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Chemical Reaction and Slip Condition Effects on MHD Oscillatory Flow Through a Porous Medium

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Abstract: We investigate the combined effects of chemical reaction and slip condition on unsteady heat and mass transfer to MHD oscillatory flow through a channel filled with saturated porous medium. Exact solution of the governing equations for fully developed flow is obtained in closed form. The effects of various material parameters like Hartmann number, radiation parameter, Reynolds number, magnetic Reynolds number, Grashof number, modified Grashof number, porosity parameter, and frequency of the oscillation are discussed on flow variables and presented by graphs.

Key words: Chemical reaction, MHD, oscillatory flow, porous medium, slips condition

INTRODUCTION

Applications of combined heat and mass transfer flow with chemical reaction play important role in the design of chemical processing equipment, formation and dispersion of fog, distribution of temperature and moisture over agricultural fields and groves of fruit trees damage of crops due to freezing, food processing and cooling of towers. Investigation of periodic flow through a porous channel is important from practical point of view because fluid oscillations may be expected in many magnetohydrodynamics devices and natural phenomena, where fluid flow is generated due to oscillating pressure gradient or due to vibrating walls. Abdussattar and Alam (1995) investigated MHD free convective heat and mass transfer flow with hall current and constant heat flux through a porous medium. Alam *et al.* (2009) analyzed transient MHD free convective heat and mass transfer flow with thermophoresis past a radiate inclined permeated plate in the presence of variable chemical reaction and temperature dependent viscosity. Mansour and Aly (2009) presented the effects of chemical reactions and radiation on MHD free convective heat and mass transfer from a horizontal cylinder of elliptic cross section saturated porous media with considering suction or injection. Muhaimin and Hashim (2009) presented variable viscosity and thermophoresis effects on Darcy mixed convective heat and mass transfer past a porous wedge in the presence of chemical reaction. Seddeek and Almushigeh (2010) investigated effects of radiation and variable viscosity on MHD free convective flow and mass transfer over a stretching sheet with chemical reaction. Muthucumaraswamy (2010) examined chemical reaction effects on vertical oscillating plate with variable temperature. In all these investigations "no-slip" boundary condition is considered for the velocity field. However in some application e.g. in micro fluidic and nano fluidic devices where the surface to volume ratio is large, the slip behavior is more typical and slip boundary condition is usually used for the velocity field (Darhuber and Troian, 2005).

There exist many physical reasons for the slip over hydrophobic surfaces among which are: molecular slip (Blake, 1990) and small dipole, moment of polar liquids (Melin *et al.*, 2004). Soundalgekar (1976) studied magnetogasdynamics flow past an infinite porous plate in slip flow regime. The effects of fluid slippage at the wall for couette flow under steady state condition for gases are studied by Marques *et al.* (2000), whereas Khaled and Vafai (2004) investigated stokes and couette flows produced by an oscillatory motion of a wall under slip boundary conditions. Makinde and Osalusi (2006) presented MHD steady flow in a channel with permeable boundaries under slip boundary conditions. Smolentsev (2009) investigated three types of MHD flow problems assuming hydrodynamic slip condition at the interface between the electrically conducting fluid and insulating walls which gives: Hartmann flow, fully developed flow in a rectangular duct, and quasi two dimensional turbulent flow. Mehmood and Ali (2007) presented the effect of slip condition on unsteady MHD oscillatory flow of a viscous fluid in a planer channel.

The aim of the present study is to investigate the effects of chemical reaction and slip condition on unsteady heat and mass transfer flow in a channel filled with porous medium, and other parameters on the flow

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like Hartmann number, porosity parameter, radiation parameter, thermal Grashof number, modified Grashof number, Reynolds number, magnetic Reynolds number and frequency of the oscillation.

