First Order Reactant in Magneto-hydrodynamic Turbulence Before the Final Period of Decay for the Case of Multi-point and Multi-time in a Rotating System

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Abstract: Following Deissler's approach, the decay of MHD turbulence at times before the final period for the concentration fluctuations of a dilute contaminant undergoing a first order chemical reaction in a rotating system for the case of multi-point and multi-time is studied. Here two and three point correlations between fluctuating quantities have been considered and the quadruple correlations are neglected in comparison to the second and third order correlations. The correlation equations are converted to spectral form by taking their Fourier transforms. Finally, integrating the energy spectrum over all wave numbers, the solution is obtained and this solution gives the Decay law of magnetic energy for the concentration fluctuations before the final period in a rotating system for the case of multi-point and multi-time.

Key words: MHD turbulence, first order reactant, rotating system and decay before the final period

INTRODUCTION

The essential characteristic of turbulent flows is that turbulent fluctuations are random in nature. In geophysical flows, the system is usually rotating with a constant angular velocity. Such large-scale flows are generally turbulent. When the motion is referred to axes, which rotate steadily with the bulk of the fluid, the coriolis and centrifugal force must be supposed to act on the fluid. The coriolis force due to rotation plays an important role in a rotating system of turbulent flow, while the centrifugal force with the potential is incorporated into the pressure. (Kishore and Dixit,1979) studied the effect of coriolis force on acceleration covariance in turbulent flow. (Kishore and Singh, 1984) considered the effect of Coriolis force on acceleration covariance in turbulent flow with rotational symmetry. (Dixit and Upadhyay, 1989) considered the effect of Coriolis force on acceleration covariance in MHD turbulent dusty flow with rotational symmetry. (Kishore and Golsefied,1988) discussed the effect of coriolis force on acceleration covariance in MHD turbulent flow of a dusty incompressible fluid. (Funada et al., 1978) considered the effect of coriolis force on turbulent motion in presence of strong magnetic field. (Kishore and Sarker,1990) studied the rate of change of vorticity covariance in MHD turbulence in a rotating system. (Sarker, 1997) studied the thermal decay process of MHD turbulence in a rotating system.

(Deissler, 1958,1960) developed a theory “decay of homogeneous turbulence for times before the final period”. Using Deissler's theory, (Loffler and Deissler,1961) studied the decay of temperature fluctuations in homogeneous turbulence before the final period. In their approach they considered the two and three-point correlation equations and solved these equations after neglecting fourth and higher order correlation terms. Using Deissler theory, (Kumar and Patel, 1974) studied the first-order reactant in homogeneous turbulence before the final period of decay for the case of multi-point and single-time correlation. (Kumar and Patel, 1975) extended their previous problem for the case of multi-point and multi-time concentration correlation. (Patel, 1976) also studied in detail the same problem to carry out the numerical results. (Sarker and Kishore,1991) studied the decay of MHD turbulence at time before the final period using (Chandrasekher, 1951). (Sarker and Islam,2001) studied the decay of MHD turbulence before the final period for the case of multi-point and multi-time. (Sarker and Azad, 2004) studied the Decay of MHD turbulence before the final period for the case of multi-point and multi-time in a rotating system. (Islam and Sarker, 2001) also studied the first order reactant in MHD turbulence before the final period of decay for the case of multi-point and multi-time.

In this work, following Deissler theory we studied the magnetic field fluctuation of concentration of a dilute contaminant undergoing a first order chemical reaction in MHD turbulence before the final period of decay for the case of multi-point and multi-time in a rotating system. Here, we have considered the two-point, two-time and three-point, three-time correlation equations and solved these equations after neglecting the fourth-order correlation terms. Finally we obtained the decay law for magnetic field energy fluctuation of concentration of dilute contaminant undergoing a first order chemical reaction in MHD turbulence for the case of multi-point and multi-time in a rotating system is obtained. If the fluid is non-rotating, the equation reduces to one obtained earlier by (Islam and Sarker, 2001).
Basic Equations: The equations of motion and continuity for viscous, incompressible MHD turbulent flow in a rotating system are given by

\[
\frac{\partial u_i}{\partial t} + \frac{\partial}{\partial x_k}(u_i u_k - h_k h_k) = -\frac{\partial \omega}{\partial x_i} + \nu \frac{\partial^2 u_i}{\partial x_k \partial x_k} - 2 \epsilon_{mkl} \Omega_m u_i
\]

(1)

\[
\frac{\partial h_k}{\partial t} + \frac{\partial}{\partial x_k}(h_k u_k - u_k h_k) = \frac{\partial}{\partial x_k}(h_k u_k - u_k h_k) - \frac{\partial}{\partial x_k}(h_k u_k - u_k h_k)
\]

(2)

Here, \(u_i\) is the turbulence velocity component; \(h_k\) is the magnetic field fluctuation component; \(w(\hat{x}, t)\) is the total MHD pressure + hydrodynamic pressure; \(\rho\) is the fluid density; \(\nu\) is the Kinematic viscosity; \(\lambda = \nu / \mu\) is the magnetic diffusivity; \(\mu\) is the magnetic Prandtl number; \(x_k\) is the space coordinate; the subscripts can take on the values 1, 2 or 3 and the repeated subscripts in a term indicate a summation; \(\Omega_m\) is the constant angular velocity component; \(\epsilon_{mkl}\) is the alternating tensor.

