Research Journal of Mathematics and Statistics 2(2): 56-68, 2010
ISSN: 2040-7505

Submitted Date: November 27, 2009 Accepted Date: December 21, 2010 Published Date: June 25, 2010

First Order Reactant in MHD Turbulence Before the Final Period of Decay for the Case of Multi-Point and Multi-Time in a Rotating System in Presence of Dust Particle

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Abstract: Following Deissler's theory, the decay for the concentration fluctuation of a dilute contaminant undergoing a first order chemical reaction in MHD turbulence at times before the final period in a rotating frame in presence of dust particle for the case of multi-point and multi-time is studied and have considered correlations between fluctuating quantities at two and three point. Two and three point correlation equations are obtained and the set of equations is made to determinate by neglecting the quadruple correlations in comparison to the second and third order correlations. The correlation equations are converted to spectral form by taking their Fourier transforms. Finally we obtained the decay law of magnetic energy for the concentration fluctuations before the final period in a rotating frame in presence of dust particle for the case of multi-point and multi-time by integrating the energy spectrum over all wave numbers.

Key words: Decay before the final period, dust particle, first order reactant, MHD turbulence

INTRODUCTION

Funada *et al*. (1978) considered the effect of coriolis force on turbulent motion in presence of strong magnetic field with the assumption that the coriolis force term is balanced by the geostropic wind approximation. Sarker and Islam (2001a) studied the decay of dusty fluid turbulence before the final period in a rotating system. Kishore and Sinha (1988) studied the rate of change of vorticity covariance in dusty fluid turbulence. Sinha (1988) also studied the effect of dust particles on the acceleration covariance of ordinary turbulence. Deissler, (1958, 1960) developed a theory "decay of homogeneous turbulence for times before the final period".

Using Deissler's theory, Loefter and Deissler (1961) studied the decay of temperature fluctuations in homogeneous turbulence before the final period. In their approach they considered the two and three-point correlation equations and solved these equations after neglecting fourth and higher order correlation terms. Using Deissler theory, Kumar and Patel (1974) studied the first-order reactant in homogeneous turbulence before the final period of decay for the case of multi-point and single-time correlation. Kumar and Patel (1975) extended their problem for the case of multi-point and multi-time concentration correlation. Patel (1976) also studied in detail the same problem to carry out the numerical results. Sarker and Kishore (1991) studied the decay of MHD turbulence at time before the final period using Chandrasekhar (1951) and Sarker and Islam (2001b) studied the decay of MHD turbulence before the final period for the case of multi-point and multi-time. Azad and Sarker (2003) studied the Decay of dusty fluid MHD turbulence before the final period in a rotating system for the case of multi-point and multi-time. Islam and Sarker (2001) also studied the first order reactant in MHD turbulence before the final period of decay for the case of multi-point and multi-time.

In this study, following Deissler's theory we have studied the magnetic field fluctuation of concentration of a dilute contaminant undergoing a first order chemical reaction in MHD turbulence before the final period of decay for the case of multi-point and multi-time in a rotating system in presence of dust particle. Here, we have considered the two-point, two-time and three-point, three-time correlation equations and solved these equations after neglecting the fourth-order correlation terms. Finally we obtained the decay law for magnetic field energy fluctuation of concentration of dilute contaminant undergoing a first order chemical reaction in MHD turbulence for the case of multi-point and multi-time in a rotating system in presence of dust particle is obtained. If the fluid is clean and the system is non-rotating, the equation reduces to one obtained earlier by Islam and Sarker (2001).

MATERIALS AND METHODS

Basic equations: The equations of motion and continuity for viscous, incompressible dusty fluid MHD turbulent flow in a rotating system are given by:

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$$\frac{\partial u_i}{\partial t}+\frac{\partial}{\partial x_k}\left(u_i u_k-h_i h_k\right)= -\frac{\partial w}{\partial x_i}+\nu\frac{\partial^2 u_i}{\partial x_k \partial x_k}- 2\in_{mkl}\Omega_m u_i+f\left(u_i-v_i\right) \tag{1}$$

$$\frac{\partial h_i}{\partial t}+\frac{\partial}{\partial x_k}\left(h_i u_k-u_i h_k\right)=\lambda\frac{\partial^2 h_i}{\partial x_k \partial x_k} \tag{2}$$

$$\frac{\partial v_i}{\partial t}+v_k\frac{\partial v_i}{\partial x_K}=-\frac{K}{m_s}\left(v_i-u_i\right) \tag{3}$$

with

$$\frac{\partial u_i}{\partial x_i}=\frac{\partial v_i}{\partial x_i}=\frac{\partial h_i}{\partial x_i}=0 \tag{4}$$

Here, u_i, turbulence velocity component; h_i, magnetic field fluctuation component; v_i, dust particle velocity component; $w(\hat{x},t)=\frac{P}{\rho}+\frac{1}{2}\left\langle h^2\right\rangle+\frac{1}{2}\left|\hat{\Omega}\times\hat{x}\right|^2$, total MHD pressure $p(\hat{x},t)$, hydrodynamic pressure; ρ, fluid density; ν, Kinematic viscosity; $\lambda=\nu/P_M$, magnetic diffusivity; P_M, magnetic prandtl number; x_k, space co-ordinate; the subscripts can take on the values 1, 2 or 3 and the repeated subscripts in a term indicate a summation; Ω_m, constant angular velocity component; $\in_{mkl}$, alternating tensor; $f=\frac{KN}{\rho}$, dimension of frequency ; N, constant number density of dust particle $m_S=\frac{4}{3}\pi R_s^3\rho_s$, mass of single spherical dust particle of radius R_s; ρ_s, constant density of the material in dust particle.

Two-Point, Two-Time Correlation and Spectral Equations: Under the condition that (I) the turbulence and the concentration magnetic field are homogeneous (ii) the chemical reaction has no effect on the velocity field and (iii) the reaction rate and the magnetic diffusivity are constant, the induction equation of a magnetic field fluctuation of concentration of a dilute contaminant undergoing a first order chemical reaction at the points p p′ and separated by the vector $\hat{r}$ could be written as

$$\frac{\partial h_i}{\partial t}+u_k\frac{\partial h_i}{\partial x_k}-h_k\frac{\partial u_i}{\partial x_k}=\lambda u_k\frac{\partial^2 h_i}{\partial x_k \partial x_k}-Rh_i \tag{5}$$

and

$$\frac{\partial h_i'}{\partial t'}+u_k'\frac{\partial h_i'}{\partial x_k'}-h_k'\frac{\partial u_i'}{\partial x_k'}=\lambda u_k\frac{\partial^2 h_i'}{\partial x_k' \partial x_k'}-Rh_i' \tag{6}$$

Where, R is the constant reaction rate.

Multiplying Eq. (5) by h'_j and Eq. (6) by h_i and taking ensemble average, we get

$$\frac{\partial\left\langle h_i h_j'\right\rangle}{\partial t}+\frac{\partial}{\partial x_k}\left[\left\langle u_k h_i h_i'\right\rangle-\left\langle u_i h_k h_j'\right\rangle\right]= \lambda\frac{\partial^2\left\langle h_i h_j'\right\rangle}{\partial x_k \partial x_k}-R\left\langle h_i h_j'\right\rangle \tag{7}$$

and

$$\frac{\partial\left\langle h_i h_j'\right\rangle}{\partial t'}+\frac{\partial}{\partial x_k'}\left[\left\langle u_k' h_i h_j'\right\rangle-\left\langle u_j' h_i h_k'\right\rangle\right]= \lambda\frac{\partial^2\left\langle h_i h_j'\right\rangle}{\partial x_k' \partial x_k'}-R\left\langle h_i h_j'\right\rangle \tag{8}$$

Angular bracket $\langle - - - - - \rangle$ is used to denote an ensemble average.

