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Cost Analtsis for a Nuclear Power Plant with Standby Redundant Reactor Vessel

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Abstract: In this study, the authors have studied about cost estimation of nuclear power generation plant. Supplementary variables have been used for mathematical formulation of the model. Laplace transforms is being utilize to solve the mathematical equations. Some particular cases and asymptotic behaviour of the system have also been derived to improve practical importance of the model. Expressions for availability and cost function have been computed. A numerical example together with its graphical illustration has been appended at the end to highlight important results.

Key words: Asymptotic behaviour, availability function, markovian process, MTTF, reliability function, supplementary variables

INTRODUCTION

Over 16% of the world's electricity is produced from nuclear energy, more than from all sources worldwide in 1960. A nuclear reactor produces and controls the release of energy from splitting the atoms of elements such as uranium and plutonium. In a nuclear power reactor, the energy released from continuous fission of the atoms in the fuel as heat is used to make steam. The steam is used to drive the turbines, which produce electricity (as in most fossil fuel plants). There are several components common to most types of reactors:

Fuel: Usually pellets of uranium oxide (UO_2) arranged in tubes to form fuel rods. The rods are arranged into fuel assemblies in the reactor core.

Moderator: This is material, which slows down the neutrons released from fission so that they cause more fission. It is usually water, but may be heavy water or graphite.

Control rods: These are made with neutron-absorbing material such as cadmium, hafnium or boron, and are inserted or withdrawn from the core to control the rate of reaction, or to halt it. (Secondary shutdown systems involve adding other neutron absorbers, usually in the primary cooling system).

Coolant: A liquid or gas circulating through the core so as to transfer the heat from it. In light water reactors the moderator functions also as coolant.

Pressure vessel or pressure tubes: Usually a robust steel vessel containing the reactor core and moderator/coolant, but it may be a series of tubes holding the fuel and conveying the coolant through the moderator.

Steam generator: Part of the cooling system where the heat from the reactor is used to make steam for the turbine.

In this research, the authors have studied about cost estimation (Nagraja *et al.*, 2004) of nuclear power generation plant. Nuclear reactor, system configuration has been shown in Fig 1 and 2, respectively. The whole power plant has been divided into four subsystems namely A, B, C, D and E. The subsystem A is reactor vessel and it creates heat energy through fissioning of atoms. This energy goes to subsystem C, through coolant. This subsystem C is a heat exchanger and converts the heat into steam. Now this steam moves to subsystem D, a turbine, and starts to rotate it. This subsystem D is connected with generator (Subsystem E), which products electric power on rotating of turbine. In last, electric energy produces by generator, can be stored for further utilization. In this model, the authors have taken one standby redundant reactor vessel. So, the subsystem A has two standby redundant units A_1 and A_2 (Chung, 1988). On failure of main unit A_1, we can online standby unit A_2 through a switching device B. The whole system gets fail if any of its subsystems stop working. All failures follow exponential time distribution whereas all repairs follow general time distribution. State- transition diagram has been shown in Fig. 3.

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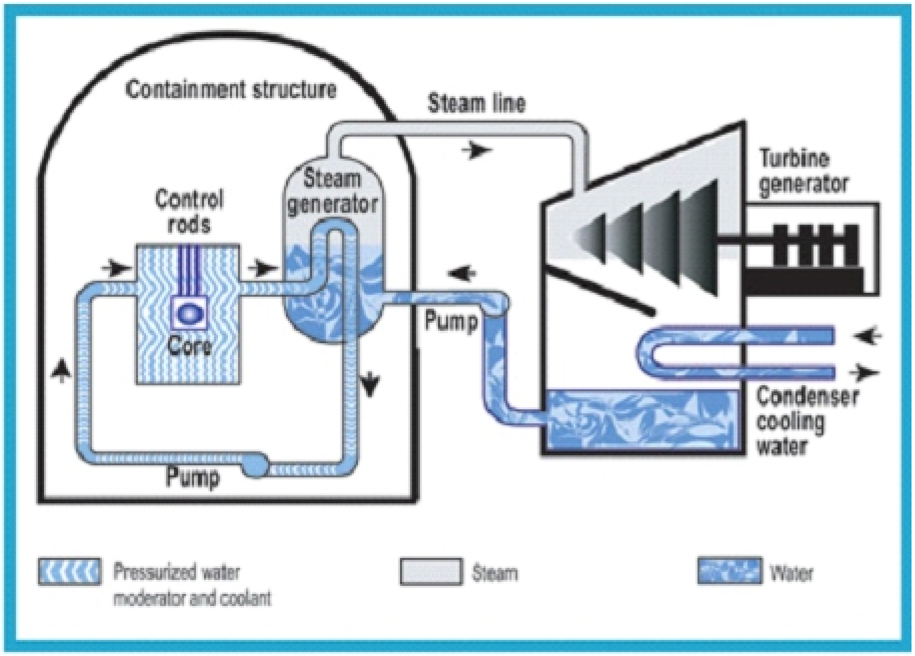

