Research Journal of Mathematics and Statistics 2(2): 69-74, 2010
ISSN: 2040-7505
Maxwell Scientific Organization, 2010
Submitted Date: March 09, 2010 Accepted Date: March 27, 2010 Published Date: June 25, 2010

Reliability Measures for a System Having Two-dissimilar Cold Standby Units with Random Check and Priority Repair

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Abstract: In this study, it was considered a cold standby system with two dissimilar units for evaluation of its reliability parameters. These two units are named as priority unit (P-unit) and standby unit (S-unit). The main working unit is P-unit but on failure of this unit we may online S-unit through an imperfect switching device. This S-unit is not efficient to fulfill the requirements similar to P-unit. In this study, the authors have been used a random check for S-unit during its non-operation period. Mathematical model of the system has been solved with the help of Laplace transform. Head-of-line policy has been used for repair purpose. Asymptotic behaviour of system and some particular cases have been computed to improve practical utility of the model. Reliability function, availability function and MTTF of considered system have been obtained. At the last, we appended a numerical illustration together with its graphical representation to highlight important results of this study.

Key words: Asymptotic behaviour, availability function, markovian process, MTTF, reliability function, supplementary variables

INTRODUCTION

In this research, authors have analyzed a two units cold standby system (Chung, 1988) for evaluation of various reliability measures. These two units are named as priority unit (P-unit) and standby unit (S-unit). Such type of system can be seen easily in daily life. For example, we may take air conditioner as P-unit and cooler as S-unit. On failure of air conditioner we may use cooler to maintain room temperature. Cooler is not efficient as compared to air conditioner to keep the room cool and dry. On failure of air conditioner we may use cooler but it is possible that at the time of need we find that cooler is not working due to non-operation for a long period. Therefore, it is essential to check randomly the cooler during operable condition of air conditioner. Thus, this study is useful in analyzing the working behaviour of two units cold standby systems (Pandey *et al*., 1995).

Since the system under consideration is Non-Markovian, the authors have used supplementary variables (Barlow *et al*., 1965) to convert this in Markovian (Sharma *et al*., 2005a). Transition-state diagram of the system has been shown in Fig. 1. Table 1 gives details of various system states shown in Fig. 1. The following assumptions are associated with this study:

- Initially the whole system in new and operable
- Failures follow exponential time distribution and are S-independent
- Repairs follow general time distribution and are perfect
- Head-of-line policy has been used for repair purpose
- Switching device used to online S-unit is imperfect
- Random inspection of S-unit is required in its standby position
- P-unit and S-unit are dissimilar and arranged in cold standby position

MATERIALS AND METHODS

This study was conducted at Department of Mathematics, N.A.S. (PG) College, Meerut, India during May 2009. The results obtained are studied at Department of Mathematics, D.J. College of Engineering and Technology, Modinagar, Ghaziabad, India during June 2009.

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Table 1: State description of considered system. The flow of these states has shown in Fig. 1.

S.No.	State	Probability	P-unit	S-unit device	Switching	System state
1	S_1	$P_{0,S}$	Operable	Standby	Off	Operable
2	S_2	$P_{F,0}$	Failed	Operable	On	Degraded
3	S_3	$P_{F,F}$	Failed	Failed	On	Failed
4	S_4	$P_{0,C}$	Operable	Random Check	Off	Operable
5	S_5	$P_{0,F}$	Operable	Failed	Off	Operable
6	S_6	P_{SW}	Failed	Operable	Failed	Failed

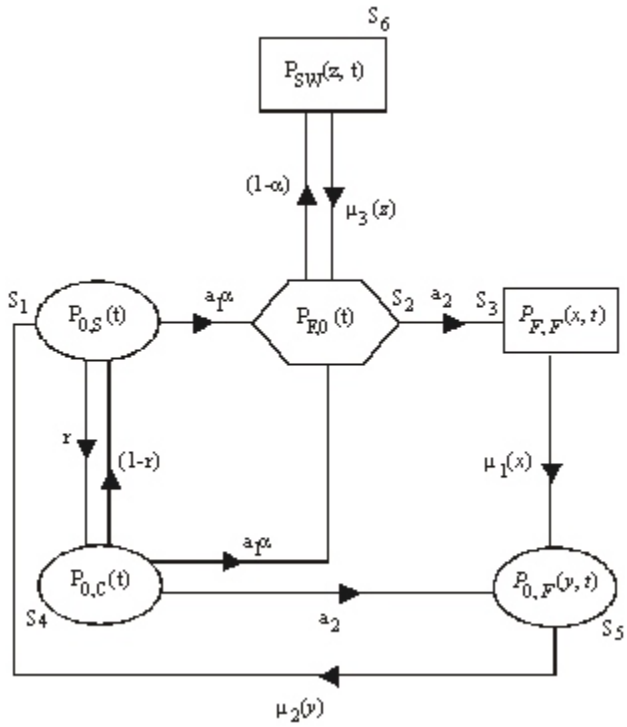

Fig. 1: Represents the transition of all possible states of considered system. The states are detailed in Table 1