Using the transformations

$$\frac{\partial}{\partial x_k}=-\frac{\partial}{\partial r_k},\frac{\partial}{\partial x_k'}=\frac{\partial}{\partial r_k} \qquad \left(\frac{\partial}{\partial t}\right)t'=\left(\frac{\partial}{\partial t}\right)\Delta t-\frac{\partial}{\partial \Delta t},\frac{\partial}{\partial t'}=\frac{\partial}{\partial \Delta t} \tag{9}$$

into Eq. (7) and (8), we obtain:

$$\frac{\partial\left\langle h_i h_j'\right\rangle}{\partial t}+\frac{\partial}{\partial r_k}\left[\left\langle u_k' h_i h_j'\right\rangle-\left\langle u_j' h_i h_k'\right\rangle\right](\hat{r},\Delta t,t) -\frac{\partial}{\partial r_k}\left[\left\langle u_k h_i h_j'\right\rangle-\left\langle u_i h_k h_j'\right\rangle\right](\hat{r},\Delta t,t) =2\lambda\frac{\partial^2\left\langle h_i h_j'\right\rangle}{\partial r_k \partial r_k}-2R\left\langle h_i h_j'\right\rangle \tag{10}$$

and

$$\frac{\partial\langle h_i h_j'\rangle}{\partial\Delta t}+\frac{\partial}{\partial r_k}\left[\langle u_k' h_i h_j'\rangle-\langle u_j' h_i h_k'\rangle\right](\hat{r},\Delta t,t) = \lambda\frac{\partial^2\langle h_i h_j'\rangle}{\partial r_k\partial r_k}-R\langle h_i h_j'\rangle \tag{11}$$

Using the relations of (Chandrasekhar, 1951)

$$\langle u_k h_i h_j'\rangle=-\langle u_k' h_i h_j'\rangle,\quad \langle u_j' h_i h_k'\rangle=\langle u_i h_k h_j'\rangle$$

Eq. (10) and (11) become:

$$\frac{\partial\langle h_i h_j'\rangle}{\partial t}+2\frac{\partial}{\partial r_k}\left[\langle u_k' h_i h_j'\rangle-\langle u_i' h_k h_j'\rangle\right] = 2\lambda\frac{\partial^2\langle h_i h_j'\rangle}{\partial r_k\partial r_k}-2R\langle h_i h_j'\rangle \tag{12}$$

and

$$\frac{\partial\langle h_i h_j'\rangle}{\partial t}+\frac{\partial}{\partial r_k}\left[\langle u_k' h_i h_j'\rangle-\langle u_i' h_k h_j'\rangle\right] = \lambda\frac{\partial^2\langle h_i h_j'\rangle}{\partial r_k\partial r_k}-R\langle h_i h_j'\rangle \tag{13}$$

Now we write Eq. (12) and (13) in spectral form in order to reduce it to an ordinary differential equation by use of the following three-dimensional Fourier transforms:

$$\langle h_i h_j'\rangle(\hat{r},\Delta t,t) = \int_{-\infty}^{\infty}\langle \psi_i \psi_j'\rangle(\hat{K},\Delta t,t)\exp\left[\hat{i}(\hat{K}.\hat{r})\right]d\hat{K} \tag{14}$$

and

$$\langle u_i h_k h_j'\rangle(\hat{r},\Delta t,t) = \int_{-\infty}^{\infty}\langle \alpha_i \psi_k \psi_j'\rangle(\hat{K},\Delta t,t)\exp\left[\hat{i}(\hat{K}.\hat{r})\right]d\hat{K} \tag{15}$$

Interchanging the subscripts i and j then interchanging the points p and p′ gives

$$\langle u_k' h_i h_j'\rangle(\hat{r},\Delta t,t) = \langle u_k h_i h_j'\rangle(-\hat{r},-\Delta t,t+\Delta t) \int_{-\infty}^{\infty}\langle \alpha_i \psi_k \psi_j'\rangle(-\hat{K},-\Delta t,t+\Delta t)\exp\left[\hat{i}(\hat{K}.\hat{r})\right]d\hat{K} \tag{16}$$

where $\hat{K}$ is known as a wave number vector and $d\hat{K}=dK_1\,dK_2\,dK_3$. The magnitude of has the dimension 1/length and can be considered to be the reciprocal of an eddy size. Substituting of Eq. (14) to (16) into Eq. (12) and (13) leads to the spectral equations

$$\frac{\partial\langle \psi_i \psi_j'\rangle}{\partial t}+2\left[\lambda k^2+R\right]\langle \psi_i \psi_j'\rangle = 2ik_k\left[\langle \alpha_i \psi_k \psi_j'\rangle(\hat{K},\Delta t,t) - \left[\langle \alpha_k \psi_i \psi_j'\rangle(-\hat{K},-\Delta t,t+\Delta t)\right]\right. \tag{17}$$

and

$$\frac{\partial\langle \psi_i \psi_j'\rangle}{\partial\Delta t}+\left[\lambda K^2+R\right]\langle \psi_i \psi_i'\rangle = ik_k\left[\langle \alpha_i \psi_k \psi_j'\rangle(\hat{K},\Delta t,t) - \left[\langle \alpha_k \psi_i \psi_j'\rangle(-\hat{K},-\Delta t,t+\Delta t)\right]\right. \tag{18}$$

The tensor Eq. (17) and (18) becomes a scalar equation by contraction of the indices i and j

$$\frac{\partial\langle \psi_i \psi_j'\rangle}{\partial t}+2\left[\lambda K^2+R\right]\langle \psi_i \psi_i'\rangle = 2ik_k\left[\langle \alpha_i \psi_k \psi_i'\rangle(\hat{K},\Delta t,t) - \left[\langle \alpha_k \psi_i \psi_j'\rangle(-\hat{K},-\Delta t,t+\Delta t)\right]\right. \tag{19}$$

and

$$\frac{\partial\langle \psi_i \psi_j'\rangle}{\partial\Delta t}+\left[\lambda K^2+R\right]\langle \psi_i \psi_i'\rangle = ik_k\left[\langle \alpha_i \psi_k \psi_i'\rangle(\hat{K},\Delta t,t) - \left[\langle \alpha_k \psi_i \psi_j'\rangle(-\hat{K},-\Delta t,t+\Delta t)\right]\right. \tag{20}$$

The terms on the right side of Eq. (19) and (20) are collectively proportional to what is known as the magnetic energy transfer terms.