Mathematical formulation: Consider a two-dimensional free and force convection effects on unsteady incompressible laminar MHD radiative heat and mass transfer flow with temperature oscillating in a channel filled with saturated porous medium. It is assumed that the fluid has a small electrical conductivity, the electromagnetic force produced is very small and a uniform magnetic field. It is also assumed that the effect of viscous dissipation is negligible in the energy equation and there is a first order chemical reaction between the diffusing species and the fluid. The x-axis is taken along the plate in the vertical upward direction and the y-axis is taken normal to the plate. All the fluid properties are considered constant except the influence of the density variation in the buoyancy term, according to the classical Boussinesq approximation. Under these assumptions along with Boussinesq approximations for incompressible fluid model, the equations governing the motion are given as:

$$\frac{\partial u'}{\partial t'} = -\frac{1\partial p}{\rho \partial x'} + v\frac{\partial^2 u'}{\partial y'^2} - \frac{v}{k'}u' - \frac{\sigma_e B_0^2 u'}{p} g\beta^*\left(T - T_0\right) + g\beta^*\left(C - C_0\right) \tag{1}$$

$$\frac{\partial T'}{\partial t'} = \frac{k}{\rho C_p}\frac{\partial^2 T'}{\partial y'^2} - \frac{1}{\rho C_p}\frac{\partial q'}{\partial y'} \tag{2}$$

$$\frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial y'^2} - K_r^*\left(C' - C_0'\right) \tag{3}$$

$$\left.\begin{aligned} &u' - \gamma^*\frac{\partial u'}{\partial y'} = 0, \quad T' - T_\omega' \qquad on y' = 0 \\ &u' = 0, T' = T_0' + (T_w' - T_0')\cos\omega' t', on y' = 1 \end{aligned}\right\} \tag{4}$$

where u' is the axial velocity, t' is the time, T' the fluid temperature, T'_ω and T'_0 are walls temperatures P' the pressure, g the gravitational force, q′ the radiative heat flux, C' the fluid concentration, C'_ω and C'_0 are walls concentrations, D is mass diffusivity, K is the permeability of the porous medium ω' is the frequency of the oscillation, β is the coefficient of thermal expansion, β* is the coefficient of concentration expansion, B_o the electromagnetic induction, s_e the conductivity of the fluid, ρ the density of the fluid, v is the kinematics viscosity coefficient. It is assumed that the fluid is optically thin with a relatively low density and the radiative heat flux is given by:

$$\frac{\partial q'}{\partial y'} = 4\alpha^2\left(T_0' - T'\right) \tag{5}$$

where a is the mean radiation absorption coefficient. On introducing the following dimensionless quantities:

$$\left.\begin{aligned} &\mathrm{Re} = \frac{Ua}{v}, x = \frac{x}{a}, y = \frac{y'}{a}, u = \frac{u'}{U}, \theta = \frac{T - T_0'}{T_\omega - T_0}, \\ &H^2 = \frac{a^2\sigma_e B_0^2}{\rho v}, K_r = \frac{K_r^* a^2}{D}, t = \frac{t'U}{a}, \\ &p\frac{aP'}{\rho v U}, D_a = \frac{K}{a^2}, Gr = \frac{g\beta(T_\omega' - T_0')a^2}{vU}, \\ &Pe = \frac{Ua\rho C_p}{k}, N^2 = \frac{4\alpha^2 a^2}{k}, C = \frac{C' - C_0'}{C_\omega' - C_0'}, \\ &Gc = \frac{g\beta^*(C_\omega' - C_0')a^2}{vU}, Rm = -\frac{Ua}{D}, \gamma = \frac{\gamma^*}{a} \end{aligned}\right\} \tag{6}$$

In Eq. (1) to (4), leads to:

$$\mathrm{Re}\frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial y^2} - (s^2+H^2)u + Gr\theta + GcC \tag{7}$$

$$Pe\frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2} + N^2\theta \tag{8}$$

$$Rm\frac{\partial C}{\partial t} = \frac{\partial^2 C}{\partial y^2} - K_r C \tag{9}$$

With the boundary conditions:

$$\left.\begin{aligned} &u - \gamma\frac{du}{dy} = 0, \quad \theta = 0, \quad C = 0, \quad on\ y = 0 \\ &u = 0, \quad \theta = \cos\omega t, \quad C = \cos\omega t, \quad on\ \ y = 1 \end{aligned}\right\} \tag{10}$$

where, Gr is the thermal Grashof number, H is the Hartmann number, N is the radiation parameter, Pe is the Peclet number, Re is the Reynolds number, D_a is the Darcy number, Pr is the Prandtl number, γ is the slip parameter and s is the porous medium shape factor.

