer, is called a polynomial of degree n and the symbol x is called an indeterminate. If n=0 then it is called constant function. If n=1 then it is called polynomial of degree 1 i.e. simple linear function. If n=2 then it is called polynomial of degree 2 i.e. quadratic polynomial, etc. (Spiegel, 1992).

Mathematical Model Fitting:

A) Using the scattered plot of ASMFRs by age group in years of Bangladesh (Fig. 1 and 2), it is observed that ASMFRs can be fitted by polynomial model with respect to different ages in year. Therefore, an nth degree polynomial model is considered and the form of the model is

$$y = a_0 + \sum_{i=1}^{n} a_i x^i + u \quad \text{(Gupta and Kapoor, 1997)}$$

where, x is age group in years; y is ASMFRs;is the constant; a_i is the coefficient of x_o^i (i =1, 2, 3, ..., n) and u is the stochastic error term of the model. Here, a suitable n is chosen for which the error sum of square is minimum.

B) Using the dotted plot of forward cumulative ASMFRs by age group in years of Bangladesh (Fig. 3 and 4), it seems that forward cumulative ASMFRs follows an nth degree polynomial model with respect to different ages in year. Therefore, the form of the model is

$$y = a_0 + \sum_{i=1}^{n} a_i x^i + u$$

where, x is age group in years; y is forward cumulative ASMFRs;is the constant; is the coefficient of (i =1, 2, 3, ..., n) and u is the error term of the model. In this case, it is to be chosen a suitable n so that the error sum of square is minimum. The information on model fitting has been demonstrated in Table 2.

The software STATISTICA was used to fit these mathematical models.

Model Validation: To check how much these models are stable, the cross validity prediction power (CVPP), ρ_{cv}^2, is applied. Here,

$$\rho_{cv}^2 = 1 - \frac{(n-1)(n-2)(n+1)}{n(n-k-1)(n-k-2)}(1 - R^2);$$

Table 1: ASMFRs and its Forward Cumulative Distribution of Bangladesh

Age Group a – a+5	ASMFRs		Cumulative ASMFRs	
	1996	1999-2000	1996	1999-2000
15-19	0.393	0.273	0.393	0.273
20-24	0.267	0.26	0.66	0.533
25-29	0.163	0.179	0.823	0.712
30-34	0.092	0.11	0.915	0.822
35-39	0.061	0.065	0.976	0.887
40-44	0.02	0.019	0.996	0.906
45-49	0.003	-	0.999	-

Table 2: Information on Model Fitting

Models	Percentage of Variance Explained	Parameters	Significant Probability (p)
Model 1	99.652		
		a_0	0.00003
		a_1	0.0001
		a_2	0.000576
Model 2	97.84		
		a_0	0.00005
		a_1	0.0002
Model 3	99.956		
		a_0	0.0008
		a_1	0.00070
		a_2	0.00220
		a_3	0.005697
Model 4	99.928		
		a_0	0.0015
		a_1	0.00187
		a_2	0.0057
		a_3	0.016500

where, n is the number of cases, k is the number of regressors in the model and the cross-validated R is the correlation between observed and predicted values of the dependent variable (Stevens, 1996). The estimated CVPP corresponding to their R^2 has been summarized in Table 3.

Shrinkage of the Fitted Model: The shrinkage of the model is Shrinkage $= \left|\rho_{cv}^2 - R^2\right|$; where is cross validity prediction power & R^2 is the coefficient of determination of the model. Moreover, the stability of R^2 of the model is equal to 1- shrinkage.

F-test: The F-test is used to the model to verify the overall measure of the significance of the model as well as the significance of R^2. The formula for F-test is given as

$$F = \frac{R^2 / (m-1)}{(1-R^2) / (n-m)} \quad \text{with (m-1, n-m)}$$

degrees of freedom (d.f.);

where m is the number of parameters of the fitted model, n is the number of cases and R^2 is the coefficient of determination of the model (Gujarati, 1998).

