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Modelling the Epidemiology of Malaria: A Case Study of Wassa West District in the Western Region of Ghana

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Abstract: This study deals with modelling of the epidemiology of Malaria in the Wassa West District. The SIR model proposed by O. Kermack and Anderson Gray Mckendrick and data obtained from the District Health Management Team-DHMT, Tarkwa were used to analyze the rate of infection of Malaria in the District. It was discovered that $R_o > 0$. From the principle of R_o which states that when $R_o < 0$, the infection will die out with a certainty. But if $R_o > 0$, there will be a proper Malaria outbreak, Based on the basic reproductive rate R_o, which is greater than 0, it was observed that the force of infection of Malaria in the Wassa West District is high. The necessary recommendations for the reduction of malaria were made.

Key words: Basic reproductive rate, district health management team, epidemiology of malaria, SIR, Wassa West District

INTRODUCTION

Malaria, derived from male aria (Italian for “bad air”) and formerly called ague or marsh fever in English, is an infectious disease, which cause at least 300 million infections with humans and approximately 1.3-2 million deaths annually, mainly in the Tropics, Sub-Saharan Africa accounts for 80 to 90% of these facilities (Trigg and Kondrachine, 1998). The exact statistics are unknown because many cases occur in the rural areas where people do not have access to hospital and/or the means to afford health care. Consequently, many cases are treated at home and are not documented. Malaria is a major cause of morbidity and mortality. It ranks alongside acute respiratory infections, measles and diarrhea diseases as a major cause of mortality worldwide. Each year 2-3 million children die from *Plasmodium falciparum* malaria and up to 500 million people throughout the world suffer from malaria clinical disease (Engers and Godal, 1998).

Human Beings, have long been living with malaria, as far back as 2700BC, medical writings in China and India allude to what is most likely malaria, and the disease is also described in the writings of Homer (Bruce-Chwatt, 1998). In one of the four Vedas of the Hindus, malaria is referred to as “a disease most dreaded affliction, King of disease”. The Chinese referred to the disease as “Mother of fevers” The relationship of fever to swamps and low-lying water was also recognized by the Greeks in the sixth century and the Romans’ attempts to drain large areas of swampland was probably motivated by the desire to reduce malaria. Protective measures against mosquitoes date back at least to 484-425 BC, when Herodotus observed that in parts of Egypt above the marshes, people slept in lofty towers that mosquitoes could not reach, while those in marshlands slept under nets. Marco polo noted that the wealthier residents of the Coromandel Coast in Indian slept on bedsteads with curtains that could be closed at night.

Malaria is caused by the protozoan parasite of the genus Plasmodium (of the Phylum Apicomplexa) and the transmission vector for human malaria parasites is the female Anophheles mosquito. The protozoan parasite is of Four types: Plasmodium falciparum, Plasmodium vivax, Plasmodium malaria and Plasmodium ovale. Of these, Plasmodium falciparum is of greatest risk to non- immune humans.The Plasmodium falciparum variety of parasites account for 80% of cases and 90% off deaths. Children under the age of five and pregnant women are the most vulnerable to the severe forms of malaria. The parasites are transmitted from person to person by mosquito, of the genus Anopheles each time the infected insect takes a blood meal. The symptoms in an infected human include bouts of fever, headache, vomiting flu-like, anemia (destroying red blood cell) and malaria can kill by clogging the capillaries that carry blood to the brain (cerebral malaria) or other vital organs. On the average the incubation period of Plasmodium falciparum is about 12 days in humans.

The discovery of the malaria parasite by Laveran in 1880 was a milestone in the history of medicine, destroying the belief in the role of “telluric” or “miasmatic” factors and establishing the causal link of the

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microbes to disease (Bruce-Chwatt, 1988). At the end of the nineteenth century, Ross in India and Grassi in Italy independently described the life cycle of the malaria parasite in the birds and humans. Anopheline mosquitoes typically breed in stagnant, unpolluted surface waters. Eggs are laid on the water surface, breathing air and feeding on small suspended particles. Typically, breeding sites for different species include forest pools, irrigated fields, lakes, and temporary rainwater puddles. For this reason, anopheline mosquitoes are mainly associated with rural settings. However, there are a few exceptions where malaria vectors have adopted to city life

