Research Journal of Mathematics and Statistics 2(3): 97-100, 2010
ISSN: 2040-7505

Submitted Date: April 09, 2010 Accepted Date: July 30, 2010 Published Date: September 30, 2010

A Remark on the Solution Set of Interval Nonlinear Systems of Equations via Markov's Method

S.E. Uwamusi
Department of Mathematics, Faculty of Physical Sciences,
University of Benin, Benin City, Nigeria

Abstract: It shown that the operations on the solution set to the parametric interval non linear system of equations obtained via Markov's method may not be effective as a standard formula for the solution to algebraic systems of nonlinear equations even when the system is well conditioned. This was more so since it may produce sudden negative radius in the solution set and demonstrated by comparing note with the results from Fast Krawczyk's type method. It was found that Krawczyk's method gave quite impressive results where that of Markov's method failed.

Key words: Parametric non linear systems, Rump's mid point - radius coordinates, Newton's method, krawczyk's method, validated bounds

INTRODUCTION

Interval arithmetic or self validating methods for non linear systems demand taking into account of all system with the inclusion of round off errors.

Rump (1999, 1990, 1992) gave desirable qualities of good interval enclosures to include the following:

- To deliver rigorous results, in a computing time not too distance from a pure numerical algorithm,
- To also give proof of existence (and possibly uniqueness) of a solution.

This means that Self-validating method should be able to give account of errors of all computed results.

As pointed out by Markov (2001) it was stated that the use of interval arithmetic to a verifying solution is a delicate one. The reason being that interval arithmetic does not allow a representation by coordinates, as the interval operations, respectively interval algebraic problem cannot be generally reduced into real problems to be solved separately for each coordinates. He transformed the midpoint-radius representation for interval linear system due to (Rump, 1999) to two real systems of equal size via systems of coordinates. However (Markov, 2001) did not show if his method was deficient in producing negative radius during transformation stages. This was the major plank to which our findings are based.

We consider a nonlinear interval system of:

$$F(x)=0 \qquad (1)$$

to an equivalent interval linear system:

$$A(\hat{x}-x_k) \ni -F(x_k) \qquad (k=0, 1, \ldots) \qquad (2)$$

By parameterization of the nonlinearity, where $F:D\subset IR^n$ is a Lipschitz continuous function and $x=[\underline{x}_1,\overline{x}_1]\times[\underline{x}_2,\overline{x}_2]\times\ldots\times[\underline{x}_n,\overline{x}_n]$ being a box in R^n.

The matrix A is the Lipshitz matrix for F over the box x which is regular and positive definite.

The regularity of A can be proved using the Peron-Frobenius theorem, for a good overview (Rohn, 1993, 2006) and (Neumaier, 1990) may be good references.

Notations:

The following notations used in the study:

R^n: A set of real numbers

A real point interval $a=[a_1,a_2]=\{a\in R:a_1\leq a_2]$ is a segment of the real line

IR: The set of real point intervals. This may be represented by its end points or by the mid point and radius.

Interval method has an intrinsic property of lack of memory less property. This means that given an interval $a=[a_1,a_2]$ with order preserving < defined on it $a-a\neq 0$ unless $a=[0,0]$

The non linear system (1) can be parameterized by adding some levels of tolerance (Neumaier, 1990) so that the interval linear system (2) becomes nonlinear in the form

$$A(\varepsilon)x=-b(\varepsilon) \qquad (3)$$

Where; $A(\varepsilon) \in R^{nxn}$ and $b(\varepsilon) \in R^n$ depend affine linearly on a parameter vector $\varepsilon \in R^k$. Thus allowing ε varying within a certain range $\varepsilon \in R^k$ will compel the set of solution to all $A(\varepsilon)x = b(\varepsilon), \varepsilon \in |\varepsilon|$ to be satisfied, called the parametric solution set and is given by

$$\sum^{\varepsilon} = \sum(A(\varepsilon), b(\varepsilon), [\varepsilon]) = \{x \in R^n | A(\varepsilon).x = b(\varepsilon) \quad (4)$$

for some $\varepsilon \in [\varepsilon]\}$ This forms the major plank to which our practical applications will be discussed cf e.g., (Popova, 2002) and (Alefeld *et al*., 1998).

Following; Rohn, (2005, 2006) and Kreinovich *et al.* (1998), computing interval linear systems enclosures of the solution set (2) is an NP-hard problem.

Our main motive of study to experiment with the method of Markov (2001) which transforms a linear interval system to two systems of equal size via systems of coordinates using already known method due to Rump (1999) for the solution to the nonlinear equation with coefficients subject to uncertainty. In this study we use the basic numerical methods such as the interval LU decomposition algorithm to invert an interval matrix.