Two-Point, Two-Time Correlation and Spectral Equations: Under the condition that (i) the turbulence and the concentration magnetic field are homogeneous (ii) the chemical reaction has no effect on the velocity field and (iii) the reaction rate and the magnetic diffusivity are constant, the induction equation of a magnetic field fluctuation of concentration of a dilute contaminant undergoing a first order chemical reaction at the points \(p\) and \(p'\) separated by the vector \(\vec{r}\) could be written as

\[
\frac{\partial h_k}{\partial t} + u_k \frac{\partial h_k}{\partial x_k} = \lambda \frac{\partial^2 h_k}{\partial x_k \partial x_k} - R h_k
\]

(3)

and

\[
\frac{\partial h_k'}{\partial t'} + u_k' \frac{\partial h_k'}{\partial x_k'} = \lambda \frac{\partial^2 h_k'}{\partial x_k' \partial x_k'} - R h_k'
\]

(4)

where \(R\) is the constant reaction rate.

Multiplying equation (3) by \(h'\) and equation (4) by \(h\) and taking ensemble average, we get

\[
\frac{\partial}{\partial t} \langle h_i h_j' \rangle + \frac{\partial}{\partial x_k} \left[ \langle u_k h_i h_j' \rangle - \langle u_i h_k h_j' \rangle \right] = \lambda \frac{\partial^2}{\partial x_k \partial x_k} \langle h_i h_j' \rangle - R \langle h_i h_j' \rangle
\]

(5)

and

\[
\frac{\partial}{\partial t'} \langle h_i h_j' \rangle + \frac{\partial}{\partial x_k} \left[ \langle u_k h_i h_j' \rangle - \langle u_i h_k h_j' \rangle \right] = \lambda \frac{\partial^2}{\partial x_k' \partial x_k'} \langle h_i h_j' \rangle - R \langle h_i h_j' \rangle
\]

(6)

Angular bracket \(\langle \ldots \rangle\) is used to denote an ensemble average. Using the transformations

\[
\frac{\partial}{\partial x_k} = -\frac{\partial}{\partial x_k'} \frac{\partial}{\partial x_k'} = \frac{\partial}{\partial x_k} \left( \frac{\partial}{\partial t'} \right)\]

(7)

into equations (5) and (6), we obtain

\[
\frac{\partial}{\partial t} \langle h_i h_j' \rangle + \frac{\partial}{\partial x_k} \left[ \langle u_k h_i h_j' \rangle - \langle u_i h_k h_j' \rangle \right] = \lambda \frac{\partial^2}{\partial x_k \partial x_k} \langle h_i h_j' \rangle - R \langle h_i h_j' \rangle
\]

(8)

and

\[
\frac{\partial}{\partial t'} \langle h_i h_j' \rangle + \frac{\partial}{\partial x_k} \left[ \langle u_k h_i h_j' \rangle - \langle u_i h_k h_j' \rangle \right] = \lambda \frac{\partial^2}{\partial x_k' \partial x_k'} \langle h_i h_j' \rangle - R \langle h_i h_j' \rangle
\]

(9)

Using the relations of (Chandrasekhar, S., 1951)

\[
\langle u_k h_i h_j' \rangle = -\langle u_k h_i h_j' \rangle, \langle u_i h_k h_j' \rangle = \langle u_i h_k h_j' \rangle
\]

Equations (8) and (9) become
Now we write equations (10) and (11) in spectral form in order to reduce it to an ordinary differential equation by use of the following three-dimensional Fourier transforms:

\[
\langle h_i h_j \rangle (\hat{\mathbf{p}}, \Delta t, t) = \int_{-\infty}^{\infty} \langle \psi_i \psi_j \rangle (\mathbf{K}, \Delta t, t) \exp[i(\mathbf{K} \cdot \hat{\mathbf{p}})] d\mathbf{K}
\]

and

\[
\langle u_k h_i h_j \rangle (\hat{\mathbf{p}}, \Delta t, t) = \int_{-\infty}^{\infty} \langle \alpha_i \psi_i \psi_j \rangle (\mathbf{K}, \Delta t, t) \exp[i(\mathbf{K} \cdot \hat{\mathbf{p}})] d\mathbf{K}
\]