Three-Point, Three-Time Correlation and Spectral Equations: Similar procedure can be used to find the three-point correlation equations. For this purpose we take the momentum equation of dusty fluid MHD turbulence in a rotating system at the point P and the induction equations of magnetic field fluctuations, governing the concentration of a dilute contaminant undergoing a first order chemical reaction at p' and p'' separated by the vector $\hat{r}$ and $\hat{r}'$ as:

$$\frac{\partial u_l}{\partial t} + u_k \frac{\partial u_l}{\partial x_k} - h_k \frac{\partial h_l}{\partial x_k} = -\frac{\partial w}{\partial x_l} + \frac{\partial^2 u_l}{\partial x_k \partial x_k} - 2\in_{mkl} \Omega_m u_l + f(u_l - v_l) \tag{21}$$

$$\frac{\partial h_i'}{\partial t'} + u_k' \frac{\partial h_i'}{\partial x_k'} - h_k' \frac{\partial u_i'}{\partial x_k'} = \lambda \frac{\partial h_i'}{\partial x_k' \partial x_k'} - Rh_i' \tag{22}$$

$$\frac{\partial h_j''}{\partial t''} + u_k'' \frac{\partial h_j''}{\partial x_k''} - h_k'' \frac{\partial u_j''}{\partial x_k''} = \lambda \frac{\partial h_j''}{\partial x_k'' \partial x_k''} - Rh_j'' \tag{23}$$

Multiplying Eq. (21) by $h_i' h_j''$, Eq. (22) by $u_l h_j''$ and Eq. (23) by $u_l h_i'$, taking ensemble average, one obtains

$$\frac{\partial \langle u_l h_i' h_j'' \rangle}{\partial t} + \frac{\partial}{\partial x_k}\left[\langle u_k u_l h_i' h_j'' \rangle - \langle h_k h_l h_i' h_j'' \rangle\right] = \frac{\partial \langle w h_i' h_j'' \rangle}{\partial x_l} + \nu \frac{\partial^2 \langle u_l h_i' h_j'' \rangle}{\partial x_k \partial x_k} - 2\in_{mkl} \Omega_m \langle u_l h_i' h_j'' \rangle + f\left(\langle u_l h_i' h_j'' \rangle - \langle v_l h_i' h_j'' \rangle\right) \tag{24}$$

$$\frac{\partial \langle u_l h_i' h_j'' \rangle}{\partial t} + \frac{\partial}{\partial x_k'}\left[\langle u_l u_k' h_i' h_j'' \rangle - \langle u_l u_i' h_k' h_j'' \rangle\right] = \lambda \frac{\partial^2 \langle u_l h_i' h_j'' \rangle}{\partial x_k' \partial x_k'} - R\langle u_l h_i' h_j'' \rangle \tag{25}$$

and

$$\frac{\partial \langle u_l h_i' h_j'' \rangle}{\partial t''} + \frac{\partial}{\partial x_k''}\left[\langle u_l u_k'' h_i' h_j'' \rangle - \langle u_l u_j'' h_i' h_k'' \rangle\right] = \lambda \frac{\partial^2 \langle u_l h_i' h_j'' \rangle}{\partial x_k'' \partial x_k''} - R\langle u_l h_i' h_j'' \rangle \tag{26}$$

Using the transformations:

$$\frac{\partial}{\partial x_k} = -\left(\frac{\partial}{\partial r_k} + \frac{\partial}{\partial r_k'}\right), \quad \frac{\partial}{\partial x_k'} = \frac{\partial}{\partial r_k},$$

$$\frac{\partial}{\partial x_k''} = \frac{\partial}{\partial r_k'},$$

$$\left(\frac{\partial}{\partial t}\right)t', t'' = \left(\frac{\partial}{\partial t}\right)\Delta t, \Delta t' - \left(\frac{\partial}{\partial \Delta t} - \frac{\partial}{\partial \Delta t'}\right),$$

$$\frac{\partial}{\partial t'} = \frac{\partial}{\partial \Delta t}, \qquad \frac{\partial}{\partial t''} = \frac{\partial}{\partial t'},$$

into Eq. (24) to (26), we have

$$\frac{\partial \langle u_l h_i' h_j'' \rangle}{\partial t} - \left(\frac{\partial}{\partial r_k} + \frac{\partial}{\partial r_k'}\right)\left[\langle u_k u_l h_i' h_j'' \rangle - \langle h_k h_l h_i' h_j'' \rangle\right] + \frac{\partial}{\partial r_k}\left[\langle u_l u_k' h_i' h_j'' \rangle - \langle u_l u_i' h_k' h_j'' \rangle\right] + \frac{\partial}{\partial r_k'}\left[\langle u_l u_k'' h_i' h_j'' \rangle - \langle u_l u_j'' h_i' h_k'' \rangle\right]$$
$$= \left(\frac{\partial}{\partial r_l} + \frac{\partial}{\partial r_l'}\right)\langle w h_i' h_j'' \rangle + \nu\left(\frac{\partial}{\partial r_k} + \frac{\partial}{\partial r_k'}\right)^2 \langle u_l h_i' h_j'' \rangle + \lambda\left[\frac{\partial^2 \langle u_l h_i' h_j'' \rangle}{\partial r_k \partial r_k} + \frac{\partial^2 \langle u_l h_i' h_j'' \rangle}{\partial r_k' \partial r_k'}\right] - 2\in_{mkl} \Omega_m \langle u_l h_i' h_j'' \rangle + f\left(\langle u_l h_i' h_j'' \rangle - \langle v_l h_i' h_j'' \rangle\right) \tag{27}$$

$$\frac{\partial \langle u_l h_i' h_j'' \rangle}{\partial \Delta t} + \frac{\partial}{\partial r_k}\left[\langle u_l u_k' h_i' h_j'' \rangle - \langle u_l u_i' h_k' h_j'' \rangle\right] = \lambda \frac{\partial^2 \langle u_l h_i' h_j'' \rangle}{\partial r_k \partial r_k} - R\langle u_l h_i' h_j'' \rangle \tag{28}$$

and

$$\frac{\partial\langle u_l h_i' h_j''\rangle}{\partial \Delta t'} + \frac{\partial}{\partial r_k'}\left[\langle u_l u_k'' h_i' h_j''\rangle - \langle u_l u_j'' h_i' h_k''\rangle\right] = \lambda \frac{\partial^2 \langle u_l h_i' h_j''\rangle}{\partial r_k' \partial r_k'} - R\langle u_l h_i' h_j''\rangle \tag{29}$$

In order to convert Eq. (27)-(29) to spectral form, we can define the following six dimensional Fourier transforms:

$$\langle u_l h_i' h_j''\rangle(\hat{r},\hat{r}',\Delta t,\Delta t',t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle \phi_l \beta_i' \beta_j''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)\exp\left[\hat{i}\left(\hat{K}.\hat{r}+\hat{K}'.\hat{r}'\right)\right]d\hat{K}d\hat{K}' \tag{30}$$

$$\langle u_l u_k' h_i' h_j''\rangle(\hat{r},\hat{r}',\Delta t,\Delta t',t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle \phi_l \phi_K' \beta_i' \beta_j''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)\exp\left[\hat{i}\left(\hat{K}.\hat{r}+\hat{K}'.\hat{r}'\right)\right]d\hat{K}d\hat{K}' \tag{31}$$

$$\langle w h_i' h_j''\rangle(\hat{r},\hat{r}',\Delta t,\Delta t',t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle \gamma \beta_i' \beta_j''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)\exp\left[\hat{i}\left(\hat{K}.\hat{r}+\hat{K}'.\hat{r}'\right)\right]d\hat{K}d\hat{K}' \tag{32}$$

$$\langle u_k u_l h_i' h_j''\rangle(\hat{r},\hat{r}',\Delta t,\Delta t',t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle \phi_K \phi_l \beta_i' \beta_j''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)\exp\left[\hat{i}\left(\hat{K}.\hat{r}+\hat{K}'.\hat{r}'\right)\right]d\hat{K}d\hat{K}' \tag{33}$$