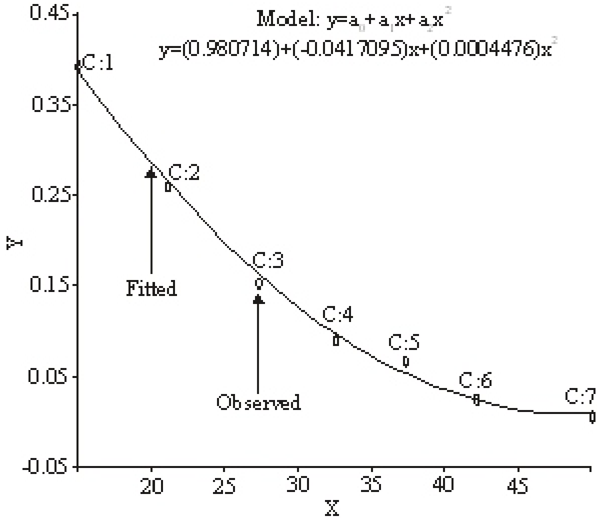

Fig. 1: Observed and Fitted ASMFRs of Bangladesh in 1996. X: Age Group in Years and Y: ASMFRs.

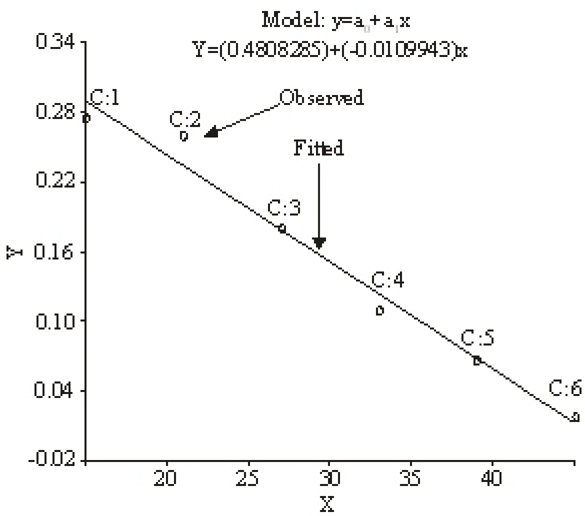

Fig. 2: Observed and Fitted ASMFRs of Bangladesh in 1999-2000. X: Age Group in Years and Y: ASMFRs

RESULTS AND DISCUSSION

The polynomial model is assumed for ASMFRs of Bangladesh. The fitted equations are as follows:

$y = 0.98071 - 0.0417095x + 0.0004476x^2$ in 1996 (1)

The coefficient of determination $R^2=0.99652$ and $\rho^2_{cv}=0.990057$. This is the polynomial of degree two, i.e. quadratic polynomial.

$y = 0.4808285 - 0.0109943x$ in 1999-2000 (2)

Giving $R^2=0.9784$ and $\rho^2_{cv}=0.958$. This is the polynomial of degree 1, i.e., simple linear regression model and,

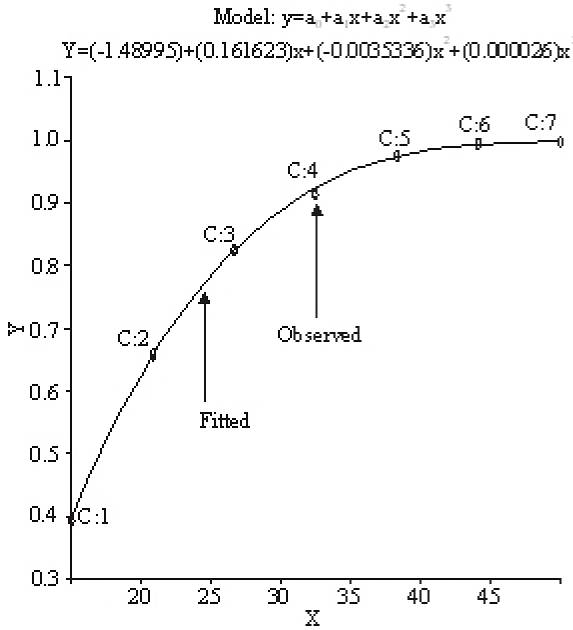

Fig. 3: Observed and Fitted Forward Cumulative ASMFRs of Bangladesh in 1996. X: Age Group in Years and Y: Forward Cumulative ASMFRs