This study describes the centered outward interval multiplication (or simply co-multiplication) due to Rump (1999) and obtain the method of Markov (2001) which reduces interval linear system of equation to real two linear systems of equal size for the unknown intervals. We compare our results obtained with those from known standard Fast Krawckzyk' method proposed in Uwamusi (2009).

MATERIALS AND METHODS

The centre stage of our investigation is the method proposed by Markov (2001). The investigation was first started in 2008 but was later abandoned by the author in the Department of Mathematics, University of Benin, Benin City, Nigeria.

We consider $[a] = [a', a'']$ and $[b] = [b', b'']$ to be two real intervals where a', b' are the centers of [a] and [b] respectively and a'', b'' are also respective radius of [a] and [b] in that order.

Associate the following operation in *I(R)* in our subsequent discussions in the form:

$$\begin{aligned} a.b &= (a'', a'').(b', b'') \\ &= (a'b', |a'|b'' + |b'|a'' + a''b'') \end{aligned} \quad (5)$$

Method (3.1) is often referred to as co-multiplication of centered outward multiplication, (Rump, 1990). The set theoretic multiplication of two intervals can now be written in the form:

$$\begin{aligned} &(a', a'') \circ (b', b'') = \\ &(a'b' + a''b'', (|b'|a'' + |a'|b'')) \; if \; a'b' \geq 0 \\ &(a'b' - a''b'', |b'|a'' + |a'|b'') \; if \; a'b' < 0 \end{aligned}$$

Using absolute value of a $\in IR$ we define $|a| = |a'| + a''$. One obtains a transformed version of interval co-multiplication in the form:

$$\begin{aligned} a.b &= (a'b', |a|b'' + |b'|a'') \\ &= (a'b', |b'|a'' + |a'|b'') \end{aligned} \quad (6)$$

The co-multiplication for intervals in (5) has bad reputation in that it leads to overestimation of the desired interval output (Markov, 2001).

The set theoretic interval multiplication is defined to be:

$$a \circ b = (a'b' + a''b'', |a'|b'' + |b'|a'') \quad (7)$$

(Markov, 2001) demonstrated that a linear system involving intervals and interval multiplication can be reduced to two real linear systems for the midpoint-radius coordinates of the intervals. This he did by rewriting earlier results due to (Rump, 1999) as follows:

Defining $A = (A', A''), \; b = (b', b'')$ and $X = (x', x'')$ the matrix vector multiplication becomes:

$$\begin{aligned} Ax &= (A'x', |A'|x'' + A''|x'| + A''x'' \\ &= (A'x', |A|x'' + A''|x'|) \end{aligned} \quad (8)$$

where, $|A| = |A'| + A''$

The interval linear system (2) has been rewritten as two real linear system of equal size by Markov (2001) in the form:

$$Ax' = b' \quad (9)$$

$$|A|x'' = b'' - A''|x'| \quad (10)$$

RESULTS AND DISCUSSION

Numerical example: Our aim is to show that Markov' s method may not after all be effective for solving a system of interval nonlinear equation as the following example demonstrates.

We consider the following numerical problem taking from Uwamusi (2009):

$$F(x) =$$
$$6x_1 - 2\cos(x_2 x_3) - 1 = 0$$
$$9x_2 + \sqrt{x_1^2 + \sin x_3 + 1.06} + 0.9 = 0$$
$$60x_3 + 3e^{-x_1 x_2} + 10\pi - 3 = 0$$

$$X^{(0)} = \begin{pmatrix} 0.1 \\ -0.1 \\ 0.1 \end{pmatrix}. \quad (11)$$

Here we take the value of ε to be equal to 10^{-2} as inflation.

We used modified Krawczyk's method obtained by Uwamusi (2009) in a manner given below as a measure of comparison of results with those of Markov (2001) and Uwamusi (2009) method is given below:

$$K[K]^{\left(k+\frac{v+1}{q}\right)} = y^{(k)} - H^{(k)} f\left(y^{\left(k+\frac{v+1}{q}\right)}\right) + \left(I - H^{(k)} F'([X]^{(k)})\right)\left(\hat{X}^{\left(k+\frac{v+1}{q}\right)} - y^{\left(k+\frac{v+1}{q}\right)}\right) \quad (12)$$

where;

$$\hat{X}^{\left(k+\frac{v+1}{q}\right)} = y^{\left(k+\frac{v}{q}\right)} - H^{(k)} f(y^{\left(k+\frac{v}{q}\right)} + (I - H^{(k)} F'(X)^{(k)}))(X^{\left(k+\frac{v}{q}\right)} - y^{\left(k+\frac{v}{q}\right)}) \quad (13)$$

k = 0, I, …, v = 0,1,…, t-1, q = 2,3, …t. t, is a positive integer.

