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Reduced Population Viscosity in Spatially Disordered, Triple Strategy Prisoner's Dilemma Games

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Abstract: Altruism in selfish groups of individuals has been explained using game theory. In this work, cooperation within a spatial evolutionary prisoner's dilemma game is studied with three strategies: cooperation, defection or Tit-for-Tat. By imposing the condition of a site diluted lattice and relaxing the condition of strong population viscosity, the emergence of cooperating and defecting island universes is observed. Under a softer condition of movement such that players may move to a lattice site of at least equal payoff we find that these defecting islands become inherently unstable and dissociate to invade the rest of the system. This subsequently leads to a re-emergence of cooperation in the entire system as the Tit-for-Tat strategies knockout any rampant unconditional defection strategies. These results are interpreted suggestively in the context of biology and sociology.

Key words: Emergent behaviour, game theory, population dynamics

INTRODUCTION

Game theory suggests that cooperation and altruism can naturally emerge in a population of otherwise selfish individuals. One game in which this can be seen is the iterated, random length prisoner's dilemma game (PD herein). The PD has gained widespread attention in many disciplines including mathematics, physics, economics, evolutionary biology and sociology (e.g., von Neumann and Morgenstern 1944; Axelrod and Hamilton, 1981; Maynard Smith, 1982; Luce and Raiffa, 1985; Weibull 1995; Hofbauer and Sigmund, 1998; Miekisz, 2004; Le and Boyd, 2007; Alves Pereira *et al*., 2008). Indeed, Donninger (1986), reports that over one thousand articles have been published about the PD in the interval of two decades.

The PD in its basic form is a simple matrix game (Table 1) where the symmetric payoff of two participants is determined from their simultaneous decisions to cooperate with or to defect against each other (Axelrod and Hamilton, 1981). A defector against a cooperator achieves the largest individual payoff, the latter's payoff being the smallest possible. Should both players decide to cooperate, they receive the largest payoff divided between them. With two defectors, however, the payoff is lower than the mutual cooperation payoff, but greater than the lowest reward (Table 1). The dilemma faced by the players is then that no matter what the opposing player does, they are individually always better off adopting a defector strategy than a cooperator strategy. Such a rational approach by both players, however, gives the result of both being worse off than if they had both adopted a cooperator strategy.There have been many extensions of the original matrix game described above. Iterated, evolutionary multi-player PD games (Trivers, 1971; Maynard Smith, 1978; Nowak and May, 1992; Nowak and May, 1993; Sigmund, 1993; Nowak *et al*. 1994a; Nowak *et al*. 1994b; Hauert, 2001; Szabó *et al*., 2005; Alonso-Sanz, 2009) have been implemented with interesting results. A strategy known as 'Tit for Tat' (TFT) emerged from Axelrod and Hamilton (1981) as possessing the highest payoff overall for an individual player (Szabó *et al*., 2000; Baek and Kim, 2008; Szolnoki *et al*., 2009). TFT works by cooperating in the first round and then imitating the opponent's previous decision in all subsequent rounds. Axelrod commented that the success of TFT was down to several factors: it never defects first; it retaliates against any defection; it retains the capacity to cooperate (i.e., is forgiving) and is highly predictable. By imposing an evolutionary aspect, successful (i.e., high scoring) populations can also grow. This is usually implemented by causing a low scoring participant to adopt the tactic of a higher scoring one. Against Cooperate Unconditionally (Cu) and Defect unconditionally (Du) strategies, TFT produces an important effect. As the Du population increases (and conversely, the Cu decreases), pay-off to the Du population falls. This provides an excellent opportunity for the TFT population to invade the Du population (Sigmund, 1993). The evolutionary model of the PD does not, however, account for the spatial distribution of participants (i.e., individuals acting with those in geographical proximity). A Spatial PD (SPD) adds this feature to the game by placing participants on a regular lattice in an m-dimensional space (Nowak and May, 1993; Nowak *et al*., 1994a, b; Hauert, 2001). Each participant

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then plays against its 2 m neighbors simultaneously. The evolutionary condition is such that each player adopts the strategy of its neighbor that possesses the highest (average) payoff. Given the importance of the TFT population and the characteristics of the evolving SPD, Szabó *et al*. (2000) studied the emergence of cooperation in a three strategy system (Cu, Du, and TFT) with the external constraint that the players adopt the Cu strategy with a probability, p. Forcing some of the population to adopt a Cu strategy does not always have the desired effect of globally promoting Cu, however. At certain values, Du regains a stronghold. Further, Vainstein and Arenzon (2001) made a novel study of the effect of including empty lattice sites in the initial distribution of a pure Du and Cu network. They find that the system is able to sustain and sometimes augment the emergence of cooperation, even within this disordered environment, as suggested by Nowak *et al*. (1994a, b). Vainstein *et al*. (2007) demonstrated that relaxing population viscosity (i.e. allowing the players to move lattice sites) could promote cooperation.