Fig. 1: Represents the Block Diagram of Nuclear Reactor

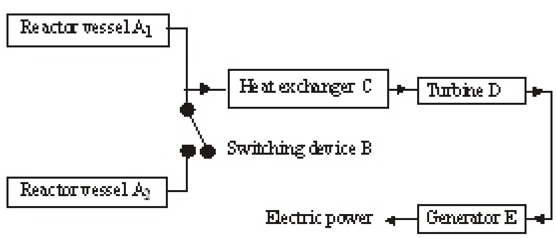

Fig. 2: Represents the system configuration of Nuclear Reactor

The following assumptions have been associated with this study:

- Initially, the whole system is new and operable.
- All Failures follow exponential time distribution and are S-independent.
- All Repairs follow general time distribution and are perfect.
- Switching device used to online standby unit of subsystem A is imperfect.
- The subsystem A can be repaired after complete failure.

MATERIAL AND METHODS

This study was conducted at Dept. of Mathematics, N.A.S. (PG) College, Meerut, India during May 2009. The results obtained are studied at Dept. of Mathematics, D.J. College of Engineering and Technology, Modinagar, Ghaziabad, India during June 2009.

In this study, the authors have been used supplementary variables technique (Gupta and Gupta, 1986) to formulate mathematical model of the considered system. Various difference-differential equations (Barlow and Proschan, 1965) have been obtained for the transition states depicted in Fig. 3. This set of difference-differential equations has solved by using Laplace transform (Nagraja *et al.*, 2004). The probabilities of the system having in different transition states (Gnedenko *et al.*, 1969) have computed. These results can be used to obtain various reliability parameters of the system having similar configurations.

Using elementary probability considerations and limiting procedure, we obtain the following set of difference-differential equations governing the behaviour of considered system, continuous in time and discrete in space:

$$\left[\frac{d}{dt} + \alpha f_A + f_C + f_D + f_E\right] P_0(t)$$

$$= \mu P_B(t) + \int_0^\infty P_A(m,t) r_A(m) dm$$

$$+ \int_0^\infty P_C(x,t) r_C(x) dx + \int_0^\infty P_D(y,t) r_D(y) dy$$

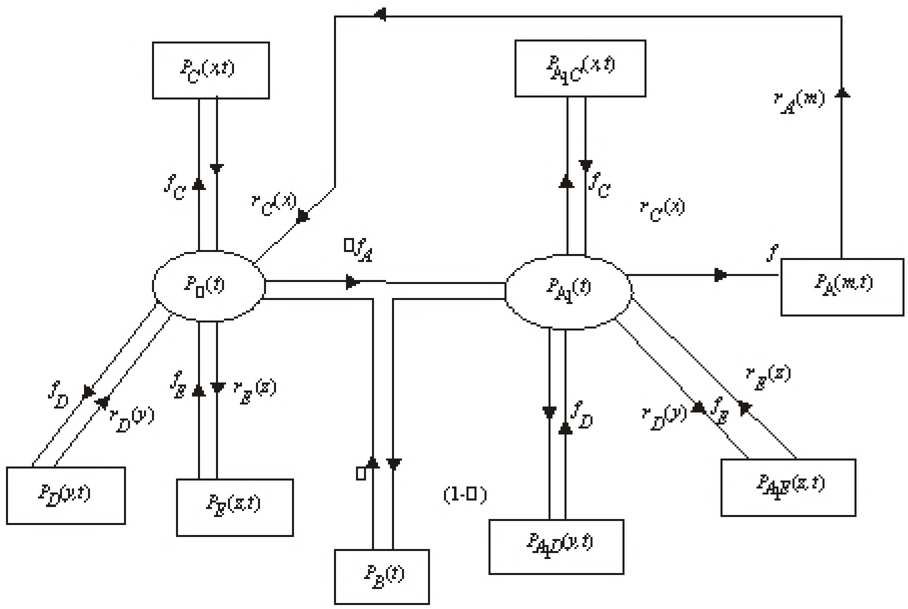

Fig. 3: Represents the transition of all possible states of considered system

$$+\int_0^\infty P_E(z,t)r_E(z)dz \tag{1}$$

$$\left[\frac{\partial}{\partial x}+\frac{\partial}{\partial t}+r_C(x)\right]P_C(x,t)=0 \tag{2}$$

$$\left[\frac{\partial}{\partial y}+\frac{\partial}{\partial t}+r_D(x)\right]P_D(y,t)=0 \tag{3}$$

$$\left[\frac{\partial}{\partial z}+\frac{\partial}{\partial t}+r_E(x)\right]P_E(z,t)=0 \tag{4}$$

$$\left[\frac{d}{dt}+f_A+f_C+f_D+f_E+(1-\alpha)\right]P_{A_1}(t)$$

$$=\alpha f_A P_0(t)+\int_0^\infty P_{A_1C}(x,t)r_C(x)dx$$

$$+\int_0^\infty P_{A_1D}(y,t)r_D(y)dy+\int_0^\infty P_{A_1E}(z,t)r_E(z)dz \tag{5}$$

$$\left[\frac{d}{dt}+\mu\right]P_B(t)=(1-\alpha)P_{A_1}(t) \tag{6}$$

$$\left[\frac{\partial}{\partial x}+\frac{\partial}{\partial t}+r_C(x)\right]P_{A_1C}(x,t)=0 \tag{7}$$

$$\left[\frac{\partial}{\partial y}+\frac{\partial}{\partial t}+r_D(y)\right]P_{A_1D}(y,t)=0 \tag{8}$$

$$\left[\frac{\partial}{\partial z}+\frac{\partial}{\partial t}+r_E(z)\right]P_{A_1E}(z,t)=0 \tag{9}$$

$$\left[\frac{\partial}{\partial m}+\frac{\partial}{\partial t}+r_A(m)\right]P_A(m,t)=0 \tag{10}$$

Boundary conditions are:

$$P_C(0,t)=f_C P_0(t) \tag{11}$$

$$P_D(0,t)=f_D P_0(t) \tag{12}$$

$$P_E(0,t) = f_E P_0(t) \tag{13}$$

$$P_{AC}(0,t) = f_C P_A(t) \tag{14}$$

$$P_{AD}(0,t) = f_D P_A(t) \tag{15}$$

$$P_{AE}(0,t) = f_E P_A(t) \tag{16}$$

$$P_A(0,t) = f_A P_A(t) \tag{17}$$

Initial conditions are:

$P_0(0) = 1$, and all other state probabilities are zero at t = 0 (18)

Taking Laplace transforms of Eq. (1) through (17) subjected to initial conditions (18) and then on solving them one by one; we obtain the following:

$$\overline{P}_0(s) = \frac{1}{F(s)} \tag{19}$$

$$\overline{P}_i(s) = \frac{f_i D_i(s)}{F(s)} \quad i = \text{C, D and E} \tag{20}$$

$$\overline{P}_A(s) = \frac{K}{F(s)} \tag{21}$$

$$\overline{P}_B(s) = \frac{(1-\alpha)K}{(s+\mu)F(s)} \tag{22}$$

$$\overline{P}_A(s) = \frac{f_i K D_i(s)}{F(s)} \quad i = \text{C, D and E} \tag{23}$$

$$\overline{P}_A(s) = \frac{f_A K D_A(s)}{F(s)} \tag{24}$$

where,

$$K = \frac{\alpha f_A}{s + f_A + f_C + f_D + f_E} \times \frac{1}{(1-\alpha) - f_C\overline{S}_C(s) - f_D\overline{S}_D(s) - f_E\overline{S}_E(s)} \tag{25}$$