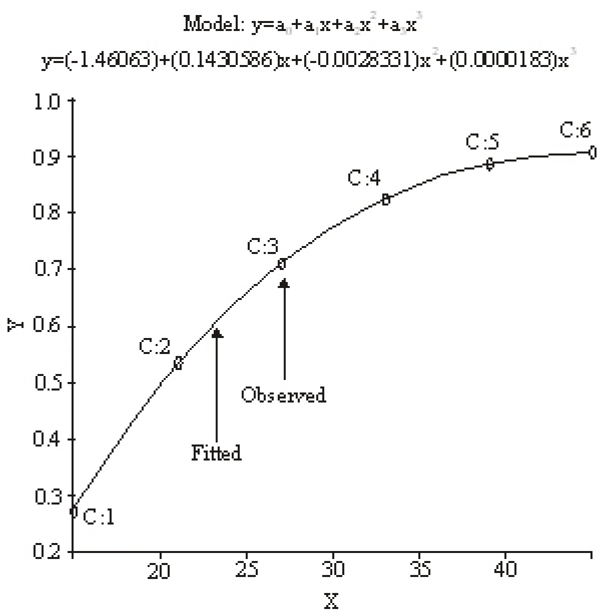

Fig. 4: Observed and Fitted Forward Cumulative ASMFRs of Bangladesh in 1999-2000. X: Age Group in Years and Y: Forward Cumulative ASMFRs

another polynomial model is assumed for forward cumulative ASMFRs of Bangladesh and the fitted equations are

$y = -1.48995 + 0.161623x - 0.0035336x^2 + 0.000026x^3$ (3) in 1996

Providing $R^2=0.99956$ and =0.997486. This is the three degree polynomial, i.e., cubic polynomial.

Table 3: Estimated Cross-Validity Prediction Power, ρ^2_{cv}, of the Predicted Equations of ASMFRs and its Forward Cumulative Distribution of Bangladesh

Models	n	k	R^2	ρ^2_{cv}	Shrinkage	Calculated F	Tabulated F
Equation 1	7	2	0.99652	0.990057	0.006463	572.71	18.0 with (2, 4) d. f.
Equation 2	6	1	0.9784	0.9580	0.02040	181.19	21.2 with (1, 4) d. f.
Equation 3	7	3	0.99956	0.997486	0.002074	2271.73	29.5 with (3, 3) d. f.
Equation 4	6	3	0.99996	0.999533	0.000427	16666.00	99.2 with (3, 2) d. f.

$$y = -1.4606+0.14306x-0.0028x^2+0.00018x^3 \qquad (4)$$

in 1999-2000

With R^2=0.99996 and ρ^2_{cv} =0.999533. That is also third degree polynomial, i.e., cubic polynomial model.

It should be mentioned here that usual models, i.e. Gompertz model, Makeham model, logistic model, log-linear model and semi-log linear model were also applied but seem to be worse fitted with respect to their shrinkages. Therefore, the results of those models were not shown here.

From the Table 2, it is shown that all the parameters of the fitted models in equations (1) - (4) are statistically significant with more than 99% of variance explained excepting the equation (2) but which is more than 97% of variance explained.

Table 3 shows that the fitted models in equations (1) - (4) are highly cross-validated and their shrinkage are 0.006463, 0.0204, 0.002074 and 0.000427, respectively. These imply that the fitted models (1), (3) and (4) will be stable more than 99% while the fitted model (2) will be stable more than 95%. Moreover, the stability of R^2 of these models is more than 99% excepting the model (2). But, the stability of R^2 of this model is more than 97%.

The calculated and tabulated values of F-test for the models (i) - (iv) are presented in the last two columns of Table 3. From the table it is found that the calculated values are more than that of the tabulated values at 1% level of significance. These results are indicated that these models and their corresponding to R^2 are highly statistically significant and hence, these are well fitted to the data set.

CONCLUSION

The mathematical models of ASMFRs and forward cumulative ASMFRs in 1996 and 1999-2000 of Bangladesh have been fitted. It is observed that the ASMFRs of Bangladesh follows 2nd degree polynomial model and simple linear regression model, respectively. On the other hand, forward cumulative ASMFRs follows 3rd degree polynomial model in both cases. Hope from this study young demographers and /or readers will be encouraged to fit mathematical models to the data in their respective research arena.

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