Malaria affects the health and wealth of nations and individuals alike. In Africa today, malaria is understood to be both disease of poverty and causing poverty. Malaria has significantly measurable direct and indirect cost and has recently been shown to be a major constraint to economic development. The direct cost of malaria include a combination of personal and public expenditure, include individual and family spending on Insecticide, Treated Mosquito Nets (ITNS), doctor fees, anti malaria drugs, transport to health facilities, support for patient and sometimes an accompanying family members during hospital stays. The indirect cost of malaria includes lost productivity or low income associated with illness or death. Another indirect cost of malaria is the human pain and suffering caused by the disease. It also hampers children schooling and social development through both absenteeism and permanent neurological. The simple presence of malaria in a community also hampers individual and national prosperity due to its influence on social and economic decisions. The risk of contracting malaria in endemic area can deter investment both internally and externally and affect individual and household decision making in many ways that have negative impact on economic productivity and growth.

Several researches have been carried out on malaria and studies have been reviewed, relevant publications are the work done by Aron and May (1982), Bailey (1982), Macdonal (1957), Ross (1911), Ross (1909), Nasell (1987), Day *et al.* (1998), Anderson and May, (1989), AAAS (1991) and Carter *et al.* (2000). However, the increase in this epidemic in the sub-African regions has created much vacuum and left much to be desired. It is in the light of the above that this study is carried out to look at modelling the epidemiology of malaria: a case study of Wassa West District in the Western Region of Ghana. This study is of great importance to the Wassa West District because a very high rainfall pattern is recorded in the western part of the nation of which the district is located.

MATHEMATICAL MODEL

Variables and parameters: We use the following notations and/or definitions for the human populations:

Susceptibles $(S)t$: The number of individuals who can be infected but have not yet contracted the malaria fever but may contract it if exposed to any mode of its transmission.

Infectives $I(t)$: The number of individuals who have been infected of malaria fever.

Recoveries $R(t)$: The number of individuals who are recovered after treatment and are immune to the disease.

Assuming in the give population at time say $t, S(t), I(t)$ and $R(t)$ denote susceptible, infective carriers and recoveries respectively.

We define the following parameters as follows:

N(t): The total human population size at time t.

β: The contact rate, which takes into account the probability of getting the disease in contact between a susceptible and an infectious object.

V: The rate of recovery from the disease by an individual.

F: The force of infection (coefficient of interaction between the susceptibles and the infectious).

Once a human being recovers from the disease, the person acquires immunity against the disease. The immunity may be temporal or permanent. The natural process of reproduction also occurs in each of the compartments. Natural death (due to other diseases) also occurs in each of the compartments.

Assumptions for the model:

- The disease is short lived and rarely fatal.
- The disease is spread by contact between parasites and the individual host.
- Individuals who recover develop immunity.

Equations of the model: In view of the above assumptions and inter-relations between the variables and parameters as described in the compartmentalized model in Fig. 1, we have the following system of differential equations:

$$\frac{ds}{dt} = -\beta IS \tag{1}$$

$$\frac{dI}{dt} = \beta IS - \nu I \tag{2}$$

$$\frac{dR}{dt} = \nu I \tag{3}$$

This model was for the first time proposed by O. Kermack and Anderson Gray Mckendrick, who had worked with the Nobel Laureate and father of Mathematical Epidemiology Ronald Ross.

Analysis of the model: Here present the results of the existence and stability analysis of the of likely steady states in a human individual as far as the parasites at different stages are concerned and the state of immunity of the individual likewise the equilibrium point.

The force of infection (F): In the above model, the function $F = \beta I$ models the transition rate from the compartment of susceptible individuals to the compartment of infectious individuals. This is the force of infection of the malaria parasites in the population. However since the population under consideration is large, it is more realistic to consider a force of infection that does not depend on the absolute number of infectious subjects, but rather on their fraction (with respect to the total population size N).
Thus:

$$F = \beta \frac{1}{N}$$

The basic reproduction number (R_o): This is also called the basic reproduction ratio. Given as:

$$R_0 = \frac{\beta}{\nu}$$

This ratio is derived as the number of new infections from a single infection in a population where all subjects are susceptible. The role of the basic reproduction number in determining the rate of infection is extremely important. In fact upon rewriting the equation of the rate of infection as follows:

$$\begin{aligned} \frac{dI}{dt} &= \frac{\beta}{\nu}.\nu IS - \nu I \\ &= R_0 \nu IS - \nu I \\ &= (R_0 S - I)\nu I \end{aligned}$$

