Research Journal of Mathematics and Statistics 2(2): 75-80, 2010
ISSN: 2040-7505

Submitted Date: March 12, 2010 Accepted Date: April 30, 2010 Published Date: June 25, 2010

Measurement of Cost Function for a Nuclear Power Plant with Parallel Redundant Reactor Vessel and Head-of-Line Repair

[1]Deepankar Sharma, [2]Jitendra Kumar Rana and [1]Sumit Gupta
[1]Department of Mathematics, D.J. College of Engineering and Technology, Modinagar, Ghaziabad, India
[2]Department of Mathematics, NAS (PG) College, Meerut

Abstract: In this research, it has studied about availability estimation of nuclear power generation plant by using the concept of parallel redundancy and head-of-line repair discipline. The supplementary variables have been used for mathematical formulation of the model. Laplace transforms is being utilize to solve the mathematical equations. Some particular cases and asymptotic behaviour of the system have also been derived to improve practical importance of the model. Expressions for availability and cost function have been computed. A numerical example together with its graphical illustration has been appended in the end to highlight important results.

Key words: Laplace transform, markovian process, supplementary variables

INTRODUCTION

Over 16% of the world's electricity is produced from nuclear energy, more than from all sources worldwide in 1960. A nuclear reactor produces and controls the release of energy from splitting the atoms of elements such as uranium and plutonium. In a nuclear power reactor, the energy released from continuous fission of the atoms in the fuel as heat is used to make steam. The steam is used to drive the turbines, which produce electricity (as in most fossil fuel plants). There are several components common to most types of reactors:

Fuel: Usually pellets of uranium oxide (UO_2) arranged in tubes to form fuel rods. The rods are arranged into fuel assemblies in the reactor core.

Moderator: This is material, which slows down the neutrons released from fission so that they cause more fission. It is usually water, but may be heavy water or graphite.

Control rods: These are made with neutron-absorbing material such as cadmium, hafnium or boron, and are inserted or withdrawn from the core to control the rate of reaction, or to halt it. (Secondary shutdown systems involve adding other neutron absorbers, usually in the primary cooling system).

Coolant: A liquid or gas circulating through the core so as to transfer the heat from it. In light water reactors the moderator functions also as coolant.

Pressure vessel or pressure tubes: Usually a robust steel vessel containing the reactor core and moderator/coolant, but it may be a series of tubes holding the fuel and conveying the coolant through the moderator.
Steam generator: Part of the cooling system where the heat from the reactor is used to make steam for the turbine.

Since, the system under consideration is Non-Markovian, the supplementary variables have been used for mathematical formulation of the model. Nuclear reactor, system configuration has been shown in Fig. 1 and 2 respectively. The whole power plant has been divided into four subsystems namely A, B, C and D. The subsystem A is reactor vessel and it creates heat energy through fissioning of atoms. This energy goes to subsystem B, through coolant. This subsystem B is a heat exchanger and converts the heat into steam. Now this steam moves to subsystem C, a turbine, and starts to rotate it. This subsystem C is connected with generator (subsystem D), which products electric power on rotating of turbine. In last, electric energy produces by generator, can be stored for further utilization. In this model, the author has taken one parallel redundant reactor vessel. So, the subsystem A has two parallel redundant units A_1 and A_2. On failure of main unit A_1, system works with unit A_2. The whole system gets fail if any of its subsystems stop working. All failures follow exponential time distribution (Lai *et al.*, 2005) whereas all repairs follow general time distribution. Head-of-line policy has been adopted for repair purpose. State- transition diagram has been shown in Fig. 3.