$$\langle h_k h_l h_i' h_j''\rangle(\hat{r},\hat{r}',\Delta t,\Delta t',t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle \beta_K \beta_l \beta_i' \beta_j''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)\exp\left[\hat{i}\left(\hat{K}.\hat{r}+\hat{K}'.\hat{r}'\right)\right]d\hat{K}d\hat{K}' \tag{34}$$

$$\langle u_l u_i' h_K' h_j''\rangle(\hat{r},\hat{r}',\Delta t,\Delta t',t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle \phi_l \phi_i \beta_i' \beta_j''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)\exp\left[\hat{i}\left(\hat{K}.\hat{r}+\hat{K}'.\hat{r}'\right)\right]d\hat{K}d\hat{K}' \tag{35}$$

$$\langle v_l h_i' h_j''\rangle(\hat{r},\hat{r},\Delta t,\Delta t',t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle \mu_l \beta_i' \beta_j''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)\exp\left[\hat{i}\left(\hat{K}.\hat{r}+\hat{K}'.\hat{r}'\right)\right]d\hat{K}d\hat{K}' \tag{36}$$

Interchanging the points P′ and P″ along with the indices i and j result in the relations

$$\langle u_l u_k'' h_i' h_j''\rangle = \langle u_l u_k' h_i' h_j''\rangle$$

By use of these facts and the Eq. (30)-(36), we can write Eq. (27)-(29) in the form:

$$\frac{\partial}{\partial t}\langle \phi_l \beta_i' \beta_j''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t) + \lambda\left[\left(1+P_M\right)\left(k^2+k'^2\right)+2P_M kk' + \frac{2R}{\lambda} + \frac{1}{\lambda}\left(2\,\epsilon_{mkl}\,\Omega_m - f\right)\right] \times \langle \phi_l \beta_i' \beta_j''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t)$$

$$\Big[i\left(k_k+k_k'\right)\langle \phi_k \phi_l \beta_i' \beta''\rangle - i\left(k_k+k_k'\right)\langle \beta_k \beta_l \beta_i' \beta_j''\rangle - i\left(k_k+k_k'\right)\langle \phi_l \phi_k' \beta_i' \beta_j''\rangle$$

$$\begin{aligned}&+i\left(k_k+k_k'\right)\left\langle \phi_l\phi_l'\beta_k\beta_j'\right\rangle \\ &-i\left(k_k+k_k'\right)\left\langle \gamma\beta_i'\beta_j'\right\rangle \\ &+i\left(k_k+k_k'\right)\left\langle \phi_l\phi_l'\beta_k\beta_j'\right\rangle \\ &-f\left\langle \mu_l\beta_k'\beta_j'\right\rangle\Big]\left(\hat{K},\hat{K}',\Delta t,\Delta t',t\right)\end{aligned} \tag{37}$$

$$\frac{\partial}{\partial \Delta t}\left\langle \phi_l\beta_i'\beta_j''\right\rangle\left(\hat{K},\hat{K}',\Delta t,\Delta t',t\right) + \lambda\left[K^2+\frac{R}{\lambda}\right]\left\langle \phi_l\beta_i'\beta_j''\right\rangle\left(\hat{K},\hat{K}',\Delta t,\Delta t',t\right)$$

$$\begin{aligned}&= -ik_k\left\langle \phi_l\phi_k'\beta_i'\beta_j''\right\rangle\left(\hat{K},\hat{K}',\Delta t,\Delta t',t\right) \\ &+ik_k\left\langle \phi_l\phi_l'\beta_k\beta_j''\right\rangle\left(\hat{K},\hat{K}',\Delta t,\Delta t',t\right)\end{aligned} \tag{38}$$

and

$$\frac{\partial}{\partial \Delta t'}\left\langle \phi_l\beta_i'\beta_j''\right\rangle\left(\hat{K},\hat{K}',\Delta t,\Delta t',t\right) + \lambda\left[K^2+\frac{R}{\lambda}\right]\left\langle \phi_l\beta_i'\beta_j''\right\rangle\left(\hat{K},\hat{K}',\Delta t,\Delta t',t\right)$$

$$\begin{aligned}&= -ik_k'\left\langle \phi_l\phi_k'\beta_i'\beta_j''\right\rangle\left(\hat{K},\hat{K}',\Delta t,\Delta t',t\right) \\ &+ik_k'\left\langle \phi_l\phi_l'\beta_k\beta_j''\right\rangle\left(\hat{K},\hat{K}',\Delta t,\Delta t',t\right)\end{aligned} \tag{39}$$

If the derivative with respect to x_l is taken of the momentum Eq. (21) for the point P, the equation multiplied by $h_i'h_j''$ and time average taken, the resulting equation:

$$-\frac{\partial^2\left\langle wh_i'h_j''\right\rangle}{\partial x_l\partial x_l} = \frac{\partial^2}{\partial x_l\partial x_k}\left(\left\langle u_l u_k h_i'h_j''\right\rangle - \left\langle h_l h_k h_i'h_j''\right\rangle\right) \tag{40}$$

Writing this equation in terms of the independent variables $\hat{r}$ and $\hat{r}'$:

$$\begin{aligned}&-\left[\frac{\partial^2}{\partial r_l\partial r_l}+2\frac{\partial^2}{\partial r_l\partial r_l'}+\frac{\partial^2}{\partial r_l'\partial r_l'}\right]\left\langle wh_i'h_j''\right\rangle \\ &=\left[\frac{\partial^2}{\partial r_l\partial r_k}+\frac{\partial^2}{\partial r_l'\partial r_k}+\frac{\partial^2}{\partial r_l\partial r_k'}+\frac{\partial^2}{\partial r_l'\partial r_k'}\right] \\ &\times\left(\left\langle u_l u_k h_i'h_j''\right\rangle - \left\langle h_l h_k h_i'h_j''\right\rangle\right)\end{aligned} \tag{41}$$

Taking the Fourier transforms of Eq. (28)

$$-\left\langle \gamma\beta_i'\beta_j''\right\rangle = \frac{\left(k_l k_k + k_l' k_k + k_l k_k' + k_l' k_k'\right)\left(\left\langle \phi_l\phi_k\beta_i'\beta_j''\right\rangle - \left\langle \beta_l\beta_k\beta_i'\beta_j''\right\rangle\right)}{k_l k_l + 2k_l k_l' + k_l' k_l'} \tag{42}$$

Equation (42) can be used to eliminate $\left\langle \gamma\beta_i'\beta_j''\right\rangle$ from Eq. (37).