It yields that for $R_o > 0$ in Eq. (4) above:

$$\frac{dI}{dt} > 0$$

Hence there will be a proper malarial outbreak in the District with an increase in the number of the infected individuals in the population (Which can reach a considerable fraction of the population).The disease free equilibrium is unstable and the host will not recover from the disease
On the contrary, if $R_o < 0$,

$$\frac{dI}{dt} < 0$$

That is independently from the initial size of the susceptible population, the disease can never cause a proper epidemic outbreak. This is the case where the human host will always recover from the disease

Equilibrium analysis:

At equilibrium: $\frac{dS}{dt} = \frac{dI}{dt} = \frac{dR}{dt} = 0$

(a) $$\frac{dS}{dt} = \frac{dI}{dt}$$

$$\begin{aligned} &- \beta IS = \beta IS - \nu I \\ &-2\beta IS = -\nu I \\ &\beta S = ½(\nu) \\ &S = \frac{\nu}{2\beta} \\ &S = (1/2) * (1/R_o) \\ &S \alpha k/R_o \end{aligned}$$

Thus, the number of susceptible increases as the basic reproduction ratio decreases and vice versa.

(b) $$\frac{dI}{dt} = \frac{dR}{dt}$$

$$\begin{aligned} &\beta IS - \nu I = \nu I \\ &\beta IS = 2\nu I \\ &\beta S = 2\nu \\ &S = \frac{2\nu}{\beta} \\ &S = (2) * (1/R_o) \\ &S \alpha k/R_o \end{aligned}$$

Thus, the number of susceptible increases as the basic reproduction ratio decreases and vice versa.

(c) $$\frac{dS}{dt} = \frac{dR}{dt}$$

$$\begin{aligned} &-\beta IS = \nu I \\ &\beta S = -\nu \\ &S = -\frac{\nu}{\beta} \\ &S = (-1) * (1/R_o) \\ &S \alpha k/R_o \end{aligned}$$

Thus, the number of susceptible increases as the basic reproduction ratio decreases and vice versa.

Existence of steady states: Let E(S*,I*,R*) be steady states such that , $\frac{dS}{dt} = \frac{dI}{dt} = \frac{dR}{dt} = 0$ then Eq. (1), (2) and (3) becomes:

$$-\beta I^*S^* = 0 \quad (4)$$
$$\beta I^*S^* - vI^* = 0 \quad (5)$$
$$vI^* = 0 \quad (6)$$

On existence of trivial equilibrium: Solving Eq. (4), (5) and (6) at (S*, I*, R*) = (0, 0, 0) and with all parameters greater than zero, $\beta, v > 0$ then $\beta, v \neq 0$. Moreover when the human host is not infected with the malaria, the individual is susceptible to the disease. This implies that there is no trivial equation and thus we cannot have (S*, I*, R*) = (0, 0, 0)

Disease free equilibrium: In this state the individual has no malaria parasites in the body. Thus we take I* = 0. Hence from Eq. (5) we have:

$$BI^*S^* - vI^* = 0$$
$$S^* = \frac{v}{\beta}$$
$$S^* = 1/R_o$$

Thus the disease free equilibrium is given as:

$$E^*\left(\frac{v}{\beta}, 0, 0\right)$$

Stability of steady states: We establish the stability of equilibrium points from the Jacobian matrix of Eq. (1) to (3) which is given as

$$J = \begin{bmatrix} -\beta I & 0 & 0 \\ \beta I & -v & 0 \\ 0 & v & 0 \end{bmatrix} * \begin{bmatrix} S \\ I \\ R \end{bmatrix}$$

$$= \begin{bmatrix} -\beta I - \lambda & 0 & 0 \\ \beta I & -v - \lambda & 0 \\ 0 & v & 0 - \lambda \end{bmatrix}$$

where λ are the Eigen values of J.