Method of solution: To solve Eq. (7) to (9), we assumed the pressure gradient, fluid velocity, fluid temperature, and fluid concentration in non-dimensional form, as:

$$-\frac{\partial p}{\partial x} = R\left(e^{i\omega t} + e^{-i\omega t}\right) \tag{11}$$

$$\begin{aligned} u(y,t) &= u_0(y)e^{i\omega t} + u_1(y)e^{-i\omega t} \\ \theta(y,t) &= \theta_0(y)e^{i\omega t} + \theta_1(y)e^{-i\omega t} \\ C(y,t) &= C_0(y)e^{i\omega t} + C_1(y)e^{-i\omega t} \end{aligned} \tag{12}$$

where R<0 for favorable pressure.

Equation (7) to (10) with the use of (11) and (12) reduce to

$$\frac{d^2u_0}{dy^2} - m_1^2 u_0 = -R - Gr\,\theta_0 - GcC_0 \tag{13}$$

$$\frac{d^2u_1}{dy^2} - m_2^2 u_1 = -R - Gr\,\theta_1 - GcC_1 \tag{14}$$

$$\frac{d^2\theta_0}{dy^2} + N_1^2\theta_0 = 0 \tag{15}$$

$$\frac{d^2\theta_1}{dy^2} + N_2^2\theta_1 = 0 \tag{16}$$

$$\frac{d^2C_0}{dy^2} - S_1^2 C_0 = 0 \tag{17}$$

$$\frac{d^2C_1}{dy^2} - S_2^2 C_1 = 0 \tag{18}$$

With boundary conditions:

$$\left.\begin{aligned} &u_0 - \gamma\frac{du_0}{dy} = 0,\ u_1 - \gamma\frac{du_1}{dy} = 0,\quad \theta_0 = 0,\quad \theta_0 = 0,\quad on\ y = 0 \\ &u_0 = u_1 = 0,\ \theta_0 = 1/2,\ \theta_1 = 1/2,\ C_0 = 1/2, C_1 = 1/2, on\ y = 1 \end{aligned}\right\} \tag{19}$$

where,

$$m_1 = \sqrt{s^2 + H^2 + \mathrm{Re}\,i\omega}, m_2 = \sqrt{S^2 + H^2 - \mathrm{Re}\,i\omega}$$

$$N_1 = \sqrt{N^2 - Pei\omega}, N_2 = \sqrt{N^2 - Pei\omega},$$

$$S_1 = \sqrt{K_r + Pei\omega}, S_2 = \sqrt{K_r - Pei\omega}$$

Equation (13) to (18) subject to boundary conditions (19) are solved and the solution for fluid temperature, fluid concentration and fluid velocity are presented in the following form:

$$\theta(y,t) = \frac{1}{2}\left[\frac{\sin N_1 y}{\sin N_1}e^{i\omega t} + \frac{\sin N_2 y}{\sin N_2}e^{-i\omega t}\right] \tag{20}$$

$$`C(y,t) = \frac{1}{2}\left[\frac{\sinh S_1 y}{\sinh S_1}e^{i\omega t} + \frac{\sinh S_2 y}{\sinh S_2}e^{-i\omega t}\right] \tag{21}$$

$$\begin{aligned} u(y,t) = {} & [A\cosh m_1 y + B\sinh m_1 y + \lambda_1 + Gr\lambda_2 - Gc\lambda_3]e^{i\omega t} \\ & [C\cosh m_2 y + D\sinh m_2 y + \xi_1 + Gr\xi_2 - Gc\xi_3]e^{-i\omega t} \end{aligned} \tag{22}$$

The rate of heat transfer across the channel's wall is given as:

$$\left.\frac{\partial\theta}{\partial y}\right|_{y=0} = \frac{1}{2}\left[\frac{N1}{\sin N1}e^{i\omega t} + \frac{N2}{\sin N2}e^{-i\omega t}\right] \tag{23}$$

The rate of mass transfer across the channel's wall is given as:

$$\left.\frac{\partial C}{\partial y}\right|_{y=0} = \frac{1}{2}\left[\frac{S_1}{\sin S_1}e^{i\omega t} + \frac{S_2}{\sin S_2}e^{-i\omega t}\right] \tag{24}$$