The main objective of this present study is to determine how the addition of the TFT strategy affects a spatially disordered lattice SPD game under a variety of different conditions. Following Szabó *et al*. (2000), Vainstein and Arenzon (2001), and Vainstein *et al*. (2007), the effect of allowing players interstitial translation in a lattice SPD game is investigated within the framework of three strategies. This effect can be interpreted as (isolated) communities whose (potentially dissatisfied) members on the periphery seek to interact, and thus gain a higher payoff, with other communities by moving away from their original positions (Cheng *et al*., 2011; Lin *et al*., 2011). We address what happens in such circumstances and draw parallels to biological and sociological application.

MATERIALS AND METHODS

The basis model for this work follows Szabó *et al*. (2000). We use a 2-dimensional lattice of side L = 256 employing periodic boundary conditions. We then define P(empty) as the probability of any given lattice site being in an empty state (i.e., no players present). Hence the probability of a lattice site containing Cu or Du strategies has an equal assignment probability of $\{1-P(\text{empty})\}/2$. Every site interacts with its 4 neighbors every game turn, with the implicit stipulation that no site plays against itself. An empty site cannot interact with its neighbors, and therefore no payoff can be received. Further, in calculating the average payoff for each site, these empty b, thus defines the temptation to defect, where $1<b<2$. Individual payoffs for each pair of players, P1 and P2, are shown in Table 1. Employing this matrix is legitimate as the specified payoffs are considered to be the average payoffs (Szabó *et al*., 2000).

Table 1: The payoff matrix for two players (P1 and P2) used in this work

P1\P2	Du	Cu	TFT
Du	0\0	b\0	0
Cu	0\b	1\1	1\1
TFT	0\0	1\1	1\1

At each Monte Carlo time step, each lattice site is inspected to find out what its average payoff is, as calculated from its interaction with their neighbors. This average payoff is then compared with all of its neighbors. If any neighbor's payoff is larger than itself then the site will update itself by imitating the neighboring site that has the largest payoff. In the case where two (or more) sites are equally better, then one is chosen randomly.

Ignoring dimensionality (which we fix at m = 2), this basis model contains two free parameters: b and P (empty). Any attempt to find an exact analytical solution for this model is fraught with difficulty. Accordingly, many authors advocate usage of mean-field approximations (Hofbauer and Sigmund, 1998; Szabó *et al*., 2000). By neglecting any spatial correlation, the dynamics of the system can be characterized by a time dependent concentration such that $C_\alpha = N_\alpha(t)L^{-2}$, where α is one of the strategies (e.g., Cu). The number of strategies at a certain time t is given by $N_\alpha(t)$. Since the presence of empty lattice sites has the effect of reducing C_α, we normalize it so that summation of all strategy concentrations equals unity. The basis model is tested with no movement permitted (i.e., infinite viscosity) using only Cu and Du strategies and varying b and P(empty). The result of increased cooperation in games with small P(empty) values and $4/3<b<3/2$ (Vainstein and Arenzon, 2001) is readily recovered and this is displayed in Fig. 1.

There are two additions to this basis model that we need to add in order to address our aims. Firstly, we need to add the TFT strategy. This is achieved in a straightforward manner according to the payoff matrix (Table 1) and seeding each initial site in our lattice with probability=1/3 of having one of the three strategies (Cu, Du, and TFT). The second addition is the lowering of the viscosity of the population - allowing movement of the participants. This is achieved in two ways. The first way - the hard condition - is that a player in the lattice is permitted to move to an adjacent lattice site only if the payoff of that site would have been larger. The second way - the soft condition - is that a player is permitted to move to an adjacent site if the payoff for that site would have been of at least equal value.