$$\text{Det (J)} = (-\beta I - \lambda) * \begin{vmatrix} -v - \lambda & 0 \\ v & -\lambda \end{vmatrix} + 0$$

$$= (-\beta I - \lambda) * (\lambda v + \lambda^2 - 0)$$

Table 1: District population with target groups: 2009

Sub-District/Zones	Communities	Population
Tarkwa West	18	34703
Tarkwa East	9	14089
Nsuta Sub-District	9	17352
Iduapriem Sub-District	11	12013
Pepesa East	9	12458
Pepesa West	7	11123
Nsuaem Sub-District	22	36334
Benso Sub-District	6	10233
Total	91	148,304

Table 2: Malaria cases in the Wassa west district

Years	Cases
2003	49192
2004	58487
2005	66387
2006	89617
2007	95266
2008	105858
2009	127109

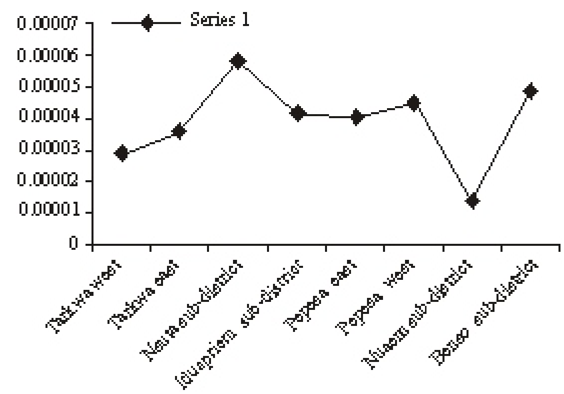

Fig 1: The Basic Reproduction Ratio (R_o) in the District when k = 1/2

It is clear that Det (J) < 0, which implies that the system is unstable. This contradicts with the initial assumption that individuals are permanently immune. Hence there is no permanent immunity in the district.

RESULTS AND DISCUSSION

Data collected from the district health management team, tarkwa: Table 1 shows the statistical analysis of the district population with the target groups 2009 and Table 2 shows Malaria cases from 2003 to 2009 in Wassa West district. More- over, the Fig. 1 to 3 is a narrative to the basic reproductive ratio.

Calculation of the basic reproduction ratio (R_o) with target groups: 2009:
This is also called the basic reproduction ratio. Given as:

$$R_o = \frac{\beta}{v}$$

From the equilibrium analysis, it was deduced that $S \alpha k/R_o$, thus R_o = k / S(t), for all k > 0 we have:

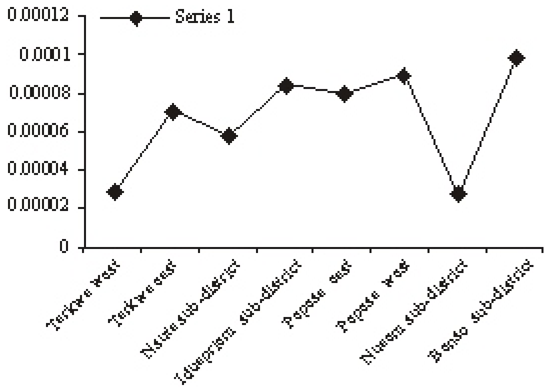

Fig. 2: The Basic Reproduction Ratio (R_o) in the District when k = 1

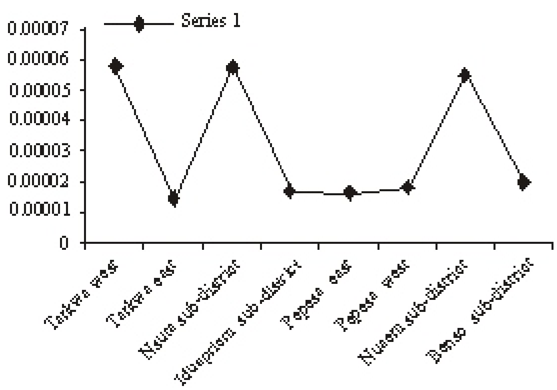

Fig. 3: The Basic Reproduction Ratio (R_o) in the District when k = 2

(i) R_o for Tarkwa West:

$$R_o = k / 34703$$
$$= 0.00002882k$$

For all K = ½, 1and 2, R_o = 0.00001441, 0.0000282 and 0.00005764, respectively.