Interchanging the subscripts \(i\) and \(j\) then interchanging the points \(p\) and \(p'\) gives

\[
\langle u_k h_i h_j \rangle (\hat{\mathbf{p}}, \Delta t, t) = \int_{-\infty}^{\infty} \langle \alpha_i \psi_i \psi_j \rangle (\mathbf{K}, \Delta t, t) \exp[i(\mathbf{K} \cdot \hat{\mathbf{p}})] d\mathbf{K}
\]

where \(\mathbf{K}\) is known as a wave number vector and \(d\mathbf{K} = dK_1 \, dK_2 \, dK_3\). The magnitude of \(\mathbf{K}\) has the dimension 1/length and can be considered to be the reciprocal of an eddy size. Substituting of equation (12) into (14) into equations (10) and (11) leads to the spectral equations

\[
\frac{\partial \langle \psi_i \psi_j \rangle}{\partial t} + 2[\lambda K^2 + R] \langle \psi_i \psi_j \rangle = 2iK_k \left[ \langle \alpha_i \psi_i \psi_j \rangle (\mathbf{K}, \Delta t, t) - \langle \alpha_k \psi_k \psi_j \rangle (\mathbf{K}, \Delta t, t + \Delta t) \right]
\]

and

\[
\frac{\partial \langle \psi_i \psi_j \rangle}{\partial \Delta t} + [\lambda K^2 + R] \langle \psi_i \psi_j \rangle = iK_k \left[ \langle \alpha_i \psi_i \psi_j \rangle (\mathbf{K}, \Delta t, t) - \langle \alpha_k \psi_k \psi_j \rangle (\mathbf{K}, \Delta t, t + \Delta t) \right]
\]

The tensor equations (15) and (16) becomes a scalar equation by contraction of the indices \(i\) and \(j\)

\[
\frac{\partial \langle \psi_i \psi_j \rangle}{\partial t} + 2[\lambda K^2 + R] \langle \psi_i \psi_j \rangle = 2iK_k \left[ \langle \alpha_i \psi_i \psi_j \rangle (\mathbf{K}, \Delta t, t) - \langle \alpha_k \psi_k \psi_j \rangle (\mathbf{K}, \Delta t, t + \Delta t) \right]
\]

and

\[
\frac{\partial \langle \psi_i \psi_j \rangle}{\partial \Delta t} + [\lambda K^2 + R] \langle \psi_i \psi_j \rangle = iK_k \left[ \langle \alpha_i \psi_i \psi_j \rangle (\mathbf{K}, \Delta t, t) - \langle \alpha_k \psi_k \psi_j \rangle (\mathbf{K}, \Delta t, t + \Delta t) \right]
\]

The terms on the right side of equations (17) and (18) are collectively proportional to what is known as the magnetic energy transfer terms.

Three-Point, Three-Time Correlation and Spectral Equations: Similar procedure can be used to find the three-point correlation equations. For this purpose we take the momentum equation of MHD turbulence in a rotating...
system at the point P and the induction equations of magnetic field fluctuations, governing the concentration of a dilute contaminant undergoing a first order chemical reaction at \( p' \) and \( p'' \) separated by the vector \( \mathbf{r} \) and \( \mathbf{r}' \) as

\[
\frac{\partial u_i}{\partial t} + u_k \frac{\partial u_i}{\partial x_k} - h_k \frac{\partial h_i}{\partial x_k} = -\frac{\partial \nu}{\partial x_i} + v \frac{\partial u_i}{\partial x_k \partial x_k} - 2 \epsilon_{mkl} \Omega_{m} \mathbf{h}_l
\]  

\[i = 1, \ldots, n\]  \tag{19}

\[
\frac{\partial h_i'}{\partial t'} - u_k' \frac{\partial h_i'}{\partial x_k'} - h_k' \frac{\partial h_i'}{\partial x_k'} = \lambda' \frac{\partial^2 h_i'}{\partial x_k' \partial x_k'} - R h_i' \]  \tag{20}

\[
\frac{\partial h_i''}{\partial t''} + u_k'' \frac{\partial h_i''}{\partial x_k''} - h_k'' \frac{\partial h_i''}{\partial x_k''} = \lambda'' \frac{\partial^2 h_i''}{\partial x_k'' \partial x_k''} - R h_i'' \]  \tag{21}

Multiplying equation (19) by \( h_i', h_{i''} \), equation (20) by \( u_i h_{i''} \), and equation (21) by \( u_i h_i'' \), taking ensemble average, one obtains

\[
\frac{\partial \langle u_i h_i' h_i'' \rangle}{\partial t} + \frac{\partial}{\partial x_k} \left[ \langle u_k u_i h_i' h_i'' \rangle - \langle h_k h_i' h_i'' \rangle \right]
\]

\[
= -\frac{\partial \nu}{\partial x_i} \frac{\partial}{\partial x_k} \left[ \langle u_k u_i h_i' h_i'' \rangle - \langle h_k h_i' h_i'' \rangle \right] + 2 \epsilon_{mkl} \Omega_{m} \langle u_i h_{i''} \rangle
\]

\[\frac{\partial^2 \langle u_i h_i' h_i'' \rangle}{\partial x_k' \partial x_k} - R \langle u_i h_i' h_i'' \rangle \]  \tag{22}