The shear stress at the lower wall of the channel is given:

$$\begin{aligned} \left.\frac{\partial u}{\partial y}\right|_{y=0} &= [Bml + \eta_1 - \eta_2]e^{i\omega t} \\ &\quad + [Dm2 + t_1 - t_2]e^{-i\omega t} \end{aligned} \tag{25}$$

where, A = a1−a2, B = b1−b2−b3−b4+b5+b6,

$$D5 = \frac{R}{m_2^2 d2}, D = D1 - D2 - D2 - D4 + D5 + D6,$$

$$C = c1 - c2, \lambda_1 = \frac{R}{m_1^2}, \lambda_2 = \frac{\sin N_1 y}{2\left(N_1^2 + m_1^2\right)\sin N_1},$$

$$\lambda_3 = \frac{\sin S_1 y}{2\left(S_1^2 + m_1^2\right)\sinh S_1}, \xi_1 = \frac{R}{m_2^2},$$

$$\xi_2 = \frac{SinN_2 y}{2\cosh m_1\left(N_1^2 + m_2^2\right)}, \xi_3 = \frac{SinS_2 y}{2\left(S_2^2 - m_2^2\right)\sinh S_1},$$

$$a1 = \frac{-B\sinh m_1}{\cosh m_1} - \frac{R}{m_1^2\cosh m_1'}$$

$$a_2 = \frac{Gr}{2\cosh m_1\left(N_1^2 + m_1^2\right)} - \frac{Gc}{2\cosh m_1\left(S_1^2 + m_1^2\right)}$$

$$b1 = \frac{Gc}{2\cosh m_1\left(S_1^2 - m_1^2\right)d1}, b2 = \frac{Gc}{2\cosh m_1\left(N_1^2 - m_1^2\right)},$$

$$b3 = \frac{\gamma GrN_1}{2\sinh N_1\left(N_1^2 - m_1^2\right)d1},$$

$$b4 = \frac{R}{m_1^2\cosh m_1 d1}, b5 = \frac{R}{m_1^2 d1},$$

$$b6 = \frac{\gamma GcS_2}{2\sinh S_2\left(S_2^2 - m_2^2\right)d2}, b6 = \frac{\gamma GcS_1}{2\sinh S_1\left(S_1^2 - m_1^2\right)d1},$$

$$d1 = \frac{\sin m_1}{\cosh m_1} + \gamma m_1, d2 = \frac{\sinh m_2}{\cosh m_2} + \gamma m_2,$$

$$c1 = \frac{-D\sinh m_2}{\cosh m_2} - \frac{R}{m_2^2\cosh m_2},$$

$$a2 = \frac{Gr}{2\cos m_2\left(N_2^2 + m_2^2\right)} - \frac{Gc}{2\cos m_2\left(S_2^2 - m_2^2\right)},$$

$$D1 = \frac{Gc}{2\cosh m_2\left(S_2^2 - m_2^2\right)d2},$$

$$D2 = \frac{Gr}{2\cosh m_2\left(N_2^2 - m_2^2\right)}, D3 = \frac{\gamma GrN_2}{2\sin N_2\left(N_2^2 - m_2^2\right)d2},$$

$$D4 = \frac{R}{m_2^2\cos m_2 d2}, D5 = \frac{R}{m_2^2 d2},$$

$$\eta_1 = \frac{GrN_1}{2\sin N_1\left(N_1^2 + m_1^2\right)}, \eta_2 = \frac{GcS_1}{2\sin S_1\left(S_1^2 - m_1^2\right)},$$

$$l_1 = \frac{GrN_2}{2\sin N_2\left(N_2^2 + m_2^2\right)}, l_2 = \frac{GrS_2}{2\sinh S_2\left(S_2^2 - m_2^2\right)}$$

RESULTS AND DISCUSSION

In order to illustrate the influence of various parameters on the velocity, temperature and concentration field, numerical calculations of the solutions, obtained in the preceding section, have been carried out for different values of the chemical reaction parameter, Reynolds number, radiation parameter, slip parameter, magnetic parameter, porosity parameter, Grashof number, frequency of the oscillation, Peclet number, modified Grashof number, and magnetic Reynolds number. We

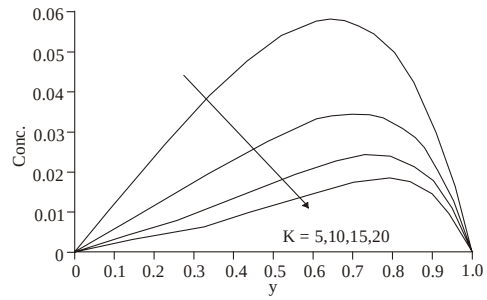

Fig. 1: Concentration profiles for different values of chemical reaction parameter