$$D_i(s) = \frac{1-\overline{S}_i(s)}{s}, \quad \forall i = C, D and E$$

and

$$\begin{aligned} F(s) = s + \alpha f_A + f_C + f_D + f_E - \frac{(1-\alpha)K\mu}{s+\mu} \\ - f_A K\overline{S}_A(s) - f_C\overline{S}_C(s) \\ - f_D\overline{S}_D(s) - f_E\overline{S}_E(s) \end{aligned} \tag{27}$$

It is worth noticing that sum of Eq. (19) through 24 = $\frac{1}{s}$

Asymptotic behaviour of the system: Using final value theorem in Laplace transform, viz, $\underset{t\to\infty}{Lim}\, P(t) = \underset{s\to 0}{Lim}\, sP(s) = P(say)$, provided the limit on left exists, in Eq. (19) through (24), we obtain the following asymptotic behaviour of considered system:

$$P_0 = \frac{1}{F'(0)} \tag{28}$$

$$P_i = \frac{f_i M_i}{F'(0)}, \quad \forall i = C, D and E \tag{29}$$

$$P_A = \frac{T}{F'(0)} \tag{30}$$

$$P_B = \frac{(1-\alpha)T}{\mu.\, F'(0)} \tag{31}$$

$$P_{Ai} = \frac{f_i T.\, M_i}{F'(0)}, \quad \forall i = C, D and E \tag{32}$$

$$P_A = \frac{f_A T.\, M_A}{F'(0)} \tag{33}$$

where $M_i = -\overline{S}_i'(0) =$ mean time to repair i^{th} subsystem:

$$F'(0) = \left[\frac{d}{ds}F(s)\right]_{s=0}$$

and

$$T = \frac{\alpha f_A}{f_A + (1-\alpha)} \tag{34}$$

A particular case is also discussed as given below:

When all repairs follow exponential time distribution: In this case, setting $\overline{S}_i(s) = r_i / (s + r_i),\ \forall i$ in Eq. (19) through (24), we can obtain the Laplace transforms of various state probabilities of Fig. 3.

Availability of considered system: Laplace transform of availability of considered system is given by:

$$\overline{P}_{up}(s) = \overline{P}_0(s) + \overline{P}_{A_1}(s)$$

or,

$$\overline{P}_{up}(s) = \frac{1}{s + \alpha f_A + f_C + f_D + f_E} \times \left[1 + \frac{\alpha f_A}{s + f_A + f_C + f_D + f_E + (1-\alpha)}\right]$$

Taking inverse Laplace transforms, we obtain:

$$P_{up}(t) = \left(1 + \frac{\alpha f_A}{(1+\alpha)(1+f_A)}\right) e^{-(\alpha f_A + f_C + f_D + f_E)t} - \frac{\alpha f_A}{(1+\alpha)(1+f_A)} e^{-(f_A + f_C + f_D + f_E + 1 - \alpha)t} \tag{35}$$

Cost function of the system: Let C_1 be the revenue per unit up time and C_2 be the repair cost per unit time, then cost function for the considered system is given by:

$$G(t) = C_1 \int_0^t P_{up}(t)\,dt - C_2 t \tag{36}$$

where,

$$\int_0^t P_{up}(t)\,dt = (1+U)\frac{1 - e^{-(\alpha f_A + f_C + f_D + f_E)t}}{\alpha f_A + f_C + f_D + f_E} - U\frac{1 - e^{-(\alpha f_A + f_C + f_D + f_E + 1 - \alpha)t}}{f_A + f_C + f_D + f_E + (1-\alpha)} \tag{37}$$

Where; $U = \dfrac{\alpha f_A}{(1-\alpha)(1+f_A)}$

Numerical illustration: For a numerical computation, let us consider the values:

$\alpha = 0.7,\ f_A = 0.002,\ f_C = 0.004,\ f_D = 0.001,$

$f_E = 0.006,\ C_1 = Rs.7.00,\ C_2 = Rs.2.00,$ and

$t = 0,1,2,---10$

Using this, we obtain graphs shown in Fig. 4 and 5, respectively.