RESULTS AND DISCUSSION

We firstly turn to the addition of the TFT strategy. If there were no empty lattice sites, one would expect that the stationary state of the system would be a trivial pure

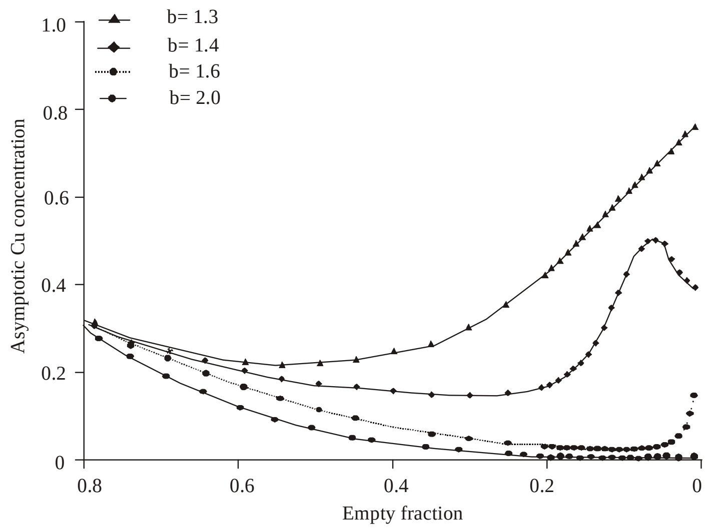

Fig. 1: Asymptotic concentration (i.e., as $t \to \infty$) of Cu strategies, as a function of the fraction of empty lattice sites, for various values of b. The points are median averages over 100 simulations where L = 100

Du state, or a Cu plus TFT state (Szabó *et al*. 2000). With any finite number of initial TFT strategies, however, the stationary state will always be a trivial cooperative one. By adding empty lattice sites, it is possible to isolate parts of the geometrical arrangement from other parts, effectively creating "island universes" that are causally disconnected from other parts of the lattice (Fig. 2).

Figure 3 displays the results of the three-strategy SPD game with no movement permitted. As the initial empty fraction approaches 100 per cent, the stationary state concentrations become $C_{Cu} = C_{Du} = C_{TFT} = 1/3$, as would be expected (cf. Vainstein *et al*., 2007). This can be explained simply by considering the limit of the empty fraction approaching unity; each filled lattice site is effectively an island universe to begin with and doesn't change.

A perhaps unexpected feature of Fig. 3 is the enhanced concentration of the Du strategy at an empty fraction of 0.8. This can be potentially explained as an initial state where there are island universes devoid of any TFT strategy. These Cu-Du islands will likely become dominated by the Du strategy. To investigate this hypothesis, we examine the size of such islands as a function of P(empty) in Fig. 4. For low empty fractions, there is typically just one sole large island that contains all of the available lattice sites. As the empty fraction increases, the mean size of the islands decreases whereby at P(empty)>0.3, the chance of having all available lattice sites connect up as one big island is fleetingly small; a $\gg 5\sigma$ event (Fig. 4). As the empty fraction approaches unity, the mean size of the islands becomes small (inset panel, Fig. 4). Moreover, an island size of $\approx$ two connected lattice sites occurs more frequently at P(empty) 0.8. For two connected lattice sites, there are nine combinations of strategy that could occur (Table 2). Out of these, the only one to evolve with time is the Cu+Du combination, which will change into a Du+Du one. Therefore, the average Du fraction in a collection of size two lattice site islands would increase from 1/3 to 4/9. Extending Table 2 to size three lattice site islands (where the geometrical arrangement can still be simplified to a linear arrangement), it is simple to deduce that the average Du fraction goes from 1/3 to 38/81. Taken in combination, these results are sufficient to explain the observed increase in Du strategy concentration at an empty fraction of ~ 0.8 where islands of such sizes dominate the lattice structure (Fig. 4).

Table 2: End state for all possible strategy combinations contained in isolated islands of size two.

Site 1 + Site 2	Du	Cu	TFT
Du	Du + Du	Du + Du	Du + TFT
Cu	Du + Du	Cu + Cu	Cu + TFT
TFT	TFT + Du	TFT + Cu	TFT + TFT

By adding in the ingredient of site movement, the island universes referred to above now become interconnected in a complex manner. Rather than each island evolving to a stationary state regardless of the state of other islands, each island now becomes part of a larger, global community.