Using the transformations into equations (22) to (24), we have

\[
\frac{\partial \langle u_i h_i' h_i'' \rangle}{\partial t} - \frac{\partial}{\partial r_k} \left[ \frac{\partial}{\partial r_k} \langle u_k u_i h_i' h_i'' \rangle - \langle h_k h_i' h_i'' \rangle \right]
\]

\[
= -\frac{\partial \nu}{\partial r_i} \frac{\partial}{\partial r_k} \left[ \langle u_k u_i h_i' h_i'' \rangle - \langle h_k h_i' h_i'' \rangle \right] + 2 \epsilon_{mkl} \Omega_{m} \langle u_i h_{i''} \rangle
\]

\[
+ \lambda \left[ \frac{\partial}{\partial r_k} \langle u_i h_i' h_i'' \rangle + \frac{\partial}{\partial r_k} \langle u_i h_i' h_i'' \rangle \right] \]  \tag{23}

\[
= \lambda \left[ \frac{\partial}{\partial r_k} \langle u_i h_i' h_i'' \rangle - \frac{\partial}{\partial r_k} \langle u_i h_i' h_i'' \rangle \right] - 2 \epsilon_{mkl} \Omega_{m} \langle u_i h_{i''} \rangle
\]

\[\frac{\partial \langle u_i h_i' h_i'' \rangle}{\partial t} + \frac{\partial}{\partial x_k'} \left[ \langle u_k u_i h_i' h_i'' \rangle - \langle h_k h_i' h_i'' \rangle \right]
\]

\[
= -\frac{\partial \nu}{\partial x_i} \frac{\partial}{\partial x_k'} \left[ \langle u_k u_i h_i' h_i'' \rangle - \langle h_k h_i' h_i'' \rangle \right] + 2 \epsilon_{mkl} \Omega_{m} \langle u_i h_{i''} \rangle
\]

\[
+ \lambda \left[ \frac{\partial}{\partial r_k} \langle u_i h_i' h_i'' \rangle - \frac{\partial}{\partial r_k} \langle u_i h_i' h_i'' \rangle \right] \]  \tag{24}

\[
= \lambda \left[ \frac{\partial}{\partial r_k} \langle u_i h_i' h_i'' \rangle - \frac{\partial}{\partial r_k} \langle u_i h_i' h_i'' \rangle \right] - 2 \epsilon_{mkl} \Omega_{m} \langle u_i h_{i''} \rangle
\]
In order to convert equation (25)-(27) to spectral form, we can define the following six-dimensional Fourier transforms:

\[
\left\langle u_i u_j h_i h_j'' \right\rangle \left( \hat{r}, \hat{r}', \Delta t, \Delta t', t \right) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left\langle \phi_i \phi_j \beta_i \beta_j' \right\rangle \left( \hat{k}, \hat{k}', \Delta t, \Delta t', t \right) d\hat{k} d\hat{k}'
\]

\[
= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left\langle \phi_i \phi_j \beta_i \beta_j' \right\rangle \left( \hat{r}, \hat{r}', \Delta t, \Delta t', t \right) d\hat{r} d\hat{r}'
\]

\[
\left\langle w_i w_j h_i h_j'' \right\rangle \left( \hat{r}, \hat{r}', \Delta t, \Delta t', t \right) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left\langle \phi_i \phi_j \beta_i \beta_j' \right\rangle \left( \hat{k}, \hat{k}', \Delta t, \Delta t', t \right) d\hat{k} d\hat{k}'
\]

By use of these facts and the equations (28)-(34), we can write equations (25)-(27) in the form

\[
\left\langle u_k u_l h_i h_j'' \right\rangle \left( \hat{r}, \hat{r}', \Delta t, \Delta t', t \right) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left\langle \phi_k \phi_l \beta_i \beta_j' \right\rangle \left( \hat{k}, \hat{k}', \Delta t, \Delta t', t \right) d\hat{k} d\hat{k}'
\]

Interchanging the points \( P' \) and \( P'' \) along with the indices \( i \) and \( j \) result in the relations

\[
\left\langle u_i u_j h_i h_j'' \right\rangle = \left\langle u_i u_j h_i h_j'' \right\rangle
\]
If the derivative with respect to $x_i$ is taken of the momentum equation (19) for the point $P$, the equation multiplied by $\hbar^j$, and time average taken, the resulting equation

$$ - \frac{\partial^2}{\partial x_i \partial x_j} \left( \langle u_k u_k' h_j h_j' \rangle - \langle h_k h_k' h_j h_j' \rangle \right) = \frac{\partial}{\partial x_i \partial x_k} $$

Writing this equation in terms of the independent variables $\tilde{F}$ and $\tilde{F}'$

$$ - \left[ \frac{\partial^2}{\partial x_i \partial x_j} + \frac{2}{\partial x_i \partial x_j'} + \frac{2}{\partial x_j \partial x'_i} \right] \langle w_k h_j \rangle $$

where

$$ \times \left( \langle u_k u_k' h_j h_j' \rangle - \langle h_k h_k' h_j h_j' \rangle \right) $$