Corresponding Author: Deepankar Sharma, Department of Mathematics, D.J. College of Engineering and Technology, Modinagar, Ghaziabad, India

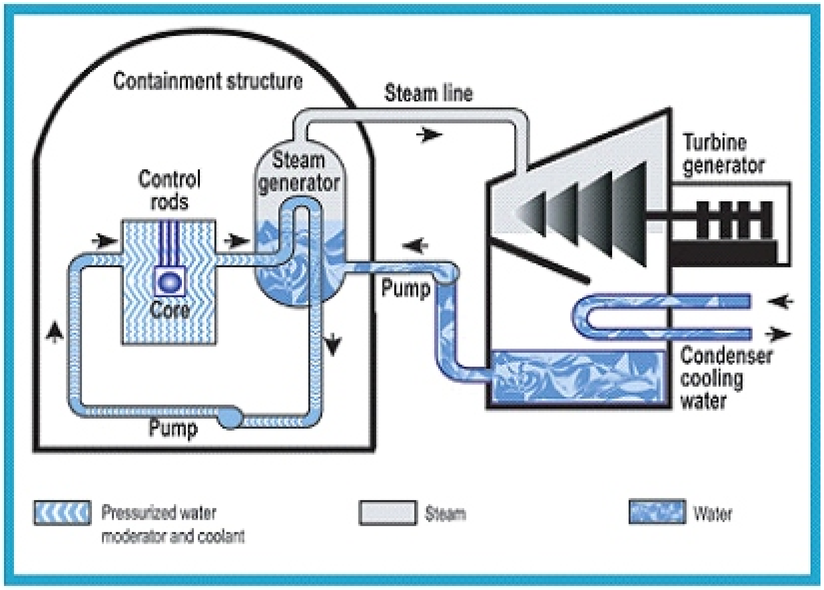

Fig. 1: Represents the block diagram of nuclear reactor

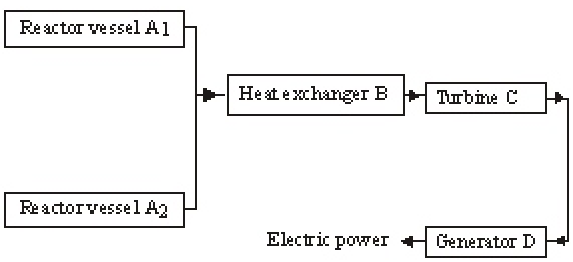

Fig. 2: Represents the system configuration of nuclear reactor

The following assumptions have been associated with this study:

- Initially, the whole system is new and operable.
- All Failures follow exponential time distribution and are S-independent.
- All Repairs follow general time distribution and are perfect.
- On failure of any one unit of subsystem A, the whole system works in reduced efficiency state.
- Head-of-line policy has been adopted for repair purpose.

MATERIALS AND METHODS

This study was conducted at Dept. of Mathematics, N.A.S. (PG) College, Meerut, India during May 2009. The results obtained are studied at Department. of Mathematics, D.J. College of Engineering and Technology, Modinagar, Ghaziabad, India during June 2009.

In this study, the authors have been used supplementary variables technique (Gupta and Agarwal, 1983) to formulate mathematical model of the considered system. Various difference-differential equations have been obtained for the transition states shown in Fig. 3. This set of difference-differential equations has solved by using Laplace transform. The probabilities of the system having in different transition states have computed. These results can be used to obtain various reliability parameters of the system having similar configurations.

Using elementary probability considerations and limiting procedure (Gupta and Agarwal, 1983), we obtain the following set of difference-differential equations governing the behaviour of considered system, continuous in time and discrete in space:

$$\left[\frac{d}{dt} + f_A + f_B + f_C + f_D\right] P_0(t) = \int_0^\infty P_{A_1}(m,t)\, r_{A_1}(m)\, dm + \int_0^\infty P_B(x,t)\, r_B(x)\, dx + \int_0^\infty P_C(y,t)\, r_C(y)\, dy + \int_0^\infty P_D(z,t)\, r_D(z)\, dz + \int_0^\infty P_{A_2}(n,t)\, r_{A_2}(n)\, dn \quad (1)$$