Consider a cooperative island consisting of only Cu or TFT strategies. There are several ways in which such a cooperative community can arise. Trivially, there may be no Du strategies present within a given island when the strategy assignment takes place. Secondly, two processes may eliminate the Du population of an island: via TFT invasion or via Du migration. Szabó *et al*. (2000) discuss the TFT invasion principal in detail. Any Du on the edge of an island may not succumb to this process; rather, it

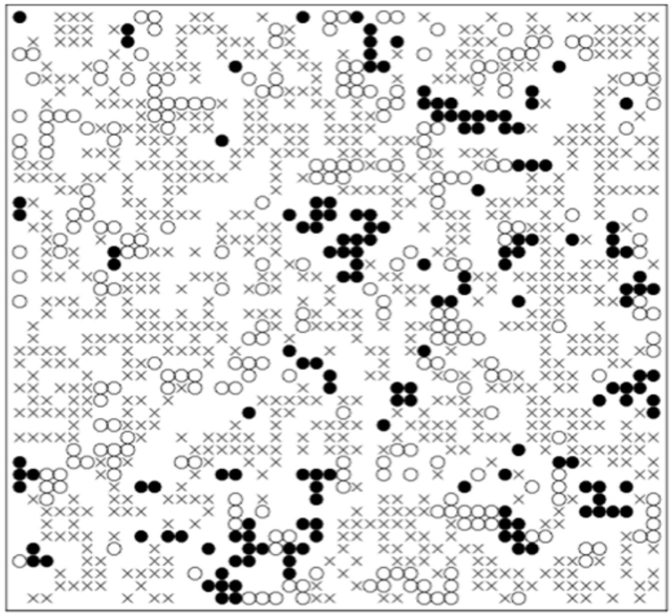

Fig. 2: Snapshot showing a 48×48 lattice subsection of an L = 256 system after 1000 Monte Carlo time steps, with P (empty) = 0.50 and b = 1.5 with zero population movement. The open circles denote Cu strategies; solid circles are Du; crosses are TFT and ‘blank’ spaces are simply the empty lattice sites. Several casually disconnected “island universes” are apparent in this configuration

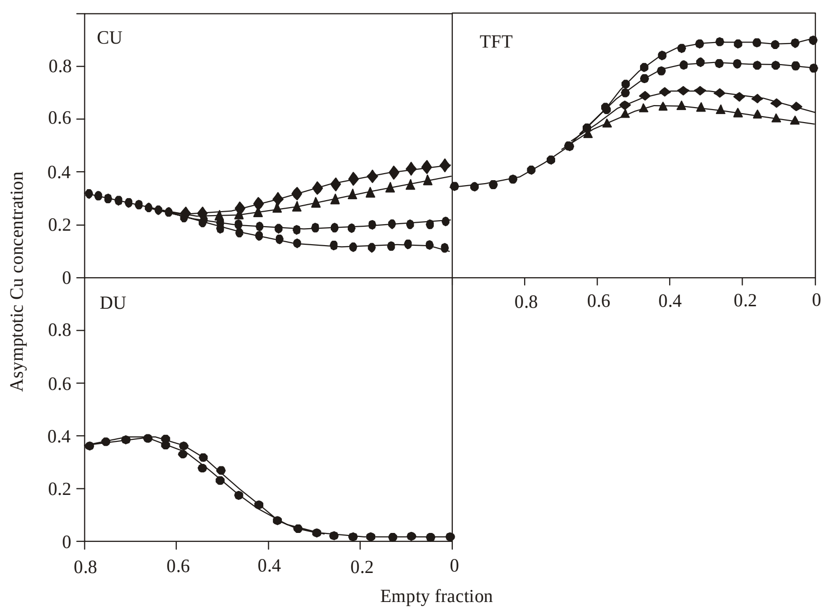

Fig. 3: Asymptotic concentration of strategies in the three-player SPD game as a function of the fraction of empty lattice sites for various values of b, with no movement. The point and line types are the same as for Fig. 1

can migrate away from the island if another island is sufficiently close (under the hard condition) or will always drift away (under the soft condition). The logic behind this is simple: a Du juxtaposed to any number of TFT strategies will obtain a zero payoff. Such a Du will have no reason to be stationary. It can obtain the same (zero) payoff by moving into an empty space regardless of the direction of the translation under the soft movement condition.