Taking the Fourier transforms of equation (26)

$$ \text{Solution for Times before the Final Period:} \text{It is known that the equation for final period of decay is obtained by considering the two-point correlations after neglecting third-order correlation terms. To study the decay for times before the final period, the three-point correlations are considered and the quadruple correlation terms are neglected because the quadruple correlation terms decays faster than the lower-order correlation terms. The term } \langle \beta_1 \beta_2 \beta_3 \rangle \text{ associated with the pressure fluctuations should also be neglected. Thus neglecting all the terms on the right hand side of equations (41) to (43)} $
Integrating equations (44) to (46) between $t_o$ and $t$, we obtain

$$k_i \langle \phi_i \beta_i^r \beta_i^t \rangle = f_l \exp \left[ -\lambda \left( 1 + P_M \right) \left( k^2 + k^2 \right) \right]$$

$$+ 2 P_M \kappa \cos \theta + \frac{2 R}{\lambda} + \frac{1}{\lambda} \left( 2 \varepsilon_{r d} \Omega_m \right) \left( t - t_o \right)$$

$$k_i \langle \phi_i \beta_i^r \beta_i^t \rangle = g_l \exp \left[ -\lambda \left( k^2 + \frac{R}{\lambda} \right) \Delta t \right]$$

and

$$k_i \langle \phi_i \beta_i^r \beta_i^t \rangle = g_l \exp \left[ -\lambda \left( k^2 + \frac{R}{\lambda} \right) \Delta t^r \right]$$

For these relations to be consistent, we have

$$k_i \langle \phi_i \beta_i^r \beta_i^t \rangle = k_i \langle \phi_i \beta_i^r \beta_i^t \rangle_o \exp \left[ -\lambda \left( 1 + P_M \right) \right]$$

$$\left( k^2 + k^2 \right) \left( t - t_o \right) + k^2 \Delta t + k^2 \Delta t^r +$$

$$2 P_M \kappa \cos \theta \left( t - t_o \right) + \frac{2 R}{\lambda} \left( t - t_o + \frac{\Delta t + \Delta t^r}{2} \right)$$

$$+ \left( \frac{2 \varepsilon_{r d} \Omega_m}{\lambda} \right) \left( t - t_o \right)$$

where $\theta$ is the angle between $\hat{K}$ and $\hat{K}'$ and

$$\langle \phi_i \beta_i^r \beta_i^t \rangle_o$$

is the value of $\langle \phi_i \beta_i^r \beta_i^t \rangle$ at $t = t_o, \Delta t = \Delta t^r = 0, \lambda = \frac{V}{P_M}$

By letting $\hat{r} = 0, \Delta t = 0$ in the equation (28) and comparing with equations (13) and (14) we get

$$\langle \phi_i \beta_i^r \beta_i^t \rangle \left( \hat{K}, \hat{K}', \Delta t, t_o \right) =$$

$$\int \left( \phi_i \beta_i^r \beta_i^t \right) \left( \hat{K}, \hat{K}', \Delta t, t_o \right) d\hat{K}'$$

Substituting equation (47) - (49) into equation (17), one obtains

$$\langle \phi_i \beta_i^r \beta_i^t \rangle \left( \hat{K}, \hat{K}', \Delta t, t_o \right) =$$

$$\int \left( \phi_i \beta_i^r \beta_i^t \right) \left( \hat{K}, \hat{K}', \Delta t, t_o \right) d\hat{K}'$$

Substituting of equation (51) in equation (50) yields

$$\frac{\partial}{\partial t} \left( \phi_i \beta_i^r \beta_i^t \right) \left( \hat{K}, \hat{K}', \Delta t, t_o \right) + 2 \lambda \left( k^2 + \frac{R}{\lambda} \right) \left( \phi_i \beta_i^t \beta_i^r \right)$$

$$- \left( \frac{2 \kappa k_i}{\lambda} \left( \phi_i \beta_i^r \beta_i^t \right) \left( \hat{K}, \hat{K}', \Delta t, t_o \right) \right) \left( -2 \pi k_i \varepsilon_{r d} \right) \left( \frac{\hat{K}}{\lambda} \right)$$

Now, $d\hat{K}'$ can be expressed in terms of $k'$ and $\theta$ as

$$-2 \pi k' \varepsilon (\cos \theta) d\theta$$

i.e.