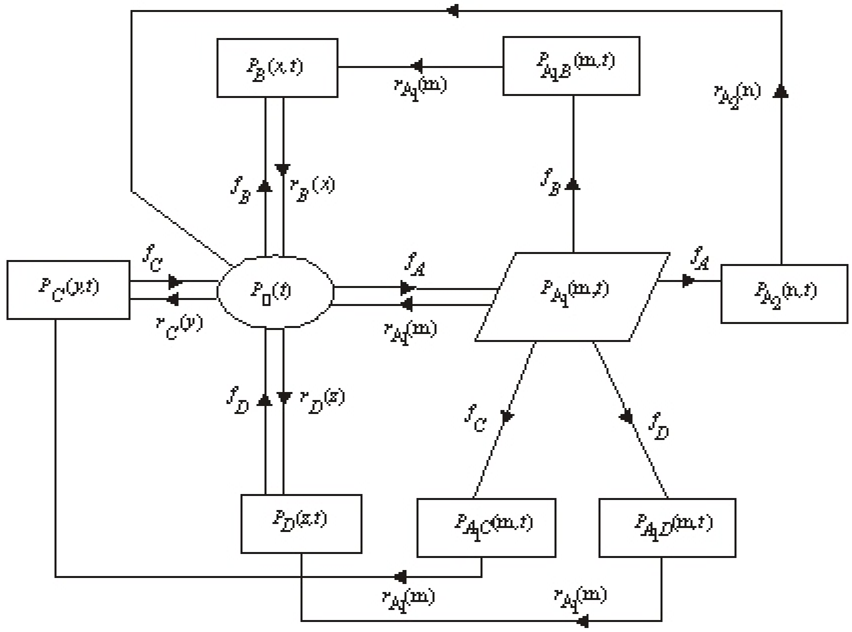

Fig. 3: Represents the transition of all possible states of considered system

$$\left[\frac{\partial}{\partial j}+\frac{\partial}{\partial t}+r_i(j)\right]P_i(j,t)=0 \tag{2}$$

where; $i = B, C, D$ and j = x, y, z respectively

$$\left[\frac{\partial}{\partial m}+\frac{\partial}{\partial t}+f_A+f_B+f_C+f_D+r_{A_1}(m)\right]P_{A_1}(m,t)=0 \tag{3}$$

$$\left[\frac{\partial}{\partial m}+\frac{\partial}{\partial t}+r_{A_1}(m)\right]P_{A_{1i}}(m,t)=f_iP_{A_1}(m,t), \quad i = B, C \text{ and } D \tag{4}$$

$$\left[\frac{\partial}{\partial n}+\frac{\partial}{\partial t}+r_{A_2}(n)\right]P_{A_2}(n,t)=0 \tag{5}$$

Boundary conditions are:

$$P_i(0,t)=f_iP_0(t)+\int_0^\infty P_{A_{1i}}(m,t)r_{A_1}(m)dm \quad \forall i = B, C \text{ and } D \tag{6}$$

$$P_{A_1}(0,t)=f_AP_0(t) \tag{7}$$

$$P_{A_{1i}}(0,t)=0, \qquad \forall i = B, C \text{ and } D \tag{8}$$

$$P_{A_2}(0,t)=f_AP_{A_1}(t) \tag{9}$$

Initial conditions are:

$$P_0(0)=1, \text{ Otherwise zero at } t=0 \tag{10}$$

Taking Laplace transforms of Eq. (1) through (9) by the use of initial conditions (10) and then on solving them one by one, we obtain

$$\bar{P}_0(s)=\frac{1}{F(s)} \tag{11}$$

$$\bar{P}_i(s)=\frac{f_iA(s)D_i(s)}{F(s)}, \quad \forall i = B, C \text{ and } D \tag{12}$$

$$\bar{P}_{A_1}(s)=\frac{f_AD_{A_1}(K)}{F(s)} \tag{13}$$

$$\bar{P}_{A_{1i}}(s)=\frac{f_if_A\left[D_{A_1}(s)-D_{A_1}(K)\right]}{(K-s)F(s)}, \quad \forall i = B, C \text{ and } D \tag{14}$$

$$\overline{P}_{A_2}(s) = \frac{f_A^2 D_{A_1}(K) D_{A_2}(s)}{F(s)} \tag{15}$$

where, $K = s + f_A + f_B + f_C + f_D$ (16)

$$A(s) = 1 + \frac{f_A}{K-s}\left[\overline{S}_A(s) - \overline{S}_{A_1}(K)\right] \tag{17}$$

and

$$F(s) = K - A(s)\left[f_B\overline{S}_B(s) + f_C\overline{S}_C(s) + f_D\overline{S}_D(s)\right] - f_A^2 D_{A_1}(K)\overline{S}_{A_2}(s) - f_A\overline{S}_{A_1}(K) \tag{18}$$