The tensor Eq. (37) to (39) can be converted to scalar equation by contraction of the indices i and j and inner multiplication by k_l:

$$\begin{aligned}&\frac{\partial}{\partial t}k_l\left\langle \phi_l\beta_i'\beta_j''\right\rangle\left(\hat{K},\hat{K}',\Delta t,\Delta t',t\right) \\ &+\lambda\left[1+P_M\right]\left(k^2+k'^2\right)+2P_M kk'+\frac{2R}{\lambda} \\ &\frac{1}{\lambda}\left(2\in_{mkl}\Omega_m-f\right)\left\langle \phi_l\beta_i'\beta_j''\right\rangle\left(\hat{K},\hat{K}',\Delta t,\Delta t',t\right) \\ &=\left(k_k+k_k'\right)\left\langle \phi_k\phi_l\beta_i'\beta_j''\right\rangle \\ &\left(\hat{K},\hat{K}',\Delta t,\Delta t',t\right)-i\left(k_k+k_k'\right)\left\langle \beta_K\beta_l\beta_i'\beta_i''\right\rangle \\ &\left(\hat{K},\hat{K}',\Delta t,\Delta t',t\right)-i\left(k_k+k_k'\right)\left\langle \phi_l\phi_k\beta_i'\beta_i''\right\rangle \\ &\left(\hat{K},\hat{K}',\Delta t,\Delta t',t\right)-i\left(k_k+k_k'\right)\left\langle \phi_l\phi_l'\beta_k\beta_i''\right\rangle \\ &\left(\hat{K},\hat{K}',\Delta t,\Delta t',t\right)-i\left(k_k+k_k'\right)\left\langle \gamma\beta_i'\beta_i''\right\rangle \\ &\left(\hat{K},\hat{K}',\Delta t,\Delta t',t\right)-f\left\langle \mu_l\beta_i'\beta_i''\right\rangle \\ &\left(\hat{K},\hat{K}',\Delta t,\Delta t',t\right)\end{aligned} \tag{43}$$

$$\frac{\partial}{\partial \Delta t'}\langle \phi_l \beta_i' \beta_i'' \rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t) + \lambda\left[K^2+\frac{R}{\lambda}\right]\langle \phi_l \beta_i' \beta_i'' \rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t) = -ik_k\langle \phi_l \phi_k \beta_i' \beta_i'' \rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t) + ik_k\langle \phi_l \phi_i' \beta_k \beta_i'' \rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t) \quad (44)$$

and

$$\frac{\partial}{\partial \Delta t'}k_l\langle \phi_l \beta_i' \beta_i'' \rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t) + \lambda\left[K'^2+\frac{R}{\lambda}\right]\langle \phi_l \beta_i' \beta_i'' \rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t) = -ik_k'\langle \phi_l \phi_k' \beta_i' \beta_i'' \rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t) + ik_k'\langle \phi_l \phi_i' \beta_k' \beta_i'' \rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t) \quad (45)$$

Solution for times before the final period: It is known that the equation for final period of decay is obtained by considering the two-point correlations after neglecting third-order correlation terms. To study the decay for times before the final period, the three-point correlations are considered and the quadruple correlation terms are neglected because the quadruple correlation terms decays faster than the lower-order correlation terms. The term $\langle \gamma \beta_i' \beta_j'' \rangle$ associated with the pressure fluctuations should also be neglected. Thus neglecting all the terms on the right hand side of Eq. (43) to (45):

$$\frac{\partial}{\partial t}k_l\langle \phi_l \beta_i' \beta_i'' \rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t) + \lambda[1+P_M](k^2+k'^2)+2P_M kk'+\frac{2R}{\lambda} + \frac{1}{\lambda}(2\epsilon_{mkl}\Omega_m - f_s)]\langle \phi_l \beta_i' \beta_i'' \rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t) = 0 \quad (46)$$

$$\frac{\partial}{\partial \Delta t}K_l\langle \phi_l \beta_i' \beta_i'' \rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t) + \lambda\left[k^2+\frac{R}{\lambda}\right]\langle \phi_l \beta_i' \beta_i'' \rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t) = 0 \quad (47)$$

and

$$\frac{\partial}{\partial \Delta t'}K_l\langle \phi_l \beta_i' \beta_i'' \rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t) + \lambda\left[k'^2+\frac{R}{\lambda}\right]\langle \phi_l \beta_i' \beta_i'' \rangle(\hat{K},\hat{K}',\Delta t,\Delta t',t) = 0 \quad (48)$$

where, $\langle \mu_l \beta_i' \beta_i'' \rangle = C\langle \phi_l \beta_i' \beta_i'' \rangle$ and 1-C=S, here C and S are arbitrary constant.

Integrating Eq. (46) to (48) between t_o and t, we obtain

$$k_l\langle \phi_l \beta_i' \beta_j'' \rangle = f_l \exp\left\{-\lambda[1+P_M](k^2+k'^2) + 2P_M kk'\cos\theta + \frac{2R}{\lambda} + \frac{1}{\lambda}(2\epsilon_{mkl}\Omega_m - fs)\right](t-t_o)\}$$

$$k_l\langle \phi_l \beta_i' \beta_i'' \rangle = g_l \exp\left[-\lambda\left(K^2+\frac{R}{\lambda}\right)\Delta t\right]$$

and $k_l\langle \phi_l \beta_i' \beta_i'' \rangle = q_l \exp\left[-\lambda\left(k'^2+\frac{R}{\lambda}\right)\Delta t'\right]$

For these relations to be consistent, we have

$$k_l\langle \phi_l \beta_i' \beta_i'' \rangle = k_l\langle \phi_l \beta_i' \beta_i'' \rangle_o \exp\{-\lambda[1+P_M](k^2+k'^2)(t-t_o) + k^2\Delta t + k'^2\Delta t' + 2P_M kk'\cos\theta(t-t_o) + \frac{2R}{\lambda}\left(t-t_o+\frac{\Delta t+\Delta t'}{2}\right) \left(\frac{2\epsilon_{mkl}\Omega_m}{\lambda} - \frac{fs}{\lambda}\right)(t-t_o)]\} \quad (49)$$

where θ is the angle between $\hat{K}$ and $\hat{K}'$ and $\langle \phi_l \beta_i' \beta_i'' \rangle_o$ is the value of $\langle \phi_l \beta_i' \beta_i'' \rangle$ at $t = t_o$, $\Delta t = \Delta t' = 0$, $\lambda = \frac{\nu}{P_M}$

By letting $\hat{r}' = 0$, $\Delta t' = 0$ in the Eq. (30) and comparing with Eq. (15) and (16) we get:

$$\langle \alpha_i \psi_k \psi_i' \rangle (\hat{K}, \Delta t, t) = \int_{-\infty}^{\infty} \langle \phi_l \beta_i' \beta_i'' \rangle (\hat{K}, \hat{K}', \Delta t, o, t) d\hat{K}' \tag{50}$$

and

$$\langle \alpha_i \psi_k \psi_i' \rangle (-\hat{K}, -\Delta t, t + \Delta t) = \int_{-\infty}^{\infty} \langle \phi_l \beta_i' \beta_i'' \rangle (-\hat{K}, \hat{K}', \Delta t, o, t) d\hat{K}' \tag{51}$$

Substituting Eq. (49) to (51) into Eq. (19), one obtains:

$$\frac{\partial}{\partial t} \langle \psi_i \psi_i' \rangle (\hat{K}, \Delta t, t) + 2\lambda \left[k^2 + \frac{R}{\lambda} \right] \langle \psi_i \psi_i' \rangle (\hat{K}, \Delta t, t)$$
$$= \int_{-\infty}^{\infty} 2ik_l \Big[\langle \phi_l \beta_i' \beta_i'' \rangle (\hat{K}, \hat{K}', \Delta t, 0, t) - \langle \phi_l \beta_i' \beta_i'' \rangle (-\hat{K}, -\hat{K}', \Delta t, 0, t) \Big]_\sigma$$
$$\exp\Big\{ -\lambda [1 + P_M] (k^2 + k'^2)(t - t_\sigma) + k^2 \Delta t + 2P_M (t - t_\sigma) kk' \cos\theta + \frac{2R}{\lambda}(t - t_\sigma + \Delta t) + \left(\frac{2 \in_{mkl} \Omega_m}{\lambda} - \frac{fs}{\lambda} \right)(t - t_\sigma) \Big] \Big\} d\hat{k} \tag{52}$$