$$d\hat{K}' = 2 \pi k' \varepsilon (\cos \theta) d\theta$$

Substituting of equation (51) in equation (50) yields

$$\frac{\partial}{\partial t} \left( \phi_i \beta_i^r \beta_i^t \right) \left( \hat{K}, \hat{K}', \Delta t, t_o \right) + 2 \lambda \left( k^2 + \frac{R}{\lambda} \right) \left( \phi_i \beta_i^t \beta_i^r \right)$$

$$- \left( \frac{2 \kappa k_i}{\lambda} \left( \phi_i \beta_i^r \beta_i^t \right) \left( \hat{K}, \hat{K}', \Delta t, t_o \right) \right) \left( -2 \pi \varepsilon_{r d} \right)$$

$$= \lambda \left( 1 + P_M \right) \left( k^2 + k^2 \right) \left( t - t_o \right)$$

$$+ k^2 \Delta t + 2 P_M \left( t - t_o \right)$$

$$+ 2 \pi \left( \frac{2 \kappa k_i}{\lambda} \left( \phi_i \beta_i^r \beta_i^t \right) \left( \hat{K}, \hat{K}', \Delta t, t_o \right) \right) \left( \frac{\varepsilon_{r d} \Omega_m}{\lambda} \right) \left( t - t_o \right)$$

$$\left( \frac{2 \kappa k_i}{\lambda} \left( \phi_i \beta_i^r \beta_i^t \right) \left( \hat{K}, \hat{K}', \Delta t, t_o \right) \right) \left( \frac{\varepsilon_{r d} \Omega_m}{\lambda} \right) \left( t - t_o \right)$$

and

$$\langle \phi_i \beta_i^r \beta_i^t \rangle \left( \hat{K}, \hat{K}', \Delta t, t_o \right) =$$

$$\int \left( \phi_i \beta_i^r \beta_i^t \right) \left( \hat{K}, \hat{K}', \Delta t, t_o \right) d\hat{K}'$$

$$\left( \frac{2 \kappa k_i}{\lambda} \left( \phi_i \beta_i^r \beta_i^t \right) \left( \hat{K}, \hat{K}', \Delta t, t_o \right) \right) \left( \frac{\varepsilon_{r d} \Omega_m}{\lambda} \right) \left( t - t_o \right)$$

$$\left( \frac{2 \kappa k_i}{\lambda} \left( \phi_i \beta_i^r \beta_i^t \right) \left( \hat{K}, \hat{K}', \Delta t, t_o \right) \right) \left( \frac{\varepsilon_{r d} \Omega_m}{\lambda} \right) \left( t - t_o \right)$$

$$\left( \frac{2 \kappa k_i}{\lambda} \left( \phi_i \beta_i^r \beta_i^t \right) \left( \hat{K}, \hat{K}', \Delta t, t_o \right) \right) \left( \frac{\varepsilon_{r d} \Omega_m}{\lambda} \right) \left( t - t_o \right)$$
In order to find the solution completely and following (Loeffler, A.L. and R.G. Deissler, 1961) we assume that

\[ ik_1 \left[ \left( \frac{\partial}{\partial t} \mathbf{F} \right) \cdot \mathbf{K} \cdot \mathbf{K}^* - \left( \frac{\partial}{\partial t} \mathbf{F}^* \right) \cdot \mathbf{K} \cdot \mathbf{K}^* \right] \delta_{0} \]

\[ = - \frac{\delta_{0}}{(2\pi)^2} \left( k^2 k^4 - k^4 k^2 \right) \]  

where \( \delta_{0} \) is a constant determined by the initial conditions. The negative sign is placed in front of \( \delta_{0} \) in order to make the transfer of energy from small to large wave numbers for positive value of \( \delta_{0} \).

Substituting equation (53) into equation (52) we get

\[ \frac{\partial}{\partial t} 2\pi \left( \psi_1 \psi_1^* \right) \cdot \mathbf{K} \cdot \mathbf{K}^* t + 2A \left[ k^2 + R / \lambda \right] \]

\[ = 2\pi \left( \psi_1 \psi_1^* \right) \cdot \mathbf{K} \cdot \mathbf{K}^* t - \delta_{0} \int_{0}^{\infty} \left( k^2 k^4 - k^4 k^2 \right) k^2 \]

\[ + 2P_M \left( t - t_0 \right) k^2 \cos \theta + \frac{2R}{\lambda} \left( t - t_0 + \Delta t / 2 \right) \]

\[ + \left( \frac{2 \varepsilon_m \Omega_{m}}{\lambda} \right) \left( t - t_0 \right) \]

Multiplying both sides of equation (54) by \( k^2 \), we get

\[ \frac{\partial E}{\partial t} + 2A k^2 E = F \]

where, \( E = 2\pi \mathbf{K}^2 \left( \psi_1 \psi_1^* \right) \). \( E \) is the magnetic energy spectrum function and \( F \) is the magnetic energy transfer term and is given by

\[ F = -2\delta_{0} \int_{0}^{\infty} \left( k^2 k^2 - k^4 k^2 \right) k^2 \]

\[ \times \left[ \int_{\lambda}^{1} \exp \left( - \frac{\lambda}{1 + P_M} (k^2 + k^2) (t - t_0) + k^2 \Delta t \right) \right. \]

\[ + 2P_M (t - t_0) k^2 \cos \theta + \frac{2R}{\lambda} \left( t - t_0 + \Delta t / 2 \right) \]

\[ + \left( \frac{2 \varepsilon_m \Omega_{m}}{\lambda} \right) \left( t - t_0 \right) \]

\[ \left. d(\cos \theta) \right) dk' \]