It is worth noticing that

Sum of Eq. (11) through (15) = $\frac{1}{s}$ (19)

Steady-state behaviour of considered system: Using final value theorem in Laplace transform (Cluzeau *et al*, 2008), viz, $\underset{t\to\infty}{Lim} P(t) = \underset{s\to 0}{Lim}\, s\overline{P}(s) = P(say)$, provided the limit on left exists, in Eq. (11) through (15), we obtain the following steady-state behaviour of considered system:

$$P_0 = \frac{1}{F'(0)} \tag{20}$$

$$P_i = \frac{f_i A(0) M_i}{F'(0)}, \quad \forall i = B, C \text{ and } D \tag{21}$$

$$P_A = \frac{f_A D_{A_1}(K-s)}{F'(0)} \tag{22}$$

$$P_{A_1} = \frac{f_i f_A\left[M_{A_1} - D_{A_1}(K-s)\right]}{(K-s)F'(0)}, \quad \forall i = B, C \text{ and } D \tag{23}$$

$$P_{A_2} = \frac{f_A^2 D_{A_1}(K-s) M_{A_1}}{F'(0)} \tag{24}$$

where, $F'(0) = \left[\frac{d}{ds}F(s)\right]_{s=0}$

$M_i = -\overline{S}_i'(0) =$ mean time to repair i^{th} failure and

$$A(0) = 1 + f_A D_{A_1}(K-s)$$

Particular case:
When all repairs follow exponential time distribution: In this case, setting $\overline{S}_i(s) = r_i/(s+r_i), \forall i$ in Eq. (11) through (15), we obtain the following Laplace transforms of various state probabilities of Fig. 3:

$$\overline{P}_0(s) = \frac{1}{E(s)} \tag{25}$$

$$\overline{P}_i(s) = \frac{f_i A_1(s)}{E(s)(s+r_i)}, \quad \forall i = B, C \text{ and } D \tag{26}$$

$$\overline{P}_A(s) = \frac{f_A}{E(s)(K+r_A)}, \tag{27}$$

$$\overline{P}_{A_1}(s) = \frac{f_i f_A}{(k-s)E(s)}\left[\frac{1}{s+r_A} - \frac{1}{K+r_{A_1}}\right], \quad \forall i = B, C \text{ and } D \tag{28}$$

and $$\overline{P}_{A_2}(s) = \frac{f_A^2}{E(s)(K-r_{A_1})(s-r_{A_2})} \tag{29}$$

where, $$A_1(s) = 1 + \frac{f_A r_{A_1}}{(s+r_{A_1})(K+r_{A_1})} \tag{30}$$

and $$K + A_1(s)\left[\frac{f_B r_B}{s+r_B} + \frac{f_C r_C}{s+r_C} + \frac{f_D r_D}{s+r_D}\right] - \frac{f_A^2 r_{A_2}}{(K+r_{A_1})(s+r_{A_1})} - \frac{f_A r_{A_1}}{K+r_A} \tag{31}$$

Availability of considered system: Laplace transform of availability of considered system is given by:

$$\overline{P}_{up}(s) = \overline{P}_0(s) + \overline{P}_{A_1}(s) = \frac{1}{s+f_A+f_B+f_C+f_D}\left[1 + \frac{f_A}{s+f_A+f_B+f_C+f_D}\right]$$

Taking inverse Laplace transform, we have:

$$P_{up}(t) = (1 + tf_A)\exp\{-(K-s)t\} \quad (32)$$

Cost function of considered system: Let C_1 be the revenue per unit up time and C_2 be the repair cost per unit time, then cost function for the considered system is given by:

$$G(t) = C_1\int_0^t P_{up}(t)dt - C_1 t \quad (33)$$

where,

$$\int_0^t P_{up}(t)dt = \left(1 + \frac{f_A}{K-s}\right) \frac{1-e^{-(K-s)t}}{(K-s)} - \frac{f_A t}{(K-s)}e^{-(K-s)t} \quad (34)$$

Numerical computation: For a numerical computation, let us consider the values:

$$f_A = 0.002, f_B = 0.006, f_C = 0.001,$$
$$f_D = 0.004, C_1 = Rs.7.00, C_2 = Rs.2.00,$$
$$and\, t = 0,1,2,---10$$

Using this, we obtain the graphs shown in Fig. 4 and 5, respectively.