Now, $d\hat{K}'$ can be expressed in terms of k′ and θ as $-2\pi k' d(\cos\theta) dk'$ (Deissler, 1960)

$$\text{i.e., } d\hat{K}' = -2\pi k' d(\cos\theta) dk' \tag{53}$$

Substituting of Eq. (53) in (52) yields

$$\frac{\partial}{\partial t} \langle \psi_i \psi_i' \rangle (\hat{K}, \Delta t, t) + 2\lambda \left[k^2 + \frac{R}{\lambda} \right] \langle \psi_i \psi_i' \rangle (\hat{K}, \Delta t, t)$$
$$= 2\int_0^{\infty} 2\pi i k_l \Big[\langle \phi_l \beta_i' \beta_i'' \rangle (\hat{K}, \hat{K}') - \langle \phi_l \beta_i' \beta_i'' \rangle (-\hat{K}, -\hat{K}', \Delta t, 0, t) \Big]_\sigma$$
$$\Big[\int_{-1}^{1} \exp\Big\{ -\lambda [1 + P_M] (k^2 + k'^2)(t - t_\sigma) + k^2 \Delta t + 2P_M (t - t_\sigma) kk' \cos\theta + \frac{2R}{\lambda}\left(t - t_\sigma + \frac{\Delta t}{2}\right) + \left(\frac{2 \in_{mkl} \Omega_m}{\lambda} - \frac{fs}{\lambda} \right)(t - t_\sigma) \Big] \Big\} d(\cos\theta) \Big] dk' \tag{54}$$

The quantity

$$\Big[\langle \phi_l \beta_i' \beta_i'' \rangle (\hat{K}, \hat{K}') - \langle \phi_l \beta_i' \beta_i'' \rangle (-\hat{K}, -\hat{K}') \Big]_\sigma$$

depends on the initial conditions of the turbulence.

In order to find the solution completely and following (Loeffler and Deissler, 1961), we assume that:

$$ik_l \Big[\langle \phi_l \beta_i' \beta_i'' \rangle (\hat{K}, \hat{K}') - \langle \phi_l \beta_i' \beta_i'' \rangle (-\hat{K}, -\hat{K}') \Big]_\sigma = \frac{-\delta_\sigma}{(2\pi)^2} (k^2 k'^4 - k^4 k'^2) \tag{55}$$

Where, δ_0 is a constant determined by the initial conditions. The negative sign is placed in front of δ_0 in order to make the transfer of energy from small to large wave numbers for positive value of δ_0.

Substituting Eq. (55) into (54) we get:

$$\frac{\partial}{\partial t} 2\pi \langle \psi_i \psi_i' \rangle (\hat{K}, \Delta t, t) + 2\lambda \left[k^2 + R/\lambda \right] 2\pi \langle \psi_i \psi_i' \rangle (\hat{K}, \Delta t, t)$$
$$= 2\delta_0 \int_0^{\infty} (k^2 k'^4 - k^4 k'^2) k'^2$$

$$\left[\int_{-1}^{1}\exp\left(-\lambda\left[1+P_M\right]\left(k^2+k'^2\right)\left(t-t_o\right)\right.\right.$$
$$+k^2\Delta t+2P_M\left(t-t_o\right)kk'\cos\theta$$
$$+\frac{2R}{\lambda}\left(t-t_o+\frac{\Delta t}{2}\right) \tag{56}$$
$$\left.\left.+\left(\frac{2\in_{mkl}\Omega_m}{\lambda}-\frac{fs}{\lambda}\right)\left(t-t_o\right)\right]\right\}d(\cos\theta)\bigg]dk'$$

Multiplying both sides of Eq. (56) by k^2, we get:

$$\frac{\partial E}{\partial t}+2\lambda k^2E=F \tag{57}$$

where, $E=2\pi k^2\langle\psi_i\psi_i'\rangle$, E is the magnetic energy spectrum function and F is the magnetic energy transfer term and is given by:

$$F=-2\delta\int_0^{\infty}\left(k^2k'^4-k^4k'^2\right)k^2k'^2$$
$$\times\left[\int_{-1}^{1}\exp\left(-\lambda\left[1+P_M\right]\left(k^2+k'^2\right)\left(t-t_o\right)\right.\right.$$
$$+k^2\Delta t+2P_M\left(t-t_o\right)kk'\cos\theta \tag{58}$$
$$+\frac{2R}{\lambda}\left(t-t_o+\frac{\Delta t}{2}\right)$$
$$\left.\left.+\left(\frac{2\in_{mkl}\Omega_m}{\lambda}-\frac{fs}{\lambda}\right)\left(t-t_o\right)\right]\right\}d(\cos\theta)\bigg]dk'$$

Integrating Eq. (58) with respect to $\cos\theta$ and we have:

$$F=-\frac{\delta_oP_M\sqrt{\pi}}{4\lambda^{3/2}\left(t-t_o\right)^{3/2}\left(1+P_M\right)^{5/2}}$$
$$\exp\left\{-\left(\frac{2\in_{mkl}\Omega_m}{\lambda}-\frac{fs}{\lambda}\right)\left(t-t_o\right)\right\}\times$$
$$\exp\left[\frac{-k^2\lambda\left(1+2P_M\right)}{1+2P_M}\left(t-t_o+\frac{1+P_M}{1+2P_M}\Delta t\right)\right.$$
$$\left.-2R\left(t-t_o+\frac{\Delta t}{2}\right)\right]\times\left[\frac{15P_Mk^4}{4P_M^2\lambda^2\left(t-t_o\right)^2\left(1+P_M\right)}\right.$$
$$+\left\{\frac{5P_M^2}{\left(1+P_M\right)^2}-\frac{3}{2}\right\}\frac{k^6}{P_M\lambda(t-to)}$$
$$\left.+\left\{\frac{P_M^3}{\left(1+P_M\right)^3}-\frac{P_M}{1+P_M}\right\}k^8\right]$$
$$-\frac{\delta_oP_M\sqrt{\pi}}{4\lambda^{3/2}\left(t-t_o+\Delta t\right)^{3/2}\left(1+P_M\right)^{5/2}}$$
$$\exp\left\{-\left(\frac{2\in_{mkl}\Omega_m}{\lambda}-\frac{fs}{\lambda}\right)\left(t-t_o\right)\right\}\times$$
$$\exp\left[\frac{-k^2\lambda\left(1+2P_M\right)}{1+2P_M}\left(t-t_o+\frac{P_M}{1+P_M}\Delta t\right)\right.$$
$$\left.-2R\left(t-t_o+\frac{\Delta t}{2}\right)\right]\times\left[\frac{15P_Mk^4}{4v^2\left(t-t_o+\Delta t\right)^2\left(1+P_M\right)}\right.$$
$$+\left\{\frac{5P_M^2}{\left(1+P_M\right)^2}-\frac{3}{2}\right\}\frac{k^6}{P_M\lambda\left(t-t_o+\Delta t\right)} \tag{59}$$
$$\left.+\left\{\frac{P_M^3}{\left(1+P_M\right)^3}-\frac{P_M}{\left(1+P_M\right)}\right\}k^8\right]$$

The series of Eq. (59) contains only even power of k and start with k^4 and the equation represents the transfer function arising owing to consideration of magnetic field at three-point and three-times.