However because of numerical calculations and obvious advantages we obtained from the method if we restrict q to be 2, since values of q higher than 2 are meaningless in terms of speed and accuracy we delighted to report our findings in terms of numerical results as follows. We have displayed all results as shown in Table 1 and 2, respectively.

CONCLUSION

In this study implemented Markov's method on a nonlinear system of equation where in we introduced some nonlinearity and thus giving rise to interval nonlinear system of equations. The problem was further reduced to solving systems of two real equations for the coordinates of unknowns of equal size. It was observed that mononicity property for parameterized functional could not prevail on Markov's method to guarantee inclusion of the sought zeros for the given problem as unexpected negative radius was seen in the fourth iterative step in our solution as iteration progressed to the sought zeros for the given system of equations. This prompted us to use the Fast Krawczyk's method proposed in Uwamusi (2009) for the same type of problem. It was observed that the results produced by fast Krawczyk's method gave quite impressive results than those of Markov's method. This is also not quite surprising, as Krawczyk's method is known as providing existence and uniqueness theorem for interval nonlinear systems of equations, (Neumaier, 1990) for an over view. It beholds on us that Markov's method applied on parameterized nonlinear equations may need to be given further investigations. As a concluding remark, the method obtained by Rump (1999) is still preferable to that of Markov's method because not only for its ease of use but also has been used in solving similar problems in the past with huge success Uwamusi (2007, 2008).

Table 1: Showing Results from using Markov's systems of coordinates

Iterations K	Results Midpoint (x_k),Rad (x_k)
1	[0.500003456,0.009998721]
	[-0.147700069,0.003486560]
	-0.525341795,0.009776923]
2	[0.49828044,0.009803227]
	[-0.202745301,0.001962700]
	[-0.538584363,0.009610373]
3	[0.505684728,0.009610847]
	-0.198426473,0.000758936]
	[-0.528939670,0.009427909]
4	[0.498143899,0.009365659]
	-0.199590833,-0.0005164821]
	[-0.528826556,0.0091988]

Table 2: Showing results from Uwamusi (2009) method as given in Eq. (12)

Iteration	Results
1	[0.498901717,0.498901710]
	[-0.199872459,-0.199873833]
	[-0.530073299,-0.530083371]
2	[0.498144782,0.498144782]
	[-0.199605179,-0.199605179]
	[-0.528826126,-0.528826126]

REFERENCES

Alefeld, G., V. Kreinovich and G. Mayer, 1998. The shape of the solution set for system of interval linear equations with dependent coefficients. Math. Nachr., 192: 23-36.

Kreinovich, V., A. Lakeyev, J. Rohn and P. Kahl, 1998. Computational complexity and feasibility of data processing and interval computations. Kluwer, Dordrecht.

Markov, S., 2001. Computation of Algebraic Solution to Interval System via System of Coordinate. In: Krämer, W. and J.W. von Gudenberg (Eds.), Scientific Computing, Validated Numeric, Interval Methods. Kluwer Academic Plenum Publishers, New York, pp: 105-114.

Neumaier, A., 1990. Interval Methods for Systems of Equations. Cambridge University Press, New York.

Popova, E.D., 2002. Quality of the solution sets of parametric-dependent interval systems. ZAMM, 82(10): 723-727.

Rump, E.D., 1992. On the solution of interval linear systems. Computing, 47: 337-353.

Rump, S.M., 1990. Rigorous sensitivity analysis for linear and non-linear equations. Math. Comp., 54: 721-736.

Rump, S.M., 1999. Fast and parallel interval arithmetic. BIT, 39(3): 534-554.

Rohn, J., 2006. Regularity of interval matrices and theorem of the Alternatives. Reliable Comp., 12: 99-105.

Rohn, J., 2005. Linear equations mid point preconditioning may produce a 100% overestimation for arbitrarily narrow data even in case n = 4. Reliable Comp., 11: 129-135.

Rohn, J., 1993. Inverse interval Matrix. SIAM J. Numer. Anal., 30(3): 864-870.

Uwamusi, S.E., 2007. Towards the acceleration of Rump's fast and parallel circular interval arithmetic for enclosing solution of nonlinear systems of equations. Sci. Res. Essay, 2(11): 476-481.

Uwamusi, S.E., 2008. On the interval hull of solution sets of parametrised nonlinear equations. Sci. Res. Essay, 3(9): 183-189.

Uwamusi, S.E., 2009. Modified-accelerated Krawczyk's algorithm. Int. J. Phys. Sci., 4(2): 47-52.