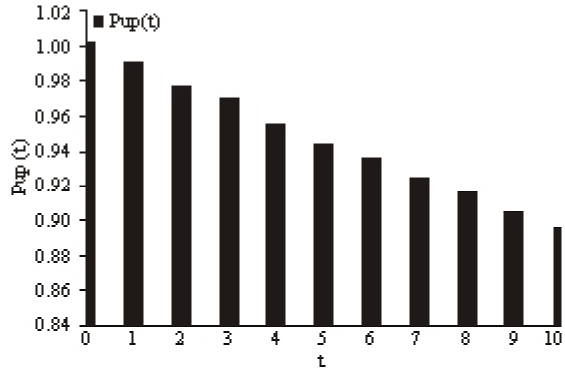

Fig. 4: The way availability of considered system decreases with the increase in time

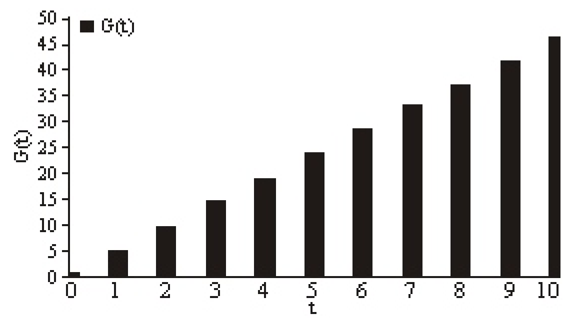

Fig. 3: The way profit function of considered system increases with the increase in time

RESULTS AND DISCUSSION

In this study, we have evaluated availability and cost functions (Gupta and Agarwal, 1983) have for the considered system by employing Supplementary variables technique (Tian *et al.*, 2008). Also, we have computed asymptotic behavior and a particular case to improve practical utility of the system. Gupta and Agarwal (1983) have done the cost analysis of redundant complex system but no care was given to switching over device. On failure of switching device, the whole system can also fail. So, for the better analysis we must have to consider the concept of imperfect switching. Thus we have done some better analysis of considered system of practical utility.

Figure 4 shows the value of availability function at different time points. Analysis of Fig. 4 reveals that availability of considered system decreases approximately in constant manner with the increase in time.

Figure 5 shows the value of cost function at various time points. Critical examination of Fig. 5 yields that cost function for system increases constantly with time and after a long period it become stationary.

CONCLUSION

In conclusion, we observe that we could improve system's overall performance by using the concept of redundancy. Also, for accuracy of reliability parameters (Zhimin *et al.*, 2005) we must have to consider the concept of priority repairs. By using the structural redundancy, we obtain the better values of availability and cost function for considered system.

NOTATIONS

f_i : Failure rate of subsystem i, where i=A,C,D and E

$r_i(j)\Delta$: First order probability that i^{th} failure can be repaired within time interval $(j, j+\Delta)$, conditioned that it was not repaired up to the time j.

$P_0(t)$: Pr {System is all operable at time t}.

$P_A(m,t)\Delta$: Pr {System is operable through A_2 unit while unit A_1 has already failed}. Elapsed repair time lies in the interval $(m, m+\Delta)$.

$P_i(j, t)\Delta$: Pr {System is failed due to failure of subsystem i}. Elapsed repair time lies in the interval $(j, j+\Delta)$.

$P_{A_1}(j,t)\Delta$: Pr {System is failed due to failure of subsystem i while unit A_1 has already failed}. Elapsed repair time for subsystem i lies in the interval $(j, j+\Delta)$.

$\bar{P}(s)$: Laplace transform of function $P(t)$.

$S_i(j)$: $r_i(j)\exp.\left\{-\int r_i(j)dj\right\}, \quad \forall i \text{ and } j$

$D_i(s)$: $\left[1-\bar{S}_i(s)\right]/s, \quad \forall i$

M_i : $-\bar{S}_i'(0) =$ Mean time to repair i^{th} failure

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