Integrating equation (56) with respect to \( \cos \theta \) and we have

\[ F = \frac{\delta_{0} P_M \sqrt{\pi}}{4 \lambda^{3/2} (t - t_0)^{3/2} (1 + P_M)^{5/2}} \exp \left( - \frac{2 \varepsilon_m \Omega_{m}}{\lambda} \right) \left( t - t_0 \right) \exp \left( - \frac{k^2 \lambda (1 + P_M)}{1 + P_M} \right) \]

\[ - 2R \left( t - t_0 + \Delta t / 2 \right) \frac{15 P_M k^4}{4P_M \lambda^2 (t - t_0)^3 (1 + P_M)^{5/2}} \]

\[ + \frac{5P_M^2}{(1 + P_M)^2} - \frac{3}{2} \left( \frac{k^6}{P_M \lambda (t - t_0)} \right) \]

\[ + \frac{P_M^3}{(1 + P_M)^3} - \frac{P_M}{1 + P_M} \left( t - t_0 + \Delta t / 2 \right) \]  

\[ \left[ \frac{15 P_M k^4}{4v^2 (t - t_0 + \Delta t)^2 (1 + P_M)} \right. \]

\[ + \frac{5P_M^2}{(1 + P_M)^2} - \frac{3}{2} \left( \frac{k^6}{P_M \lambda (t - t_0 + \Delta t)} \right) \]

\[ + \frac{P_M^3}{(1 + P_M)^3} - \frac{P_M}{1 + P_M} \left( t - t_0 + \Delta t / 2 \right) \]  

\[ \left. \right] \]
The series of equation (57) contains only even power of $k$ and start with $k^4$ and the equation represents the transfer function arising owing to consideration of magnetic field at three-point and three-times.

If we integrate equation (57) for $\Delta t=0$ over all wave numbers, we find that

$$
\int_0^\infty Pdk = 0
$$

(58)

which indicates that the expression for $F$ satisfies the condition of continuity and homogeneity. Physically it was to be expected as $F$ is a measure of the energy transfer and the total energy transferred to all wave numbers must be zero.

The linear equation (55) can be solved to give

$$
E = \exp \left[ -2\lambda k^2 \left( t - t_0 + \frac{\Delta t}{2} \right) \right]
$$

$$
\int F \exp \left[ 2\lambda \left( k^2 + \frac{R}{\lambda} \right) \left( t - t_0 + \frac{\Delta t}{2} \right) \right] dt + J(k) \exp \left[ -2\lambda \left( k^2 + \frac{R}{\lambda} \left( t - t_0 + \frac{\Delta t}{2} \right) \right) \right]
$$

(59)

where $J(k) = \frac{N\sigma k^2}{\pi}$ is a constant of integration and can be obtained as by (Corrsin, S., 1951)

Substituting the values of $F$ from equation (57) into equation (59) gives the equation

$$
E = \frac{N\sigma k^2}{\pi} \exp \left[ -2\lambda \left( k^2 + \frac{R}{\lambda} \left( t - t_0 + \frac{\Delta t}{2} \right) \right) \right]
$$

$$
+ \frac{\delta\alpha P_M \sqrt{\pi}}{4 \lambda^{3/2} \left( 1 + P_M \right)^{3/2}} \exp \left[ -2 \epsilon_{mk} \Omega_m \left( t - t_0 \right) \right]
$$

$$
\exp \left[ \frac{-k^2 \lambda \left( 1 + 2P_M \right)}{1 + P_M} \left( t - t_0 + \frac{1 + P_M}{1 + 2P_M} \Delta t \right) \right]
$$

$$
- 2R \left( t - t_0 + \frac{\Delta t}{2} \right)
$$

$$
\left[ \frac{3k^4}{2P_M \lambda^2 (t - t_0)^{3/2}} + \frac{(7P_M - 6)k^6}{3\lambda \left( 1 + P_M \right) \left( t - t_0 \right)^{3/2}} \right] F(a)
$$

(60)

where, $F(a) = e^{-a^2} \int_0^a e^{x^2} dx$.

$$
\omega = k \sqrt{\frac{\lambda (t - t_0)}{1 + P_M}}
$$

or $k = \sqrt{\frac{\lambda (t - t_0 + \Delta t)}{1 + P_M}}$. 