If we integrate Eq. (59) for $\Delta t=0$ over all wave numbers, we find that:

$$\int_0^{\infty}Fdk=0 \tag{60}$$

which indicates that the expression for F satisfies the condition of continuity and homogeneity. Physically it was to be expected as F is a measure of the energy transfer and the total energy transferred to all wave numbers must be zero.

The linear Eq. (57) can be solved to give:

$$E = \exp\left[-2\lambda k^2\left(t - t_o + \frac{\Delta t}{2}\right)\right.$$
$$\int F \exp\left[2\lambda\left(k^2 + \frac{R}{\lambda}\right)\left(t - t_o + \frac{\Delta t}{2}\right)\right]dt$$
$$\left.+ J(k)\exp\left[-2\lambda\left(k^2 + R/\lambda\left(t - t_o + \Delta t/2\right)\right)\right]\right] \quad (61)$$

where $J(k) = \frac{N_o k^2}{\pi}$ is a constant of integration and can be obtained as by Corrsin (1951) Substituting the values of F from Eq. (59) into (61) gives the equation:

$$E = \frac{N_o k^2}{\pi}\exp\left[-2\lambda\left(k^2 + R/\lambda\right)\left(t - t_o + \Delta t/2\right)\right]$$
$$+ \frac{\delta_o P_M\sqrt{\pi}}{4\lambda^{3/2}\left(1 + P_M\right)^{1/2}}$$
$$\times \exp\left[-\left(2\in_{mkl}\Omega_m - fs\right)\left(t - t_o\right)\right]$$
$$\exp\left[\frac{-k^2\lambda\left(1 + 2P_M\right)}{1 + P_M}\left(t - t_o + \frac{1 + P_M}{1 + 2P_M}\Delta t\right)\right.$$
$$\left.-t - t_o + \frac{\Delta t}{2}\right]\left[\frac{3k^4}{2P_M\lambda^2\left(t - t_o\right)^{5/2}}\right.$$
$$+ \frac{\left(7P_M - 6\right)k^6}{3\lambda\left(1 + P_M\right)\left(t - t_o\right)^{3/2}}$$
$$- \frac{4\left(3P_M^2 - 2P_M + 3\right)k^8}{3\left(1 + P_M\right)^2\left(t - t_o\right)^{1/2}}$$
$$\left.+ \frac{8\sqrt{\lambda}\left(3P_M^2 - 2P_M + 3\right)k^9 F(\omega)}{\left(1 + P_M\right)^{5/2} P_M^{1/2}}\right] \quad (62)$$

where, $F(\omega) = e^{-\omega^2}\int_0^\infty e^{x^2}dx$

$$\omega = k\sqrt{\frac{\lambda\left(t - t_o\right)}{1 + P_M}} \text{ or } k\sqrt{\frac{\lambda\left(t - t_o + \Delta t\right)}{1 + P_M}}.$$

By setting r^=0 , j=i, $d\hat{k} = -2\pi k^2 d\left(\cos\theta\right)d\hat{k}$ and $E = 2\pi k^2\left\langle\psi_i\psi_j'\right\rangle$ in Eq. (14) we get the expression for magnetic energy decay law as:

$$\frac{\left\langle h_i h_i'\right\rangle}{2} = \int_0^\infty E dk \quad (63)$$

Substituting Eq. (62) into (63) and after integration, we get:

$$\frac{\left\langle h_i h_i'\right\rangle}{2} = \frac{N_0}{8\sqrt{2\pi}\lambda^{3/2}\left(T + \frac{\Delta T}{2}\right)^{3/2}}$$
$$\exp\left[-2R\left(T + \frac{\Delta T}{2}\right)\right]$$
$$+ \frac{\pi\delta_0}{4\lambda^6\left(1 + P_M\right)\left(1 + 2P_M\right)^{5/2}}$$
$$\exp\left[-2R\left(T + \frac{\Delta T}{2}\right)\right]\exp\left[-\left(2\in_{mkl}\Omega_m - fs\right)\right]$$
$$\times\left[\frac{9}{16T^{5/2}\left(T + \frac{1 + P_M}{1 + 2P_M}\Delta T\right)^{3/2}}\right.$$
$$+ \frac{9}{16\left(T + \Delta T\right)^{5/2}\left(T + \frac{P_M}{1 + 2P_M}\Delta T\right)^{5/2}}$$
$$+ \frac{5P_M\left(7P_M - 6\right)}{16\left(1 + 2P_M\right)T^{3/2}\left(T + \frac{1 + P_M}{1 + 2P_M}\Delta T\right)^{7/2}}$$
$$+ \frac{5P_M\left(7P_M - 6\right)}{16\left(1 + 2P_M\right)\left(T + \Delta T\right)^{3/2}\left(T + \frac{P_M}{1 + 2P_M}\Delta T\right)^{7/2}}$$

$$+\frac{35P_M\left(3P_M^2-2P_M+3\right)}{8\left(1+2P_M\right)T^{1/2}\left(T+\frac{1+P_M}{1+2P_M}\Delta T\right)^{9/2}}$$

$$+\frac{35P_M\left(3P_M^2-2P_M+3\right)}{8\left(1+2P_M\right)\left(T+\Delta T\right)^{1/2}\left(T+\frac{P_M}{1+2P_M}\Delta T\right)^{9/2}}$$

$$+\frac{8P_M\left(3P_M^2-2P_M+3\right)\left(1+2P_M\right)^{5/2}}{3.2^{23/2}\left(1+2P_M\right)^{11/2}}$$

$$\sum_{n=0}^{\infty}\frac{1.3.5\ldots\ldots(2n+9)}{n!(2n+1)2^{2n}\left(1+P_M\right)^n}\times$$

$$\left.\left.\left\{\frac{T^{(2n+1)/2}}{\left(T+\frac{\Delta T}{2}\right)^{(2n+1)/2}}+\frac{\left(T+\Delta T\right)^{(2n+1)/2}}{\left(T+\frac{\Delta T}{2}\right)^{(2n+1)/2}}\right\}\right]\right] \quad (64)$$

where $T=t-t_0$.

For $T_m = T+\frac{\Delta T}{2}$, Eq. (64) takes the form:

$$\frac{\langle h^2\rangle}{2}=\frac{\langle h_i h_i'\rangle}{2}=\exp\left[-2RT_m\right]$$

$$\left[\frac{N_0}{8\sqrt{2\pi}\,\lambda^{3/2}T_m^{3/2}}+\right.$$

$$\frac{\pi\delta_0}{4\lambda^6\left(1+P_M\right)\left(1+2P_M\right)^{5/2}}$$

$$\exp\left[-\left(2\in_{mkl}\Omega_m-f\right)\right]$$

$$\times\left[\frac{9}{16\left(T_m-\frac{\Delta T}{2}\right)^{5/2}\left(T_m+\frac{\Delta T}{1+2P_M}\right)^{5/2}}\right.$$

$$+\frac{9}{16\left(T_m-\frac{\Delta T}{2}\right)^{5/2}\left(T_m+\frac{\Delta T}{2\left(1+2P_M\right)}\right)^{5/2}}$$

$$+\frac{5P_M\left(7P_M-6\right)}{16\left(1+2P_M\right)\left(T_m-\frac{\Delta T}{2}\right)^{3/2}\left(T_m+\frac{\Delta T}{2\left(1+2P_M\right)}\right)^{7/2}}$$

$$\left.\left.+\frac{5P_M\left(7P_M-6\right)}{16\left(1+2P_M\right)\left(T_m+\frac{\Delta T}{2}\right)^{3/2}\left(T_m-\frac{\Delta T}{2\left(1+2P_M\right)}\right)^{7/2}}+---\right]\right] \quad (65)$$

This is the decay law of magnetic energy fluctuations of concentration of a dilute contaminant undergoing a first order chemical reaction before the final period for the case of multi-point and multi-time in MHD turbulence in a rotating system in presence of dust particle.