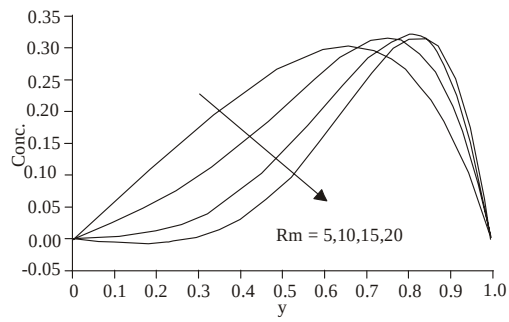

Fig. 2: Concentration profiles for different values of the magnetic Reynolds number

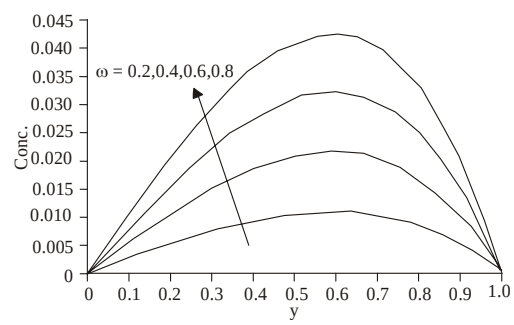

Fig. 3: Concentration profiles for different values of epsilon

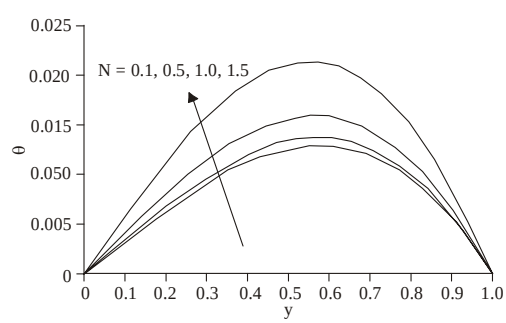

Fig. 4: Temperature profiles for different values of the Radiation

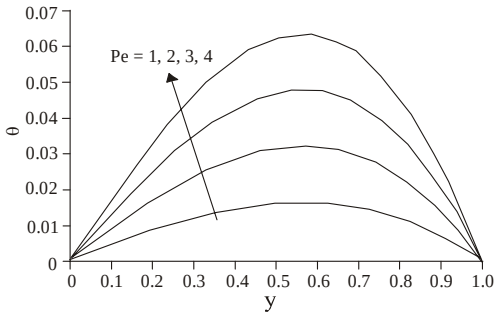

Fig. 5: Temperature profiles for different values of the Peclet number

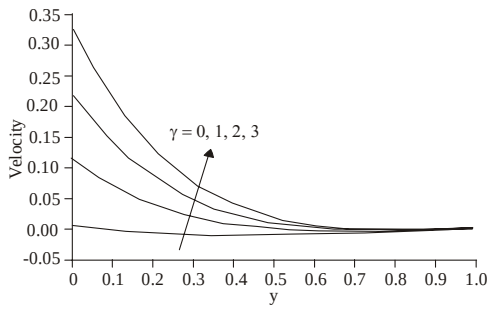

Fig. 6: Temperature profiles for different values of the frequency of the oscillation