In this study, the authors have been used supplementary variables technique (Gupta *et al*., 1986) to formulate mathematical model of the considered system. Various difference-differential equations have been obtained for the transition states depicted in Fig. 1. This set of difference-differential equations has solved by using Laplace transform (Nagraja *et al*., 2004). The probabilities of the system having in different transition states have computed. These results can be used to obtain various reliability parameters of the system having similar configurations.

Using probability consideration and limiting procedure, we obtain the following set of difference-differential equations, governing the behaviour of considered system, continuous in time and discrete in space:

$$\left[\frac{d}{dt}+a_1\alpha+r\right]P_{0,S}(t) = (1-r)P_{0,C}(t)+\int_0^\infty P_{0,F}(y,t)\mu_2(y)dy \tag{1}$$

$$\left[\frac{d}{dt}+a_2+(1-\alpha)\right]P_{F,0}(t) = a_1\alpha\left[P_{0,S}(t)+P_{0,C}(t)\right]+\int_0^\infty P_{SW}(z,t)\mu_3(z)dz \tag{2}$$

$$\left[\frac{\partial}{\partial x}+\frac{\partial}{\partial t}+\mu_1(x)\right]P_{F,F}(x,t)=0 \tag{3}$$

$$\left[\frac{d}{dt}+a_1\alpha+a_2+(1-r)\right]P_{0,C}(t)=rP_{0,S}(t) \tag{4}$$

$$\left[\frac{\partial}{\partial y}+\frac{\partial}{\partial t}+\mu_2(y)\right]P_{0,F}(y,t)=0 \tag{5}$$

$$\left[\frac{\partial}{\partial z}+\frac{\partial}{\partial t}+\mu_3(z)\right]P_{SW}(z,t)=0 \tag{6}$$

Boundary conditions are:

$$P_{F,F}(0,t)=a_2P_{F,0}(t) \tag{7}$$

$$P_{0,F}(0,t)=\int_0^\infty P_{F,F}(x,t)\mu_1(x)dx+a_2P_{0,C}(t) \tag{8}$$

$$P_{SW}(0,t)=(1-\alpha)P_{F,0}(t) \tag{9}$$

Initial conditions are:

$$P_{0,S}(0)=1, \text{ otherwise zero.} \tag{10}$$

Taking Laplace transforms of Eq. (1) through (9) subjected to initial conditions (10) and then on solving (Gnedenko *et al*., 1969) them one by one, we obtain:

$$\overline{P}_{0,S}(s)=\frac{1}{B(s)} \tag{11}$$

$$\overline{P}_{F,0}(s)=\frac{A(s)}{B(s)} \tag{12}$$

$$\overline{P}_{F,F}(s)=\frac{a_2A(s)D_1(s)}{B(s)} \tag{13}$$

$$\overline{P}_{0,C}(s)=\frac{M}{B(s)} \tag{14}$$

$$\overline{P}_{0,F}(s)=\frac{a_2}{B(s)}\left[M+A(s)\overline{S}_1(s)\right]D_2(s) \tag{15}$$

$$\overline{P}_{SW}(s)=\frac{(1-\alpha)A(s)D_3(s)}{B(s)} \tag{16}$$

where, $$M=\frac{r}{s+a_1\alpha+a_2+(1-r)} \tag{17}$$

$$A(s)=\frac{a_1\alpha(1+M)}{s+a_2+s(1-\alpha)D_3(s)} \tag{18}$$

and $$B(S)=s+a_1\alpha+r-(1-r)M-a_2\left[M+A(s)\overline{S}_1(s)\right]\overline{S}_2(s) \tag{19}$$

Verification: it is worth noticing that

$$P_{0,S}(s)+P_{F,0}(s)+P_{F,F}(s)+P_{0,C}(s)+P_{0,F}(s)+P_{SW}(s)=\frac{1}{s}$$

$$\Rightarrow L\left\{P_{0,S}(t)+P_{F,0}(t)+P_{F,F}(t)+P_{0,C}(t)+P_{0,F}(t)+P_{SW}(t)\right\}=L\{1\}$$

$$\Rightarrow P_{0,S}(t)+P_{F,0}(t)+P_{F,F}(t)+P_{0,C}(t)+P_{0,F}(t)+P_{SW}(t)=1 \tag{20}$$