RESULTS AND DISCUSSION

In this paper, we have evaluated availability and cost function (Pandey and Mendus, 1995) for the considered system by employing Supplementary variables technique (Sharma *et al*., 2005). Also, we have computed asymptotic behavior and a particular case to improve practical utility of the system. Gupta and Gupta, (1986) have done the cost analysis of redundant complex system but no care was given to switching over device. On failure of switching device, the whole system can also fail. So, for the better analysis we must have to consider the concept of imperfect switching. Thus we have done some better analysis of considered system of practical utility.

Figure 4 shows the values of availability function at different time points. Analysis of Fig. 4 reveals that availability of considered system decreases approximately in constant manner with the increase in time.

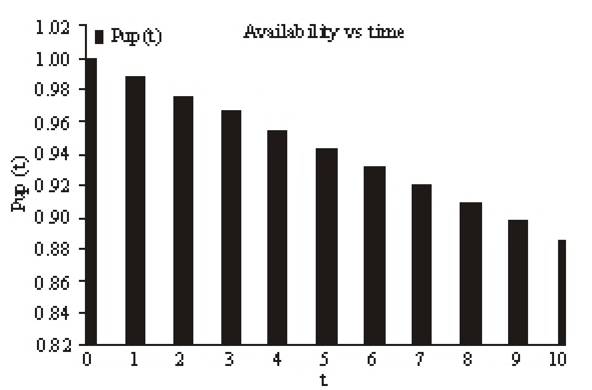

Fig. 4: The way availability of considered system decreases with the increase in time

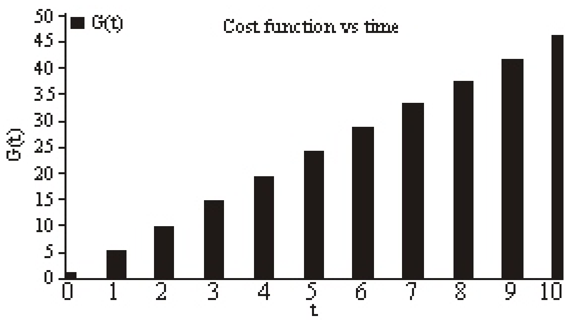

Fig. 5: The way cost function of considered system increases with the increase in time

Figure 5 shows the values of cost function at various time points. Critical examination of Fig. 5 yields that cost function for system increases constantly with time and after a long period it become stationary.

CONCLUSION

In conclusion, we observe that we could improve system's overall performance by using standby redundancy. Also, for accuracy of reliability parameters (Sharma, and Jyoti, 2005) we must have to consider the concept of imperfect switching device to changeover the standby unit. By using the concept of redundancy, we obtain the better values of availability and cost function for considered system.

NOTATIONS

f_i : Failure rate of subsystem i, where i=A,C,D and E

$(1-\alpha)$: Failure rate of switching device B.

$r_i(j)\Delta$: First order probability that i^{th} failure can be repaired within time interval $(j, j+\Delta)$, conditioned that it was not repaired up to the time j. where i = A,C,D,E and j=m,x,y,z respectively.

$P_0(t)$: Pr {System is all operable at time t}.

$P_A(t)$: Pr {System is operable through A_2 unit while unit A_1 has already failed}.

$P_B(t)$: Pr {System is failed due to failure of switching device B}.

$P_i(j,t)\Delta$: Pr {System is failed due to failure of subsystem i}. Elapsed repair time lies in the interval $(j, j+\Delta)$.

$P_{Ai}(j,t)\Delta$: Pr {System is failed due to failure of subsystem i while unit A_1 has already failed}. Elapsed repair time for subsystem i lies in the interval $(j, j+\Delta)$.

μ : Repair rate of switching device.

$\overline{P}(s)$: Laplace transform of function $P(t)$.

$S_i(j)$: $r_i(j)\exp\left(-\int r_i(j)dj\right), \forall i \text{ and } j$

$D_i(s)$: $\left[1-\overline{S}_i(s)\right]/s, \forall i$

M_i : $-\overline{S}_i(0)$ = Mean time to repair i^{th} failure.

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