RESULTS AND DISCUSSION

In Eq. (65) we obtained the decay law of magnetic energy fluctuations of a dilute contaminant undergoing a first order chemical reaction before the final period considering three-point correlation terms for the case of multi-point and multi-time in MHD turbulence in presence of dust particle in a rotating system.

If the fluid is non-rotating and clean then, f = 0, the Eq. (65) becomes:

$$\frac{\langle h^2\rangle}{2}=\exp\left[-2RT_m\right]$$

$$\left[\frac{N_0}{8\sqrt{2\pi}\,\lambda^{3/2}T_m^{3/2}}+\right.$$

$$\frac{\pi\delta_0}{4\lambda^6\left(1+P_M\right)\left(1+2P_M\right)^{5/2}}$$

$$\times\left[\frac{9}{16\left(T_m-\frac{\Delta T}{2}\right)^{5/2}\left(T_m+\frac{\Delta T}{1+2P_M}\right)^{5/2}}\right.$$

$$+\frac{9}{16\left(T_m-\frac{\Delta T}{2}\right)^{5/2}\left(T_m+\frac{\Delta T}{2\left(1+2P_M\right)}\right)^{5/2}}$$

$$+\frac{5P_M(7P_M-6)}{16(1+2P_M)\left(T_m-\frac{\Delta T}{2}\right)^{3/2}\left(T_m+\frac{\Delta T}{2(1+2P_M)}\right)^{7/2}}$$

$$\left.\left.+\frac{5P_M(7P_M-6)}{16(1+2P_M)\left(T_m+\frac{\Delta T}{2}\right)^{3/2}\left(T_m-\frac{\Delta T}{2(1+2P_M)}\right)^{7/2}}+---\right]\right] \quad (66)$$

which was obtained earlier by Islam and Sarker (2001).

If we put $\Delta T = 0$, $R = 0$, in Eq. (66) we can easily find out

$$\frac{\langle h^2\rangle}{2}=\frac{\langle h_i h_i'\rangle}{2}=\frac{N_0 T^{-3/2}}{8\sqrt{2\pi}\lambda^{3/2}}+\frac{\pi\delta_0}{4\lambda^6(1+P_M)(1+2P_M)^{5/2}}T^{-5}\left\{\frac{9}{16}+\frac{5}{16}\frac{P_M(7P_M-6)}{1+2P_M}+----\right\} \quad (67)$$

which is same as obtained earlier by Sarker and Kishore (1991).

This study shows that due to the effect of rotation of fluid in the flow field with chemical reaction of the first order in the concentrarion the magnetic field fluctuation in dusty fluid MHD turbulence in rotating system for the case of multi-point and multi-time i,e.the turbulent energy decays more rapidly than the energy for non-rotating clean fluid and the faster rate is governed by $\exp\left[-(2\epsilon_{mkl}\Omega_m - f\delta)\right]$. Here the chemical reaction in MHD turbulence for the case of multi-point and multi-time causes the concentration to decay more they would for non-rotating clean fluid and it is governed by $\exp\left[-(2RT_m + 2\epsilon_{mkl}\Omega_m - f\delta)\right]$. The first term of right hand side of Eq. (65) corresponds to the energy of magnetic field fluctuation of concentration for the two-point correlation and the second term represents magnetic energy for the three-point correlation. In Eq. (65), the term associated with the three-point correlation die out faster than the two-point correlation. If higher order correlations are considered in the analysis, it appears that more terms of higher power of time would be added to the Eq. (65). For large times the last term in the Eq. (65) becomes negligible, leaving the -3/2 power decay law for the final period.

ACKNOWLEDGMENT

The authors would like to express their heartiest thanks to the 'Maxwell Scientific Organization' for published this article in 'Research Journal of Mathematics and Statistics'. The authors also express their gratitude to the Department of Applied Mathematics, University of Rajshahi for providing them all facilities regarding this article.

REFERENCES

Azad, M.A.K. and M.S.A. Sarker, 2003. Decay of dusty fluid MHD turbulence before the final period in a rotating system for the case of multi-point and multi-time. Bihar Math. Soc., 23: 1-20.

Chandrasekhar, S., 1951. The invariant theory of isotropic turbulence in magneto hydrodynamics. Proc. Roy. Soc. London, A204: 435-449.

Corrsin, S., 1951. On the spectrum of isotropic temperature fluctuations in isotropic turbulence. J. Appl. Phys., 22: 469-473.

Deissler, R.G., 1958. On the decay of homogeneous turbulence before the final period. Phys. Fluid., 1: 111-121.

Deissler, R.G., 1960. Decay law of homogeneous turbulence for time before the final period. Phys. Fluid., 3: 176-187.

Funada, T., Y. Tuitiya and M. Ohji, 1978. The effect of coriolis force on turbulent motion in presence of strong magnetic field. J. Phys. Soc. Japan, 44: 1020-1028.

Islam, M.A. and M.S.A. Sarker, 2001. First order reactant in MHD turbulence before the final period of deay for the case of multi-point and multi-time. Indian J. Pure Appl. Math., 32(8): 1173-1184.

Kumar, P. and S.R. Patel, 1974. First order reactant in homogeneous turbulence before the final period for the case of multi-point and single time. Phys. Fluid., 17: 1362.

Kishore, N. and A. Sinha, 1988. Rate of change of vorticity covariance in dusty fluid turbulence. Astrophys. Space Sci., 146: 53.

Kumar, P. and S.R. Patel, 1975. First order reactant in homogeneous turbulence before the final period for the case of multi-point and multi-time. Int. J. Eng. Sci., 13: 305-315.

Loefter, A.L. and R.G. Deissler, 1961. Decay of temperature fluctuations in homogeneous turbulence before the final period. Int. J. Heat Mass Transfer, 1: 312-324.

Patel, S.R., 1976. First order reactant in homogeneous turbulence-Numerical results. Int. J. Eng. Sci., 14: 75.

Sarker, M.S.A. and N. Kishore, 1991. Decay of MHD turbulence before the final period. Int. J. Eng. Sci., 29: 1479-1485.

Sarker, M.S.A. and M.A. Islam, 2001a. Decay of MHD turbulence before the final period for the case of multi-point and multi-time. Ind. J. Pure Appl. Math., 32(7): 1065-1076.

Sarker, M.S.A. and M.A. Islam, 2001b. Decay of dusty fluid turbulence before the final period in a rotating system. J. Math. Sci., 16: 35-48.

Sinha, A., 1988. Effect of dust particles on the acceleration covariance of ordinary turbulence J. Sci. Res., B.H.U., India, 38: 7.