Some particular cases are also discussed as given below:

When all repairs follow exponential time distribution: In this case, setting $\overline{S}_i(j)=\mu_i/(j+\mu_i)$, $\forall$ *i* and *j*, in Eq. (11) through (16), we obtain the following Laplace transforms of states probabilities:

$$\overline{P}_{0,S}(s) = \frac{1}{E(s)} \tag{21}$$

$$\overline{P}_{F,0}(s) = \frac{F(s)}{E(s)} \tag{22}$$

$$\overline{P}_{F,F}(s) = \frac{a_2 F(s)}{E(s)(s+\mu_1)} \tag{23}$$

$$\overline{P}_{0,C}(s) = \frac{M}{E(s)} \tag{24}$$

$$\overline{P}_{0,F}(s) = \frac{a_2}{E(s)}\left[M + F(s)\frac{\mu_1}{s+\mu_1}\right]\frac{1}{(s+\mu_2)} \tag{25}$$

$$\overline{P}_{SW}(s) = \frac{(1-\alpha)F(s)}{E(s)(s+\mu_3)} \tag{26}$$

Where, has been mentioned in equation (17),

$$F(s) = \frac{a_1\alpha(1+M)}{s+a_2+s\dfrac{(1-\alpha)}{(s+\mu_3)}} \tag{27}$$

and

$$E(s) = s+a_1\alpha+r-(1-r)M - a_2\left[M+\frac{F(s)\mu_1}{s+\mu_1}\right]\frac{\mu_2}{s+\mu_2} \tag{28}$$

When switching device used, is perfect: In this case, put in Eq. (11) through (16), we can obtain the required results.

When there is no random check: In this case, put $r = 0$ in Eq. (11) through (16), we can obtain the required results.

Reliability and M.T.T.F. analysis: We have from Eq. (11),

$$\overline{R}(s) = \frac{1}{s+a_1\alpha+r}$$

$$\therefore \quad R(t) = L^{-1}\left\{\overline{R}(s)\right\} = \exp\left\{-(a_1\alpha+r)t\right\} \tag{29}$$

Also, M.T.T.F. $= \lim_{s\to 0} \overline{R}(s)$

$$= \frac{1}{a_1\alpha+r} \tag{30}$$

Availability of considered system: We obtain from Eq. (11) and (12):

$$\overline{P}_{up}(s) = \frac{1}{s+a_1\alpha+r}\left[1+\frac{a_1\alpha}{s+a_2+1-\alpha}\right]$$

$$\therefore P_{up}(t) = L^{-1}\left\{\overline{P}_{up}(s)\right\}$$

$$= \left[1+\frac{a_1\alpha}{a_2+(1-\alpha)-a_1\alpha-r}\right]\exp\left\{-(a_1\alpha+r)t\right\} - \frac{a_1\alpha}{a_2+(1-\alpha)-a_1\alpha-r}\exp\left\{-(a_2+1-\alpha)t\right\} \tag{31}$$

Also, $P_{down}(t) = 1 - P_{up}(t)$ (32)

Numerical illustration: For a numerical illustration, let us consider the values:
$a_1 = 0.003$, $a_2 = 0.008$, $r = 0.3$, $\alpha = 0.6$ and $t = 0, 1, 2, ..., 12$. Using these values in Eq. (29), (30) and (31) we compute in Fig. 2-4, respectively.

RESULTS AND DISCUSSION

In this paper, we have evaluated reliability function; availability function and mean time to failure of considered two units cold standby system under random check. Also, we have computed some particular cases to enhance practical utility of the system. The authors have considered a numerical example of practical problem and used the results obtained in this study. The results obtained have been shown below in the Fig. 2-4.

Pandey *et al.* (1995) and Sharma and Sharma (2005b) have evaluated the reliability of complex redundant systems but no care was given to random check to non-priority unit and therefore the results obtained in this study are much better than previous one. By making the random checks to non-priority unit, we have done a better analysis of practical situations.