43
By setting $\hat{\epsilon} = 0$, $i=d$, $d\hat{k} = -2\pi \delta^2 d(\cos \theta) d\hat{k}$ and $E = 2\pi \delta^2 \left( \psi_i \psi^j \right)$ in equation (12) we get the expression for magnetic energy decay law as

$$\langle h_i h^i \rangle = \frac{\pi \delta_0}{8 \sqrt{2} \pi \alpha^{3/2} (T + \Delta T^{\frac{3}{2}})^{3/2}} \exp \left[ -2 R(T + \Delta T^{\frac{3}{2}}) \right] \exp \left[ -\left( 2 \epsilon_{mkd} \Omega_m \right) \right]$$

Substituting equation (60) into equation (61) and after integration, we get

$$\langle h_i h^i \rangle = \frac{N_0}{8 \sqrt{2} \pi \alpha^{3/2} (T + \Delta T^{\frac{3}{2}})^{3/2}} \exp \left[ -2 R(T + \Delta T^{\frac{3}{2}}) \right] \exp \left[ -\left( 2 \epsilon_{mkd} \Omega_m \right) \right]$$

where $T=t-t_0$. For $T_m = T + \Delta T^{\frac{3}{2}}$, equation (62) takes the form

$$\langle h^2 \rangle = \frac{\langle h_i h^i \rangle}{2} = \exp \left[ -2 R_{T_m} \right] \frac{N_0}{8 \sqrt{2} \pi \alpha^{3/2} T^{3/2}} + \frac{\pi \delta_0}{4 \alpha^2 (1 + 2 P_M)^{5/2}} \exp \left[ -\left( 2 \epsilon_{mkd} \Omega_m \right) \right]$$

Substituting equation (60) into equation (61) and after integration, we get

$$\langle h_i h^i \rangle = \frac{N_0}{8 \sqrt{2} \pi \alpha^{3/2} (T + \Delta T^{\frac{3}{2}})^{3/2}} \exp \left[ -2 R(T + \Delta T^{\frac{3}{2}}) \right] \exp \left[ -\left( 2 \epsilon_{mkd} \Omega_m \right) \right]$$

where $T=t-t_0$. For $T_m = T + \Delta T^{\frac{3}{2}}$, equation (62) takes the form

$$\langle h^2 \rangle = \frac{\langle h_i h^i \rangle}{2} = \exp \left[ -2 R_{T_m} \right] \frac{N_0}{8 \sqrt{2} \pi \alpha^{3/2} T^{3/2}} + \frac{\pi \delta_0}{4 \alpha^2 (1 + 2 P_M)^{5/2}} \exp \left[ -\left( 2 \epsilon_{mkd} \Omega_m \right) \right]$$

where $T=t-t_0$. For $T_m = T + \Delta T^{\frac{3}{2}}$, equation (62) takes the form

$$\langle h^2 \rangle = \frac{\langle h_i h^i \rangle}{2} = \exp \left[ -2 R_{T_m} \right] \frac{N_0}{8 \sqrt{2} \pi \alpha^{3/2} T^{3/2}} + \frac{\pi \delta_0}{4 \alpha^2 (1 + 2 P_M)^{5/2}} \exp \left[ -\left( 2 \epsilon_{mkd} \Omega_m \right) \right]$$
This is the decay law of magnetic energy fluctuations of concentration of a dilute contaminant undergoing a first order chemical reaction before the final period for the case of multi-point and multi-time in MHD turbulence in a rotating system.

**RESULTS AND DISCUSSION**

In equation (63) we obtained the decay law of magnetic energy fluctuations of a dilute contaminant undergoing a first order chemical reaction before the final period considering three-point correlation terms for the case of multi-point and multi-time in MHD turbulence in a rotating system.

If the system is non-rotating then $\Omega = 0$, the equation (63) becomes

$$\frac{\langle h^2 \rangle}{2} = \exp\left[-2RT_{M}\right] + \frac{\pi \delta_0}{\sqrt{2\pi \chi T_{M}^3}} \left( \frac{\Delta T}{T_{M}} \right)^{3/2} \left( \frac{T_{M} + \Delta T/2}{2} \right)^{3/2}$$

$$+ \left( \frac{5P_{M}(7P_{M} - 6)}{16(1 + 2P_{M})} \right)^{3/2} \left( \frac{T_{M} + \Delta T/2}{2(1 + 2P_{M})} \right)^{3/2}$$

which was obtained earlier by (Sarker, M.S.A. and N. Kishore, 1991).

This study shows that due to the effect of rotation of fluid in the flow field with chemical reaction of the first order in the concentration the magnetic field fluctuation in MHD turbulence in a rotating system for the case of multi-point and multi-time i.e. the turbulent energy decays more rapidly than the energy for non-rotating fluid and the faster rate is governed by $\exp\left[-\left(2 + \frac{\epsilon_{mkl}}{\Omega_{M}}\right)\right]$. Here the chemical reaction ($R^{10}$) in MHD turbulence for the case of multi-point and multi-time causes the concentration to decay more than it would for non-rotating system and it is governed by $\exp\left[-2RT_{M} + \frac{\epsilon_{mkl}}{\Omega_{M}}\right]$.

The first term of right hand side of equation (63) corresponds to the energy of magnetic field fluctuation of concentration for the two-point correlation and the second term represents magnetic energy for the three-point correlation. In equation (63), the term associated with the three-point correlation die out faster than the two-point correlation. If higher order correlations are considered in the analysis, it appears that more terms of higher power of time would be added to the equation (63). For large times the last term in the equation (63) becomes negligible, leaving the -3/2 power decay law for the final period.

**REFERENCES**