$$R_o > 0$$

(ii) R_o for Tarkwa East:

$$R_o = k / 14089$$
$$= 0.0000709774k$$

For all K = ½, 1and 2, R_o = 0.0000354887, 0.0000709774 and 0.000014195 4, respectively.

$$R_o > 0$$

(iii) R_o for Nsuta Sub-District:

$$R_o = k / 17352$$
$$= 0.000057630k$$

For all K = ½, 1and 2, R_o = 0.000028815, 0.000057630 and 0.00011526, respectively.

$$R_o > 0$$

(iv) R_o for Iduapriem Sub-District:

$$R_o = k / 12013$$
$$= 0.0000832432k$$

For all K = ½, 1and 2, R_o = 0.0000416216, 0.0000832432 and 0.0001664864, respectively.

$$R_o > 0$$

(v) R_o for Pepesa East:

$$R_o = k / 12458$$
$$= 0.00008026971k$$

For all K = ½, 1and 2, R_o = 0.000040134855, 0.0000802697 and 0.0001605394, respectively.

$$R_o > 0$$

(vi) R_o for Pepesa West:

$$R_o = k / 11123$$
$$= 0.0000899038k$$

For all K = ½, 1and 2, R_o = 0.0000449519, 0.0000899038 and 0.0001798076, respectively.

$$R_o > 0$$

(vii) R_o for Nsuaem Sub-District:

$$R_o = k / 36334$$
$$= 0.00002752243k$$

For all K = ½, 1and 2, R_o = 0.00001376122, 0.000027522 and 0.0000550449, respectively.

$$R_o > 0$$

(viii) R_o for Benso Sub-District:

$$R_o = k / 10233$$
$$= 0.000097723053k$$

For all K = ½, 1and 2, R_o = 0.0000488615, 0.0000977231 and 0.0001954461, respectively.

$$R_o > 0$$

Hence the Basic Reproduction Ratio (R_o), for the Wassa West District is given as:

$R_o = k / 148304$
$= 0.00000674291k$

For all k = ½, 1, and 2, R_o = 0.00000337146, 0.0000067429 and 0.000013486, respectively.

Hence there is a proper malarial outbreak in the District with an increase in the number of the infected individuals in the population (Which can reach a considerable fraction of the population). The disease free equilibrium is unstable and the host will not recover from the disease.

Interpretation of the results:

- The number of new infections increases over the years in this population of which every individual is susceptible and thus the basic reproduction ratio is positive (that is $R_o > 0$). Hence there will be a proper malaria outbreak in the district with an increase in the number of the infected individuals in the population (Which can reach a considerable fraction of the population).
- The force of infection increases over the years and thus the number of susceptible individuals in the population decreases making the relation $S \alpha \frac{K}{Ro}$ valid
- It is seen that from 2003 to 2009, the number of infected individuals keep increasing verifying the fact that as $t \to \infty$, $S(t) \to 0$ and $I(t) \to N$.
- Considering 2009, out of a total population of 148304, 127109 people contracted the disease which is about 90% of the population. This shows a very high level of infection in the population and testifies the fact that every individual in the population is susceptible.

CONCLUSION

In conclusion i will like to put forward the following to help check the rate of infection if not to eradicate the malaria in the district;

- As a major cause of morbidity and mortality in Ghana and the Wassa West District to be precise, malaria must be addressed.
- Given the huge prevalence of *malaria* in Ghana at large, all patients should always be treated for *malaria* first.
- Early treatment is the key. Early recognition of symptoms by patients and early diagnosis by physicians will prevent complications.

RECOMMENDATION

To eradicate malaria fever from the Wassa West District, the following recommendations should be taken into consideration;

- Malaria prevention must be a public health priority.
- The government should provide inhabitants of the Wassa West District a clean environment and clean drainage system.
- Vaccines against malaria fever should be made available by the District Health Management Team and should be used everywhere in the Wassa West District.
- The battle against malaria can only be won through prevention.
- The public should undertake practises which can limit malaria transmissions by trying to reduce the number of mosquitoes and mosquito bites through the use of bed nets, spraying and wearing long sleeves shirts and trousers to reduce the amount of exposed skin.
- The public should remove potential mosquito breeding sites and so reduce the number of mosquitoes.

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