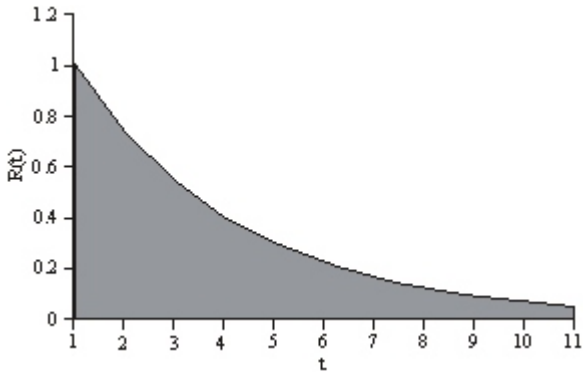

Fig. 2: The way reliability of the considered system decreases with the increase in time

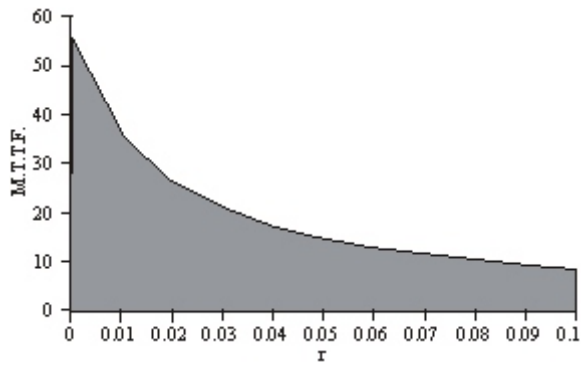

Fig. 3: The way mean time to failure of the considered system decreases with the increase in failure rate r

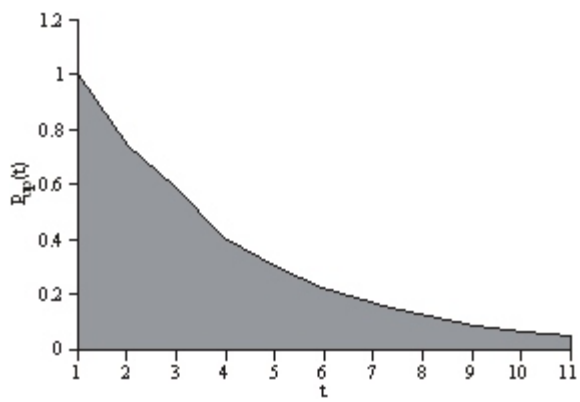

Fig. 4: The way reliability of the considered system decreases with the increase in time

Figure 2 shows the values of reliability function w.r.t. time t. Analysis of Figure 3 reveals that reliability of considered system decreases approximately in constant manner. Ther are no sudden jumps in the values of reliability function.

Figure 3 shows the values of MTTF w.r.t. failure rate r. This graph represents that MTTF of considered system decreases catastrophically in the beginning but thereafter it decreases smoothly.

Figure 4 shows the values of availability function w.r.t. time t. This shows that availability of the considered system decreases constantly with increase in time.

CONCLUSION

In conclusion, we observed that we could improve system's overall performance by using the random checks to non-priority unit. By using cold standby and random check, reliability, availability and mean time to failure remains better as compared to simple system. This study is very useful for the practical systems having similar configurations. The results obtained in this study can be directly implemented to similar systems.

NOTATIONS

The list of notations used throughout this study, is as follows:

a_1, a_2	:	Failure rates of P-unit and S-unit, respectively.
α	:	Successful operational rate of switching device.
r	:	Rate of random check of S-unit.
$\mu_3(z)$	:	Repair rate of switching device.
$\mu_1(x)\Delta / \mu_2(y)\Delta$	:	First order probability that P-units/S-unit can be repaired in time interval $(x, x+\Delta)/(y, y+\Delta)$, conditioned that it was not repaired upto time x/y.
$P_{0,S}(t)$	:	Pr {at time t, the P-unit is operable and S-unit is kept as standby}.
$P_{F,0}(t)$	:	Pr {at time t, the P-unit is failed and S-unit is operable}.
$P_{SW}(t)$	:	Pr {at time t, switching device has failed}.
$P_{0,C}(t)$	:	Pr {at time t, P-unit is operable and S-unit is going through random check}.
$P_{0,F}(y, t)$	:	Pr {at time t, S-unit is failed in duration of its random check}. Elapsed repair time lies in the interval.
$P_{F,F}(x, t)$	:	Pr {at time t, both units have failed}. Elapsed repair time lies in the interval $(y, y+\Delta)$.
$\overline{P}(S)$	:	Laplace transform (L.T.) of function $P(t)$.

$S_i(j)$: $r_i(j)\exp\left\{-\int r_i(j)d_j\right\}$

$D_i(j)$: $1-\overline{S}_i(j)/j$

G (t) : Cost function of considered system.

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