Any island that manages to rid itself of the Du strategy will become a stable island community of Cu and TFT strategies under the hard condition. These island are

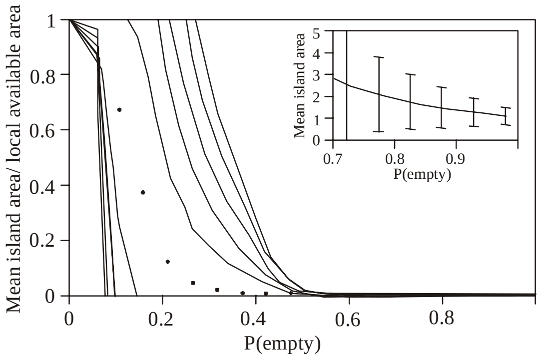

Fig. 4: Mean size of islands normalized to the total amount of available (i.e., non-empty) lattice sites as a function of P(empty). The innermost contour represents a 1σ error with each outward contour 1σ more. The inset panel displays the mean size of the islands in units of lattice sites for large P(empty) values with 1σ error bars

stable because there is no impetus for any member to relocate: A lone Cu or TFT on the periphery of such a community will always generate a better payoff by staying where it is than moving out into empty space. Finally, under the soft condition, a member of a cooperative society that finds itself on the edge of the island may move paralle l to the edge of the island. All payoffs are equal along the edge, so, for this member no position along an island's edge is preferred over another.Conversely, an island purely consisting of Du strategies is inherently unstable under the soft movement condition of this model. All of the Du strategies will score a zero payoff within such an island. Therefore all members of this island can obtain at least the same payoff by moving into empty space. Given sufficient time all members of such islands will move away, starting with those on the periphery and eventually reaching those at the centre of the island.

With the dissolution of Du islands a certainty given sufficient time, the final stationary state of the entire lattice under the soft movement condition will be a cooperative one consisting of Cu and TFT strategies only. The reason for this is the behavior of the TFT strategies: any large system with an initial, finite population of TFT strategies will asymptotically move towards a cooperative state.

The model developed in this work has the potential to be applied to many problems of an interdisciplinary nature. One can envision the model as being representative of a viral outbreak. If the Du population corresponds to the virus, then the virus moves about until it comes into contact with a Cu (say, a healthy cell with no immunity). It then infects the cell (the Cu becomes a replicant Du owing to the larger payoff). If the infected Cu is part of a larger system (a Cu island universe) then the entire island may become infected (turned into a Du population). This will continue to occur (via the Du island dissolution process outlined above) until the virus comes into contact with an immunized cell (a TFT). Depending upon the characteristics of the model, specifically the critical payoff value, the virus will either move along (Du migration) or be neutralized (turned into a TFT).

Sociologically, the dissolution of Du islands can potentially be equated to a societal collapse. Under the strain of a collapse, individualistic behavior dominates all members of the given island. Seeing their condition as no worse off if they move elsewhere provides the impetus for the dissolution of the island. The Du strategies move randomly until they find a more stable society to join (and presumably become a TFT member of this new community). If the new member finds itself on an edge of the new community that it has joined, it will move along that edge in search of a better position (larger payoff owing to more neighbors willing to cooperate).

CONCLUSION

This study has examined the effect of empty lattice sites in a three-strategy spatial evolutionary prisoner's dilemma game. Without the presence of any empty lattice sites and the TFT strategy, the model produces results that are similar to earlier studies (Szabó *et al*., 2000; Vainstein and Arenzon, 2001). With any number of TFT strategies, the stationary state emerges to be a cooperator-like one consisting of only Cu and TFT. Imposing empty lattice sites has the effect of isolating strategies from one another. With sufficient number of spaces, island universes can be formed. These island universes are either pure defector states or cooperator-like, depending upon whether TFT strategies were absent in the initial state (for the former) or had a finite number of neighboring TFT strategies (for the latter).

Introducing movement (i.e., lowering the viscosity of the players) re-connects the islands in a complex manner. With the hard condition that moving must generate a larger payoff, two types of stable Island evolve: cooperative and defecting, separated by channels of empty space. Under the soft condition that a move can be made if the prospective payoff at least equals the current payoff, Du islands will dissociate from themselves and invade the cooperative islands. This subsequently leads to a re-emergence of cooperation in the entire system as the TFT strategies knock out any rampant Du strategies.

There are still open questions for this model. Modification of the conditions for movement is one avenue of investigation, as currently the strategies stationed at each lattice site are somewhat myopic because they only examine those sites immediately adjacent to them. It would be interesting to examine what could happen if the strategies were not so myopic in their sight and movement range. Combining this with a different mixture of strategies (other than Cu, Du, plus

TFT) also merits urgent investigation, as does the imposition of external constraints (cf. Szabó *et al*., 2000) that could model the death of old strategies and the birth of new ones.

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