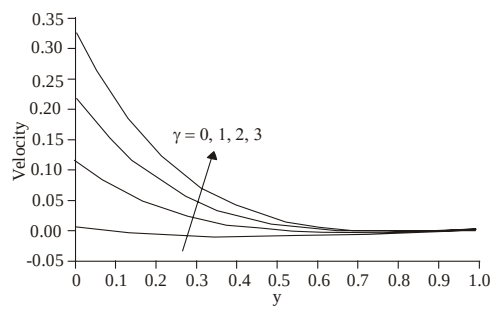

Fig. 7: Velocity profiles for different values of the slip parameter

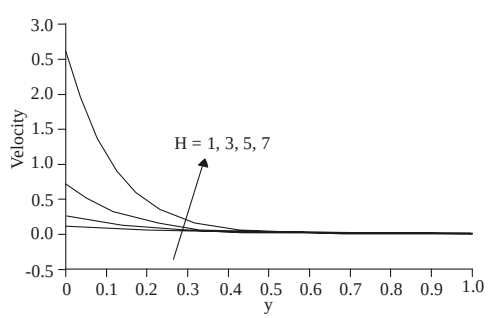

Fig. 8: Velocity profiles for different values of the Hartmann number

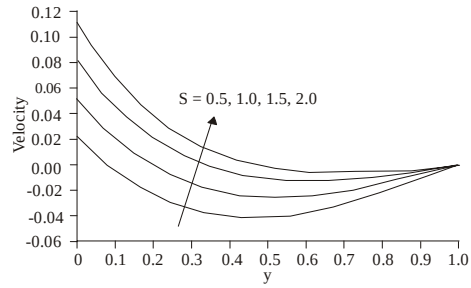

Fig. 9: Velocity profiles for different values of the porosity parameter

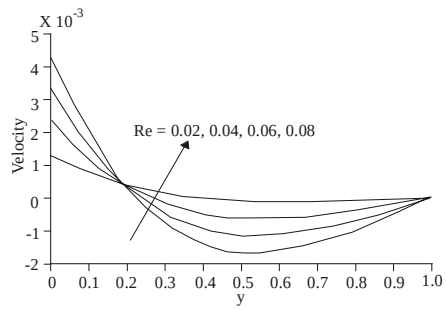

Fig. 10: Velocity profiles for different values of the Reynolds number

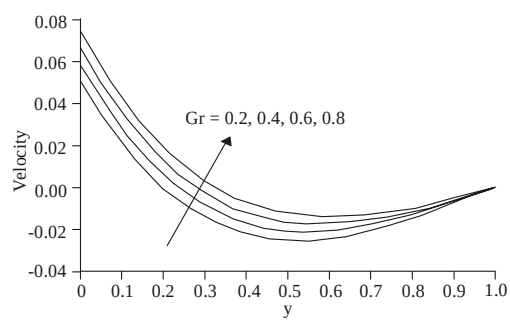

Fig. 11: Velocity profiles for different values of the Grashof number

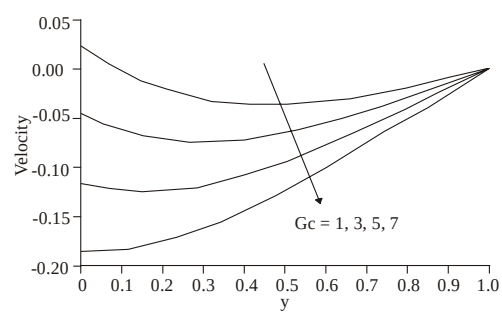

Fig. 12: Velocity profiles for different values of the modified Grashof number

made use of the following parameter values except otherwise indicated.

R e = 1, N = 1, Kr = 1, Pe = 1, Rm = 1, R = -1, S = 1, H = 1, Gr = 0.2, Gc = 0.2, $\omega t = \pi/2$, $\omega = 0.1$, $\gamma = 1$

The concentration profiles for different values of the chemical reaction parameter (K = 5, 10, 15, 20), magnetic Reynolds number (Rm = 5, 10, 15, 20) and frequency of the oscillation are shown in Figs. 1, 2 and 3, respectively. It is observed that the concentration decreases with increasing chemical reaction parameter or magnetic Reynolds number and increase with increasing frequency of the oscillation. The temperature profiles for different values of the radiation parameter (N = 0.1, 0.5, 1.0, 1.5), Peclet number (Pe = 1, 2, 3, 4) and frequency of the oscillation are presented in Figs. 4, 5 and 6 respectively. It is observed that the temperature profiles increases with increasing radiation parameter, Peclet number and frequency of the oscillation. The velocity profiles for different values of the slip parameter (46 = 0, 1, 2, 3), magnetic parameter (H = 1, 3, 5, 7), porosity parameter (S = 0.5, 1.0, 1.5, 2.0), Reynolds number (Re=0.02, 0.04, 0.06, 0.08), Grash of number (Gr = 0.2, 0.4, 0.6, 0.8) and modified Grashof number (Gc = 1, 3, 5, 7) are shown in Fig. 7, 8, 9, 10, 11 and 12, respectively. It is observed that the velocity increases with increasing slip parameter, magnetic parameter, porosity and Grashof number, and decreases with increasing Reynolds number or modified Grashof number.

CONCLUSION

This study investigated the effects of chemical reaction and slip condition on transient MHD heat and mass transfer flow through a channel filled with porous medium. The results reveal among other things that the velocity increases with increasing slip parameter, magnetic parameter or Grashof number, and decreases with increasing Reynolds number or modified Grashof number. The concentration profiles increases with increasing frequency of the oscillation, and decreases with increasing chemical reaction parameter and magnetic Reynolds number.

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